

MAE 222
Mechanics of Fluids
Princeton University

Assignment # 8

March 8, 1998

Due on Wednesday, 2PM, April 15, 1998

Chapter 8. Viscous Flow in Pipes. In the Chapter 6, we were introduced to the Navier-Stokes equations (6.120a,b,c) on pages 258-259. There are only a few exact analytical solutions of this set of *partial differential equations* known to this civilization. Unlike the linear Laplace equation which has tons of exact solutions (using the principle of superposition), the nonlinear Navier-Stokes equation has few known exact solutions, and (because of the nonlinearity) the principle of superposition does not hold. Hence, almost all interesting and important problems have no analytical solutions available. This chapter provides mostly empirical experimental results for flow in straight circular pipes (the problem Professor Moody worked on). And the methodology of dimensional analysis is exploited to get the most mileage out of the available experimental results.

§8.1, General Characteristics of Pipe Flow. In §8.1.1, we are introduced to the phenomena of laminar and turbulent flow, and the role of Reynolds number R_e in characterizing such a flow. Here are the wisdoms gleaned from all the experiments that were ever done on round, straight pipes (regardless of smoothness of inner surface within reason):

if $R_e < 2100$, the flow is laminar; if $R_e > 4000$, the flow is turbulent; if R_e is somewhere inbetween, it depends. Here the Reynolds number R_e is based on “average velocity” V in the pipe and the diameter D of the pipe. This conclusion is valid when the fluid involved is air, water, milk, volka, or tomato juice (so long as they have been certified as Newtonian fluid). In §8.1.2, we have the additional gems of wisdom: the length required for the

pipe flow to become “fully developed” (starting from the “entrance” and going downstream, the velocity profile no longer changes with the streamwise position) is given by (8.1) for laminar flow and (8.2) for turbulent flow. Hence, the Moody diagram is really valid only if the pipe length of interest is much larger than this length.

§8.2. Fully Developed Laminar Flow. This is one of the few exact solutions of the Navier Stokes equations known (Hagen-Poiseuille flow). We assume the flow to be strictly steady (thus, we *assumed* it to be non-turbulent), and we confine our attention to fully developed flow in a straight, circular pipe with relatively smooth surface. What is the problem to be solved? We would like to find out how the *volume flow rate* Q can be obtained, given the pressure difference Δp between two stations ℓ (feet or meters) apart, in a round pipe with diameter D , and for a fluid with viscosity μ . Following a solution procedure very similar to your last homework problem, the answer is given by (8.9). The velocity profile in the pipe is given by (8.7). Note that the flow velocity at the solid wall is assumed to stick to the solid wall (see comments just above (8.7)). This assumption is called the “NO-SLIP” condition and is an empirical assumption for Newtonian fluids (When you brush your teeth in the morning, you will notice that most tooth paste are not Newtonian and the no-slip condition is not satisfied).

Let us introduce the following dimensionless parameters:

$$\Pi_1 \equiv \frac{\Delta p}{\frac{1}{2}\rho V_c^2} \frac{D}{\ell} = f = \text{friction factor}, \quad (1)$$

$$\Pi_2 \equiv \frac{\rho V_c D}{\mu} = R_e = \text{Reynolds number}. \quad (2)$$

Then (8.9) can be rewritten as follows:

$$f = \frac{64}{R_e}. \quad (3)$$

Note that the solution “appears” to be valid for all Reynolds number. Look at the Moody Diagram on page 337. It is only valid when the flow is laminar! If you used this formula for R_e larger

than 4000, you would have earned the contempts by all educated engineers!

Note that this formula is independent of roughness (provided the roughness is small).

§8.3. Fully Developed Turbulent Flow. For Turbulent flow, there is no known analytical solutions. All knowledge on turbulent flow are empirical. What does the time-averaged “mean” velocity profile look like in a turbulent pipe flow? It is shown in Fig. 8.9 on page 335. The empirical formula are given by (8.15) on page 335. The value of n in (8.15) is a function of R_e .

§8.4. Dimensional Analysis . Here comes the Moody diagram. With it, you have the empirical database to make predictions on Newtonian fluid flows in any very long, round straight pipe in the world. Look at the effect of roughness on the Moody Diagram as characterized by $\frac{\epsilon}{D}$, and look at Table 8.1 on page 338 on the expected values of ϵ for various pipes. In §8.4.2 and §8.4.3, the dimensional analysis tool is extended to include other interesting engineering scenarios, such as “minor loses” and non-circular conduits.

Note that for turbulent flow, the friction factor (as defined) depends quite weakly on Reynolds number and is quite sensitive to the roughness.

§8.5. Pipe Flow Examples. Examples of how to use the empirical data provided in this chapter.

§8.6. Pipe Flowrate Measurements. You have a pipe, and you want to *measure* the flow rate, how would you do it? Here, three popular flow meters are introduced. If we assume the flow to be inviscid, and assume the shape of the streamtube to be a simple minded one, we can get the “theoretical results.” Well, testing the theoretical formula in the lab, we find they are off by a dimensionless factor. This section presents the empirical data on the correction factors. Note the correction factors in Figs. 8.25 and 8.26 are very nearly one, while that in Fig. 8.23 is quite different from one. Can you guess why this is so?

Problems at end of Chapter 8, page 368:

- Problem 8.4.

- Problem 8.9.
- Problem 8.19
- Problem 8.24 I will have talked about this problem in class.
- Problem 8.31
- Problem 8.40 Follow example 8.7 on page 356, using eq.(5.57) on page 186.
- Problem 8.68 Use the venturi meter discharge coefficient.

Use email or the newsgroup to ask questions.