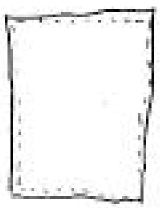
Homework #6 Solutions (MAE 224)

(1.27) CO_2 gas is in a tank.

<u>Find:</u> mass of gas (kg) and molar specific volume ($m^3/kmol$)



0.3 kmole CO_2 in tank $= n$

Tank volume $= 2.5\ m^3 = \forall$

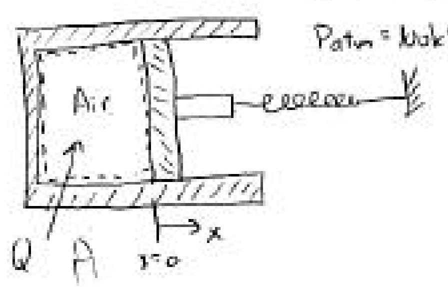
$MW_{CO_2} = 44\ kg/kmole$

$$m_{CO_2} = n_{CO_2} \cdot MW_{CO_2} = 0.3\,kmole \cdot \frac{44\,kg}{kmole} = \boxed{13.2\ kg\ CO_2}$$

$$\bar{v} = \left(\frac{n}{\forall}\right)^{-1} = \left(\frac{0.3\,kmole}{2.5\,m^3}\right)^{-1} = \boxed{8.3\ m^3/kmol = \bar{v}}$$

(226) Air is in a horizontal piston-cylinder assembly. A spring is mounted on the outside of the piston and initially exerts no force. The air expands slowly (by heating) until a final position is reached.

Find: Final pressure (kPa) and work done by air on the piston (kJ)



$P_1 = P_{atm} = 100\,kPa$

$V_1 = 2\times10^{-3}\,m^3$

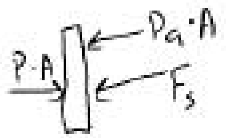
Piston area $= 18\times10^{-3}\,m^2$

$V_2 = 3\times10^{-3}\,m^3$ (final volume)

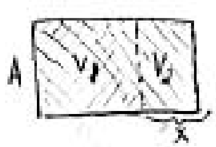
$F_{spring} = kx$ where $k = 16.2\times10^3\,N/m$

Assumption: no piston friction
system stays in thermodynamic equilibrium

consider force balance on piston



$$P\cdot A = P_a A + F_s = P_a A + kx$$



$$x\cdot A = V_2 - V_1 \rightarrow x = \frac{V_2 - V_1}{A}$$

$$P_2 = P_a + \frac{k}{A}\frac{(V_2-V_1)}{A} = 100\,kPa + \frac{16.2\times10^3\,N/m\,(3\times10^{-3}m^3 - 2\times10^{-3}m^3)}{(18\times10^{-3}m^2)^2}$$

$$\boxed{P_2 = 150\,kPa}$$

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2}\left[P_a + \frac{k}{A^2}(V - V_1)\right]dV$$

$$= P_a(V_2 - V_1) + \frac{k}{A^2}\left(\frac{1}{2}V_2^2 - V_2V_1 + \frac{1}{2}V_1^2\right)$$

$$= 100\,kPa\,(3-2)\times10^{-3}m^3 + \frac{16.2\times10^3\,N/m}{(18\times10^{-3}m^2)^2}\left(\frac{1}{2}3^2 - 3\cdot2 + \frac{1}{2}2^2\right)\times10^{-6}\,m^6 = \boxed{0.125\,kJ}$$

(169) A Gas goes through a thermodynamic cycle

$1 \to 2$: constant P

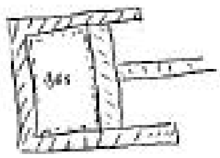
$2 \to 3$: pV = const compression

$3 \to 1$: constant volume

Find: a) sketch cycle on p-V diagram

b) calculate net work for cycle in kJ

c) calculate heat transfer for $1 \to 2$ in kJ



$p_1 = p_2 = 1.4$ bars

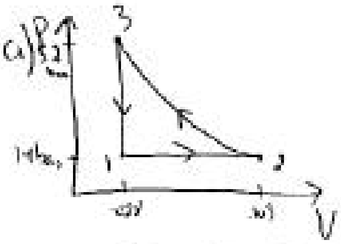
$V_1 = 0.028 m^3 \quad W_{12} = 10.5 kJ$

$U_3 = U_2$

$U_1 - U_3 = -26.4 kJ$

Assumption: gas is a closed system
ignore kinetic + potential energy

a)



c) $Q - W = \Delta E = 0$ for cycle

$\Delta E = \Delta U = 0 = \Delta U_{12} + \cancel{\Delta U_{23}}^{0} + \Delta U_{31}$

