

MAE 222  
**Mechanics of Fluids**  
Princeton University

**Assignment # 5**

March 4, 1998

Due on Wednesday, 2PM, March 11, 1998

**First Mid-Term Next Friday**  
One Hour, Closed Book

**Chapter 5. Finite Control Volume Analysis** , continued.

On page 171, YMO provided neat formula to find the torque and shaft power for a rotating machine. There is a  $\pm$  sign in all the equations. YMO also provided a good suggestion on how to pick the sign in the paragraph between eq.(5.29) and eq.(5.30). It is a good suggestion.

**§5.3, First Law of Thermodynamics** —The Energy Equation.

- §5.3.1-5.3.2. First of all, we need to know what the First Law of Thermodynamics is in English. Note that as given on top of page 176, it concerns a *system*. It basically says: whatever net amount of energy, either heat or work, that goes into a *system* is stored in the *system* as “stored energy.” Hence, “energy is conserved.”

The most important assertion of the First Law of Thermodynamics is really not “energy is conserved,” but that the amount of stored energy of a material “in thermodynamic equilibrium” is solely a function of the “state variables” of the material. What is a state variable? Whatever that describes the state of the material, such as its temperature, its density, its pressure, etc. You have a kilogram of hot air, and I have a kilogram of cold air. Whose air has more stored energy? You, of course.

After fully appreciating what the First Law says, we then need to understand what “new” concepts it brings. Consider a

control volume consisting of a streamtube, with flow entering at station #1 and exiting at station #2. Assume there is no heat or work flux across the streamtube “sides.” OK. How many of you think the influx of internal energy plus kinetic energy plus potential energy equals to the outflux of internal energy plus kinetic energy plus potential energy? Well, all those who voted yes were wrong!

The big deal in energy equation of fluid mechanics is that **enthalpy** defined by eq.(5.45), instead of internal energy is the variable of significance in steady flow problems.

The point that must be understood is: a fluid glob (my system) which is moving deforms its boundary surface as it moves. Work is done by the external pressure acting on the boundary surface. We must include this in the work calculation!

- §5.3.3 The Bernoulli’s equation comes from Newton’s Second Law, and it is valid only under certain assumptions (you must know them by heart!). The energy equation comes from the First Law of Thermodynamics, and is valid under much less restrictive assumptions. They have the same dimension.

What this section tells you is: when something is going on in the fluid flow that is frictional or generally not ideal, heat is generated, and the “available energy” is decreased. Bernoulli’s constant is a measure of available energy for incompressible flows.

- §5.3.4 In most problems, we make the one-dimension simplification: when flows is crossing a certain area  $A$ , we assume the flow velocity perpendicular to the area to be a constant,  $\bar{V}$ . When density is consider a constant, the mass flow rate is just  $\dot{m} = \rho\bar{V}A$ . How is  $\bar{V}$  defined when  $V$  is not a constant? You just do the integral for the mass flux using  $V$ , and equate that to  $\rho\bar{V}A$ . Now what is the kinetic energy flux? It would be nice to represent it as  $\alpha\dot{m}\bar{V}^2/2$ . How do you find  $\alpha$ ? You just do the integral for the kinetic energy flux, and equate . . . .

The mid-term will include materials up to the end of Chapter 5. There will be sample mid-term exams posted on the course web page. Next wednesday, we have a session devoted to review and questions.

**Problems** at end of Chapter 5:

- Problem 5.31. Use Bernoulli's equation and x-momentum balance.
- Problem 5.33. Assume the jet nozzles are very small.
- Problem 5.38. Follow Example 5.11. Make sure you get the sign of the shaft power right (see page 171).
- Problem 5.49. If there is no frictional loss, the Bernoulli's constant at point B is the same as that at point A. But now there is frictional loss, and you are given the formula for the loss. So it seems a solvable problem.
- Problem 5.53. Aha! We are not using Bernoulli's equation here! (Its a gas, and it is "expanding," so it is certainly not incompressible!)
- Problem 5.69. Assume zero gravity and incompressible flow. Assume that the static pressure at station #1 also prevails at the exit plane of the narrow section, and is a constant across the whole wide section. Assume the static pressure and the velocity across station #2 are uniform. Now draw a control volume (watch for this in class), and make the mass flux balance and apply the x-momentum balance. Then compute the change in the Bernoulli's constant. Why is the Bernoulli's constant not a constant here?

This home work will not be graded in time for you to use it to study for the mid-term. So the next wednesday session we will go over any questions you may have.

Use email or the newsgroup to ask questions.