

MAE 222
Mechanics of Fluids
Princeton University

Assignment # 4

February 25, 1998

Due on Wednesday, 2PM, March 4, 1998

Chapter 5. Finite Control Volume Analysis :

§5.1, Conservation of Mass, the Continuity Equation . The *continuity equation* is a statement of the law of conservation of mass. The law is stated by eq.(5.1) in terms of a system, and it eventually is expressed by eq.(5.5) in terms of a control volume. What is a control volume? It is a volume, fixed in space, that YOU have arbitrarily chosen to study.

Let me emphasize the concept of flux (YMO does not emphasized it to the degree I would like to see). We have:

$$\alpha \text{ flux} = \text{amount of } \alpha \text{ per unit time} = \int \int_{CS} \mathbf{n} \cdot (\alpha \mathbf{V}) dA, \quad (1)$$

where α is something with the dimension of xxx per unit volume, \mathbf{n} is the unit normal of a surface element dA of the control surface CS. If xxx is “volume,” then $\alpha = 1$, If xxx is “mass,” then $\alpha = \text{density} = \text{mass per unit volume}$, if xxx is “energy,” then $\alpha = \text{energy per unit volume}$, if xxx is “ x -momentum,” then $\alpha = x\text{-momentum per unit volume}$, if xxx is “XYZ.” then $\alpha = \text{XYZ per unit volume}$. The physical dimension of “XYZ flux” is XYZ per unit time.

In the continuity equation, the “storage” term (the unsteady term) is zero (or negligible) if we are considering a steady flow problem. Look at example 5.3, where the selected control volume is shown in fig. E5.3 on page 144. Is this a steady flow problem? No, if you stare at the tank, you will see the free surface moving up! What about the storage term? Yes, sir! it participates here. Whatever

that came into the control volume and didn't go out is stored in the control volume.

Usually, control volumes are *fixed in space*. If for some reason you want to express some relation using a moving, non-deforming control volume, you can always first obtain the control-volume-fixed-in-space result, then convert it to the moving coordinate using a mathematical transformation. This is the mission of §5.1.3 and example 5.4. How much water flux is coming out the sprinkler? Obviously, it is the water flux computed using the relative velocity of the water with respect to the sprinkler nozzle!

§5.2-5.2.2, Newton's Second Law : The linear momentum Equations.

Using Reynolds Transport Theorem (which allows us to find the time rate of change of a scalar of a system), we arrived at eq.(5.17), which is expressed in terms of a control volume fixed in space. Eq.(5.17) is a vector equation. The first term on the left hand side is obviously a vector. The term on the right hand side is obviously a vector. The second term on the left hand side must obviously be a vector! Make sure you can write down the (x,y,z) component of this equation when Cartesian coordinates are used. And we are ready for applications! In the examples, the flow velocity at interested stations are usually assume "uniform." Thus all the integrations needed for the fluxes are done simply using arithmetics. You should learn to choose your control volume intelligently. Remember, the \mathbf{F} 's on the right hand side of eq.(5.17) represent the *external* forces acting on your control volume. Remember, *external*. All of the examples in this section are fun, but example 5.9 is more fun than others.

§5.2.3-5.2.4, Moment-of-Momentum Equations. Here comes the cross product again! But we know the cross product! Remember the right hand rule? The cross product $\mathbf{A} \times \mathbf{B}$ is a vector, point in the direction given by the right hand rule (stick your four fingers in the direction of \mathbf{A} , and bend them to point in the direction of \mathbf{B} . Your thumb now points in the direction you are looking for). The magnitude of this vector is $|A||B| \sin \theta$ where θ is the angle your four right hand fingers had to bend in the previous exercise.

Actually, the magnitude of this cross product is just the magnitude of A times the *component* of \mathbf{B} in the direction perpendicular to \mathbf{A} (you may switch the two vectors). So, if there is a force \mathbf{F} acting at point \mathbf{r} which is a position vector measured from the origin, then the magnitude of the torque (or moment) of this force about the origin is F times the moment arm, which in vector form is represented by $\mathbf{r} \times \mathbf{F}$.

Are you comfortable with eq.(5.18)? Where does it come from? (did good old Newton ever said that?). Is it a result derivable from Newton's second law? If you don't know, ask me in class. Once you accept the system statement as given by eq.(5.18), then Reynolds Transport Theorem gives us eq.(5.21), which is a vector equation. And we are off to do examples!

§5.3, First Law of Thermodynamics The Energy Equation.

We probably could not do justice to this section this week.

Problems at end of Chapter 5:

- Problem 5.1. Just to make sure you know how to compute the mass flux over the six surfaces.
- Problem 5.5. Draw a control volume which encloses the hydraulic jump. Assume steady flow, and assume the flow velocity to be uniform at the entrance and the exit stations.
- Problem 5.16. Be intelligent when you choose the control volume. You are looking for the anchoring force. So the control volume should be chosen so that the anchoring force is an *external* force.
- Problem 5.23. Obviously, this is a “two-dimensional” problem—the “problem” has unit width in the direction normal to the paper. Use a very large (rectangular) control volume so that you can justify the statement that the static pressure on the surface of this large control volume is the undisturbed value (14.7 psia). For this two-dimensional rectangular control volume, there are *four* surfaces to worry about. You need to worry about x -momentum fluxes on all FOUR surfaces. Yes, FOUR. Top and bottom too! Watch for my help on this problem in class.

- Problem 5.34. Straightforward moment of momentum problem. Follow example 5.10.

Check the pu.mae.222 newsgroup from time to time. Send me feedback via email if appropriate.