MAE 224 Mid-Term
Spring 1999
Elements of Thermodynamics and Fluid Mechanics

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Closed-book, 50 minutes.

Friendly Advice

Take a look at all the problems, and budget your time intelligently. For word problems, give SHORT, SUCCINCT answers.

You may not have time to do all the problems. Complete as many as you can. Do those you have the best command first, and then try those "more difficult" ones after you have done the easy ones.

You are expected to draw diagrams and define your symbols (except the very standard ones such as p and ρ, x, y, z, g, etc.), as usual.

Problems

1. You should not spend more than 2 minutes on each of the following:

   1a (5 points) Restate in English Newton's $\mathbf{F} = m\mathbf{a}$ in terms of a control volume for the steady flow case. You may include a mathematical formula, but English is what I want.

   1b (5 points) What is the physical meaning of D/Dt in English, the substantial derivative?

1c (5 points) Give the substantial derivative formula in Cartesian coordinates.

1d (5 points) What are the conditions for the validity of the Bernoulli's equation?

1e (5 points) What are the conditions needed for you to conclude that a flow of interest is irrotational?

1f (5 points) How do we justify the neglect of viscosity in practical flow problems?

2. (10 points; you should not spend more than 5 minutes on this one) You are interested in the hydraulic jump problem. You would like to present data, either theoretical or experimental, so that can be used by engineers who want to find y_2, the height of the free surface after a (steady) hydraulic jump, as a function of y_1, U_1 (the velocity in front of the hydraulic jump), and g (the gravitational constant with dimension of acceleration). Show the matrix of dimensions. How many Π's are there in this problem? (I don't want the Π's, just how many).

3. (30 points; should take about 10 minutes) Consider a two-dimensional open-channel flow-over-a-smooth-bump problem. The vertical coordinate is measured from the altitude of the top of the bump. The free surface at minus infinity (upstream) is H meters (above the top of the bump), and the bottom there is very, very, very deep. The free suface at plus infinity (downstream) is below the top of the bump. Find the volume flow rate (per unit length into the paper) as a function of H and g.

4. (30 points; should take about 10 minutes) Imagine a stream of water with velocity U issuing horizontally out of a garden hose with exit area A. Let us assume that the jet does not break up, but remains a "tube" of water in an atmosphere essentially at rest. Find the weight of the water in the "flying tube" before it strikes the ground H meters below the jet nozzle. Show your control volume!

Good luck.

Answers

1. **1a** The resultant external force acting on the control volume, including the pressure contribution, equals to the net outflux of momentum from the surface of the control volume.

   1b Substantial derivative $D(.)/Dt$ means the time rate of change of (.) following a fluid element moving with velocity (u, v, w).

   1c In Cartesian coordinates, we have

$$\frac{D(.)}{Dt} = \frac{\partial(.)}{\partial t} + u\frac{\partial(.)}{\partial x} + v\frac{\partial(.)}{\partial y} + w\frac{\partial(.)}{\partial z}. \tag{1}$$

   1d (1) Inviscid approximation is valid because Reynolds number is large and our domain of interest is outside boundary layers, (2) the flow is steady, (3) the density is constant and (4) the relationship is valid on a streamline.

   1e (1) The inviscid assumption is valid, (2) the density is a constant, and (3) the fluid elements in our domain of interest were known to have no vorticity previously.

   1f When Reynolds number is large and our domain of interest is outside boundary layers.

2. We have 4 parameters. The matrix of dimension is easily written down:

$$\begin{bmatrix} & y_1 & y_2 & U_1 & g \\ M & 0 & 0 & 0 & 0 \\ \ell & 1 & 1 & 1 & 1 \\ T & 0 & 0 & -1 & -2 \end{bmatrix} \tag{2}$$

   The rank of this 3×4 matrix is 2. So there are 2 dimensionless parameters.

3. We denote the upstream station by subscript 'o' and the station at the top of the bump by subscript '1'. The altitude of the free surface upstream is H, and at the top of the bump it is y_1. The Froude number F_o is nearly zero because the water is very deep there. At the top of the bump, we have $F_1 = 1$ because we know the flow goes supercritical after the bump.

On the surface streamline, the pressure is a constant. The Bernoulli's equation relating the two stations is then:

$$gH = \frac{u_1^2}{2} + gy_1 = gy_1(1 + \frac{F_1^2}{2}). \tag{3}$$

Substituting F_1 in this equation, we obtain $y_1 = 2H/3$. The calculation of the volume flow rate over the bump is now totally straightforward.

4. We draw a constrol volume enclosing the flying tube of water. We note that the pressure on all the boundary of the control volume is a constant—thus there is no contribution from pressure.

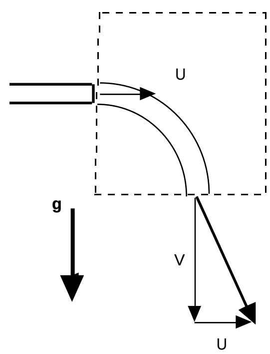

Figure 1: Diagram with control volume

We then consider the x-momentum balance (horizontal). Since there is no external force in the x direction, the net x momentum outflux must be zero. From this we conclude that the x-component of the water jet velocity at the 'exit' is U, the same as at the 'entrance.' We next consider the y-momentum balance (vertical). The external force is the gravitational pull of mother earth. It must equal to the net outflux of y momentum. At the entrance, the jet enters with zero y momentum flux. At the exit, the jet exits with $\dot{m}V$ where $\dot{m} = \rho AU$ is the mass flow rate and V is the $+y$-component of the exit velocity vector (as shown in diagram, V is negative). Applying the Bernoulli's equation along the streamline, we obtain:

$$V = -\sqrt{2gH}. \tag{4}$$

Hence, the y-force exerted by mother earth is $-\dot{m}\sqrt{2gH}$, the minus sign says it is pulling earthward.