MAE 222
Mechanics of Fluids
Princeton University

Assignment # 6

March 25, 1998

Due on Wednesday, 2PM, April 1, 1998

We will skip Chapter 6 for now, but will get back to it later in the course. We will first deal with the study of dimensional analysis, a very powerful tool for engineers.

Chapter 7. Similitude, Dimensional Analysis, and Modeling. This is important stuff.

§7.1, Dimensional Analysis. What is *Similitude*? Its a fancy word representing a very simple idea. And it is very, very popular in fluid mechanics. It is a methodology for identifying different problems being the same problem. For example, we want to know what is the pressure distribution on the wing of a big, subsonic aircraft, and I have this tiny, small wind tunnel in the MAE222 lab. Can I use the data from my small tunnel (using a scaled model) to make design decisions on the big aircraft? The answer is yes provided the 'desired answers' of these two problems can be shown to be 'similar.' Another example: you have data on the destructive power of a small atomic bomb; what would be the destructive power with a bomb with 10,000 times more energy? A major tool in such endeavors is *dimensional analysis*. The whole idea of dimensional analysis is exquisitely simple: Lets use dimensionless variables! Let the desired answers (in dimensionless form) be expressed in terms of dimensionless parameters! Having agreed to this intuitively simple idea, the big question is: how many dimensionless parameters (including the desired answers) are there in the problem? This is the job of the next section.

§7.2. Buckingham Pi Theorem. Don't be intimidated by big words. Buckingham Pi Theorem simply tells us the **number** of dimensionless parameters in the problem. **Not** what the dimensionless parameters actually are, but **how many**. If the problem has k dimensional parameters which involve r reference dimensions, then the number of dimensionless parametes is $k - r$ (provided you follow the admonishment in the last paragraph of Step 1 on page 282 in determining k, and you can find r repeating variables as instructed in first paragraph of Step 4 on page 283). Otherwise, it is less. The "full-blown" Buckingham Pi theorem is not so fragile, and it is more "idiot proof." The price of the generality is its need to involve the concept of *rank* of a certain rectangular matrix. You will notice that YMO did not present a proof or a derivation of the "method of repeating variables." I will talk about the "full blown" version in class.

§7.3-7.4. Determination of Pi Terms. Here is a step by step recipe, with example 7.1. You will encounter *Reynolds number* on page 286. A most important point being made on page 290 is that the Pi terms are *not* unique! The number of Pi terms is fixed; but what they are is not unique!!! Good engineers pick good Pi terms.

§7.5. Determination of Pi Terms by Inspection . That is the way most intelligent, educated engineers do it.

§7.6. Common Dimensionless Groups in Fluid Mechanics. Since the Pi terms are not unique, this section tells you the collection of Pi terms most fluid mechanicists have agreed to use. These Pi terms carry the name of the first person who identified its usefulness. Since all Pi terms are dimensionless, each can be interpreted as the ratio of two "effects" with the same physical dimension (or unit).

§7.7 Correlation of Experimental Data. Now that we know how to find the Pi terms, here comes how to use them! To get the most "mileage" out of your data, *Plot them in terms of your Pi's*!!!!!

§7.8 Modeling and Similitude. Once you have the relationship between the Pi terms (obtained by experiments on a model or by theoretical analysis), you can used them to make predictions!!! Here, there are some subtle but totally sensible tricks. What if the value of some of the Pi's in the real system are not exactly the same as the corresponding values of Pi's in the experiment (or the theory)? What if they are just a little bit off? (what do you think?) We will talk about this!

§7.9. Some Typical Model Studies. Here comes tons of examples.

Problems at end of Chapter 7, page 315:

- Problem 7.1.
- Problem 7.3.
- Problem 7.4. Answer the question "why would it be incorrect to include the velocity in the smaller pipe as an additional variable?" the best you can, based on the rambling discourse in **Step 1** on page 282. But wouldn't be nice if dimensional analysis can be done *without* the need for such pre-cautions! In fact, the full blown Buckingham Pi Theorem does allow total ignorance of such pre-cautions. You can merrily include the "wrong" variable, and the real Buckingham Pi theorem will sort things out and tell you how many wrong ones have been inadvertantly included!
- Problem 7.25. Once we agree that Froude number is the only Pi parameter for this problem, we can immediately make predictions based on a 1:30 scale model!
- Problem 7.32. Whoever made the measurements for the data on Fig. P7.32 was not properly educated! If he/she had only plotted them in dimensionless form!
- Problem 7.39. Obvious the same uneducated person plotted these data!

Use email or the newsgroup to ask questions.