$\Delta U_{12} = -\Delta U_{31} = -(-26.4 kJ)$

$Q - W = \Delta E = \Delta U$ for $1 \to 2$

$Q = W + \Delta U = 10.5 kJ + 26.4 kJ = \boxed{36.9 kJ = Q_{1\to 2}}$

b) $W_{12} = \int_{V_1}^{V_2} p\,dV = p_1 (V_2 - V_1)$ so $V_2 = \frac{W_{12}}{p_1} + V_1$ $\qquad W_{cycle} = W_{12} + W_{23} + \cancel{W_{31}}^{0}$

pV = const means $p = \frac{p_2 V_2}{V}$

$$W_{23} = \int_{V_2}^{V_3} p\,dV = p_2 V_2 \int_{V_2}^{V_3} \frac{dV}{V} = p_2 V_2 \ln\left(\frac{V_3}{V_2}\right) = p_1 V_2 \ln\left(\frac{V_1}{V_2}\right)$$

$$= p_1\left(\frac{W_{12}}{p_1} + V_1\right) \ln\left(\frac{V_1}{\frac{W_{12}}{p_1} + V_1}\right) = 1.4 \text{ bars}\left(\frac{10500 J}{1.4 \times 10^5 N/m^2} + 0.028 m^3\right) \ln\left(\frac{0.028 m^3}{\frac{10500 J}{1.4 \times 10^5 N/m^2} + 0.028 m^3}\right)$$

$= -18{,}800$ J $\qquad W_{cycle} = 10.5 - 18.8 = \boxed{-8.3 kJ = W}$

3.83 For what ranges of pressure and temperature can air be considered an ideal gas?

Air can be considered an ideal gas when the compressibility factor, Z, is close to 1. Figure 3.11 gives the value of Z as a function of reduced pressure, P_R, and reduced temperature, T_R. For the range of pressures shown, $Z \approx 1$ for $T_R \geq 2$. Also, $Z \approx 1$ when P_R goes to 0 (say $P_R \leq 0.25$).

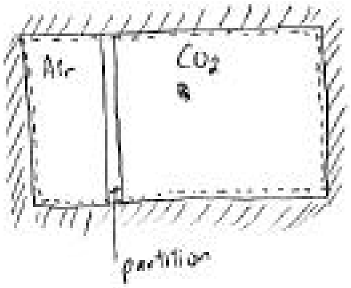
[A look at a more detailed chart, Fig A2 in the Appendix shows that Z deviates from 1 for all temperatures for $P_R > 7$-10.]

Since $P_R = \frac{P}{P_c}$ and $T_R = \frac{T}{T_c}$ and for air $T_R = 133\,K$, $P_R = 37.7$ bars (Table A1)

the above analysis indicates the ideal gas law holds for $P < 9$ bars at any temperature, and it holds above 266K for $P < 370$ bars.

(3.98) Air and CO_2 are separated by a frictionless partition inside a rigid, insulated container. The partition is free to move and it can conduct heat between the two gasses.

Find: Final T and P (in K and bars)



$m_{air} = 1\,kg \quad m_{CO_2} = 3\,kg$

$P_{1air} = 5\,bars \quad P_{1CO_2} = 2\,bars$

$T_{1air} = 350\,K \quad T_{1CO_2} = 450\,K$

Assume:
1) ideal gas
2) constant specific heats
3) ignore kinetic + potential energy
4) no energy storage in partition
5) no heat transfer to/from system
6) no work interaction with the outside of the system
7) closed system
5–7 ⇒ isolated system

apply energy equation to the entire system

$$\cancel{Q} - \cancel{W} = \Delta E = \Delta U + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$$\Delta U = (U_{air} + U_{CO_2})_2 - (U_{air} + U_{CO_2})_1 = m_{air} C_{v_{air}} T_{1air} + m_{CO_2} C_{v_{CO_2}} T_{1CO_2} - \left(m_{air} C_{v_{air}} + m_{CO_2} C_{v_{CO_2}}\right) T_f = 0$$

$$\therefore \frac{m_{air} C_{v_{air}} T_{1air} + m_{CO_2} C_{v_{CO_2}} T_{1CO_2}}{m_{air} C_{v_{air}} + m_{CO_2} C_{v_{CO_2}}} = T$$

$$= \frac{(1\,kg)(726\,J/kg\,K)(350\,K) + (3\,kg)(750\,J/kg\,K)(450\,K)}{1\,kg\,(726\,J/kg\,K) + 3\,kg\,(750\,J/kg\cdot K)} = \boxed{425.6\,K = T_f}$$

apply ideal gas to entire system at final state

$$P_f V = n R T_f$$

$$n = n_{CO_2} + n_{air} = \left(\frac{m}{MW}\right)_{CO_2} + \left(\frac{m}{MW}\right)_{air}$$

$$V = V_{initial} = V_{0_{air}} + V_{0_{CO_2}} = \left(\frac{nRT}{P}\right)_{CO_2\ initial} + \left(\frac{nRT}{P}\right)_{air\ initial}$$

