MAE 223 Homework # 10 4-28-99

(11.23) $x = x(y,w)$ , $y = y(z,w)$ , $z = z(x,w)$

$$\Rightarrow dx = \left(\frac{\partial x}{\partial y}\right)_w dy + \left(\frac{\partial x}{\partial w}\right)_y dw$$

$$dy = \left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw$$

$$\Rightarrow dx = \left(\frac{\partial x}{\partial y}\right)_w \left[\left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw\right] + \left(\frac{\partial x}{\partial w}\right)_y dw$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left[\left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw\right] + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

$$\Rightarrow \underbrace{\left[1 - \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w\right]}_{\text{This term must be zero, since } x \text{ and } w \text{ are independent variables}} dx = \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial w}\right)_x + \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

$\Rightarrow$

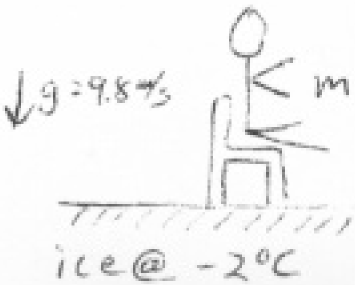
$$\boxed{\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w = 1}$$

## 11.39

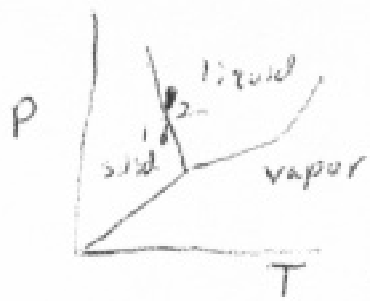
@ 0°C, $h_{if} = 333.4$ kJ/kg

$v_i = 1.0911\,(10^{-3})$ m³/kg

$v_f = 1.0002\,(10^{-3})$ m³/kg




assume the ice remains at -2°C, and $h_{if}$, $v_i$ and $v_f$ are constant with temperature around 0°C.




use Eq 11.41 (Clapeyron Eq.)

$$\Delta p = \left(\frac{h_f - h_i}{v_f - v_i}\right) \ln\left(\frac{T_2}{T_1}\right)$$

$T_1 = 273.15\,K$, $T_2 = 271.15\,K$

$\Rightarrow \Delta p = 269.54$ bar $= p$

$$p = p_{atm} + \frac{mg}{A}$$

$$\Rightarrow A = \frac{(80)(9.81)}{(268.53)(10^5)}$$

$$\boxed{A_{min} = 0.292\ \text{cm}^2}$$

11.40 $\ln P_{sat} = A - B/T$

Clapeyron Eq: $\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{T v_{fg}}$

$\frac{d}{dT}(\ln P_{sat}) = \frac{B}{T^2}$

$\Rightarrow \frac{1}{P_{sat}}\left(\frac{dP}{dT}\right)_{sat} = \frac{B}{T^2}$

$\Rightarrow \left(\frac{dP}{dT}\right)_{sat} = \frac{P_{sat}(T)\, B}{T^2}$

$\Rightarrow h_{fg} = \left(\frac{dP}{dT}\right)_{sat} T v_{fg} = \frac{P_{sat} v_{fg} B}{T}$

by Eq. 11.38: $s_{fg} = h_{fg}/T$

$\boxed{\text{a) } h_{fg} = \frac{P_{sat}(T)\, v_{fg}(T)\, B}{T}, \quad s_{fg} = \frac{P_{sat}(T)\, v_{fg}(T)\, B}{T^2}}$

Water Vapor (From Table A-2)

| $T_{sat}$ | $P_{sat}$ | $\ln P_{sat}$ | |
|---|---|---|---|
| 25 | 0.03169 | -3.452 | $= A - B/T$ |
| 30 | 0.04246 | -3.159 | $= A - B/T$ |

$\Rightarrow -0.293 = -\frac{B}{298} + \frac{B}{303} \Rightarrow B = 5{,}291$

$\Rightarrow A = -3.452 + \frac{5{,}291}{298} = 14.3$

$\ln P_{sat} \simeq -3.452 - \frac{5{,}291}{T}$

@25°C, $h_{fg} \simeq \frac{(0.03169)(10^5)(43{,}354)(5{,}291)}{298\ (10^3)} = \boxed{2{,}439}$ kJ/kg

Table A-2 shows 2,442.5 kJ/kg !

$s_{fg} \simeq \frac{h_{fg}}{T} = \boxed{8.19 \text{ kJ/kg K}}$

Table A-2 shows 8.1906 kJ/kg K !