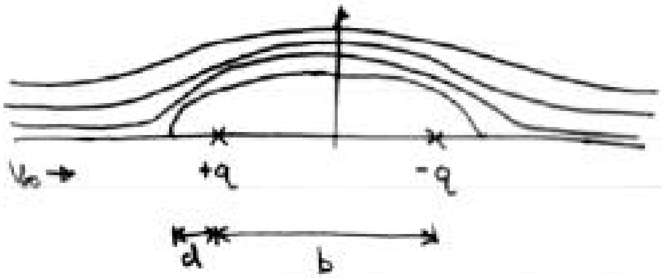
MAE 224 HW#5 Solutions

7.19)



<u>Problem</u>: Given a uniform incompressible 2-D flow V_∞ with a source and sink of strength q.

Find the potential function ϕ and the location of the upstream stagnation point.

<u>Assume</u>: Irrotational flow.

ϕ is the sum of the potential function of each feature:

$$\phi_\infty = V_\infty x$$

$$\phi_{source} = \frac{q}{2\pi} \ln \sqrt{x^2+y^2}$$

$$\phi_{sink} = \frac{-q}{2\pi} \ln \sqrt{x^2+y^2}$$

These functions are for sources + sinks centered on the origin. Here, they must be shifted by $x \pm b/2$

$$\phi = V_\infty x + \frac{q}{2\pi}\left\{ \ln\sqrt{(x+b/2)^2+y^2} - \ln\sqrt{(x-b/2)^2+y^2} \right\}$$

The stagnation point is where $u=0$. (We also know that it's on the x-axis, so $y=0$)

$$u = \frac{\partial \phi}{\partial x} = 0 = V_\infty + \frac{q}{2\pi}\left\{ \frac{x+b/2}{(x+b/2)^2+y^2} - \frac{x-b/2}{(x-b/2)^2+y^2} \right\}$$

$$u\Big|_{y=0} = V_\infty + \frac{q}{2\pi}\left\{ \frac{b}{b^2/4 - x^2} \right\} = 0 \Rightarrow x = \pm\sqrt{\frac{2qb+\pi b^2 V_\infty}{4\pi V_\infty}}$$

(continued →)

7.19) (continued)

The distance from the stagnation point to the source is just $b - x_1$ or $x_2 + b$:

$$b - \sqrt{\frac{2qb + \pi b^2 V_\infty}{4\pi V_\infty}} = d$$

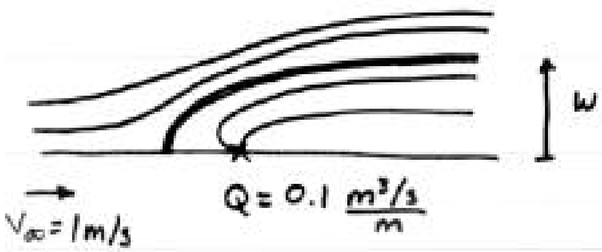
Alternate solution to part (a):

If the origin is placed at the source, the potential function changes slightly:

$$\phi = V_\infty x + \frac{q}{2\pi}\left\{ \ln\sqrt{x^2+y^2} - \ln\sqrt{(x-b)^2+y^2} \right\}$$

This won't change the answer to part (b)

7.21)



Problem: Given a source in a uniform 2-D flow ($V_\infty = 1\ m/s$, $Q = 0.1\ m^2/s$)

(a) Sketch the ½ body

(b) Find the ½ width w

(c) Find the force F_x to hold the body in place.

Assume: Irrotational flow
Incompressible flow

(b) At ∞, the velocity of the flow inside the body is the same as the velocity outside: $u = V_\infty$

Volume flow $Q = VA$ Q is 2-D, so $A = 2w$

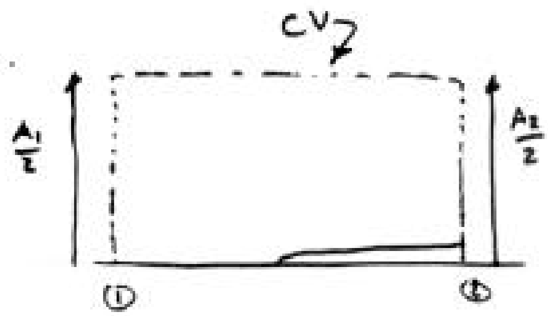
$$Q = V_\infty 2w \Rightarrow w = \frac{Q}{2V_\infty} = \underline{\underline{0.05m}}$$

(or 0.1 m if w is the whole width)

(c) Use momentum conservation, EQ (5.12)

$$F_x - \int \hat{n} p\, dA + \int \rho g\, dV = \frac{\partial}{\partial t}\int \rho \mathbf{V} d\Lambda + \int (\hat{n}\cdot\rho\bar{U})\bar{U}\, dA$$

Barrotropic, no gravity, no accumulation.



$$F_x = \int (\hat{n}\cdot\rho\bar{U})\bar{U}\, dA$$

If the CV is drawn large enough, you can see that $U_1 = U_2$.
Also $A_2 - A_1 = 2w$

$$F_x = \rho A_2 U_2^2 - \rho A_1 U_1^2 = \rho\, 2w\, U_\infty^2$$

$$F_x = (1000)(1^2)(0.1) = \underline{\underline{100\ N/m}}$$

(Or 50 N/m for the ½ body)