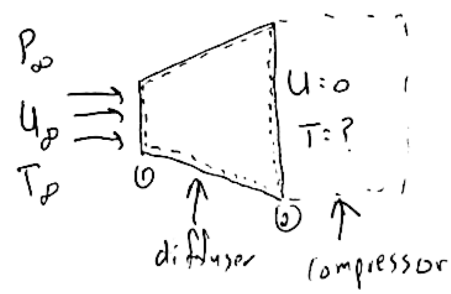
$$\therefore P_f = \frac{\left[\left(\frac{m}{MW}\right)_{CO_2} + \left(\frac{m}{MW}\right)_{air}\right] R \left[\frac{(mC_vT_1)_{air} + (mC_vT_1)_{CO_2}}{(mC_v)_{air} + (mC_v)_{CO_2}}\right]}{\left[\frac{\left(\frac{m}{MW}\right)RT_1}{P_1}\right]_{air} + \left[\frac{\left(\frac{m}{MW}\right)RT_1}{P_1}\right]_{CO_2}}$$

$$= \frac{\left(\frac{3\,kg}{44\,kg/kmol} + \frac{1\,kg}{29\,kg/kmol}\right)(8314\,J/kmol\cdot K)(425.6\,K)}{\frac{\left(\frac{1\,kg}{29\,kg/kmol}\right)\left(8314\,\frac{J}{kmol\,K}\right)(350\,K)}{5\times10^5\,N/m^2} + \frac{\left(\frac{3\,kg}{44\,kg/kmol}\right)\left(8314\,\frac{J}{kmol\,K}\right)(450\,K)}{2\times10^5\,N/m^2}}$$

$$\boxed{P_f = 2.46\,bars}$$

(4.30) Inlet to a jet engine is a diffuser which decellerates flow to zero relative velocity before entering the compressor

Find: temperature (°C) of air entering compressor



$P_\infty = 0.6$ bar
$T_\infty = 8°C = 281\,K$
$u_\infty = 1000 \frac{km}{hr} = 278\, m/s$

Assume:
- ideal gas
- neglect potential energy
- constant c_p, c_v
- adiabatic
- no work done by/on diffuser

Use steady-state energy rate balance

$$0 = \cancel{\dot{Q}} - \cancel{\dot{W}} + \dot{m}\left(h_1 + \frac{V_1^2}{2} + \cancel{gz_1}\right) - \dot{m}\left(h_2 + \frac{V_2^2}{2} + \cancel{gz_2}\right)$$

$$h_2 = h_1 + \frac{V_1^2}{2} = 281\frac{kJ}{kg} + \frac{(1000\,km/hr)^2}{2} = 281\frac{kJ}{kg} + \frac{(278\,m/s)^2}{2}\cdot\frac{1\,kJ}{1000\,J}$$

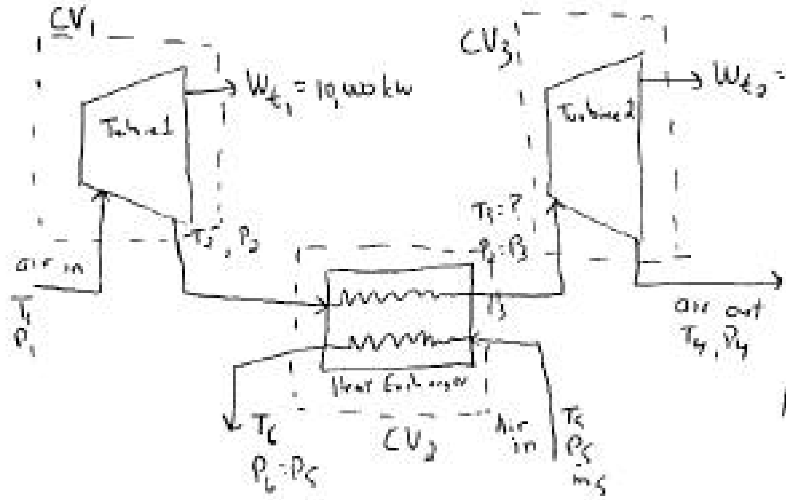
$$h_2 = 320\,kJ/kg$$

$T_2 \approx 320\,K$ by interpolation of A22

(enthalpy values taken from A22)

4.74 Air flows through two turbines and a heat exchanger. Heat transfer to the surroundings, potential and kinetic energy can be neglected. Operation is steady state.

Find: T_3 in K, and second turbine output in kW.



$T_1 = 1400$ K $P_1 = 12$ bars
$T_2 = 1100$ K $P_2 = 4$ bars
$T_4 = 980$ K $P_4 = 1$ bar
$T_5 = 1480$ K $P_5 = 1$ bar
$\dot{m}_5 = 1200$ kg/min $= 20$ kg/s
$T_6 = 1200$ K $P_6 = 1$ bar

Assume:
- steady state
- no heat loss to surroundings
- neglect KE and PE
- constant CV
- ideal gas

use steady-state energy balance

$$0 = \dot{Q} - \dot{W} + \sum \dot{m}_i\left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i}\right) - \sum \dot{m}_e\left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e}\right)$$

for CV_1: $-\dot{W}_{t_1} + \dot{m}_1 h_1 - \dot{m}_1 h_2 = 0$ $\dot{m}_1(h_1 - h_2) = \dot{W}_{t_1}$ or $\dot{m}_1 = \dfrac{\dot{W}_{t_1}}{h_1 - h_2}$

for CV_2: $0 = \dot{m}_2 h_2 - \dot{m}_3 h_3 + \dot{m}_5 h_5 - \dot{m}_6 h_6$ $\dot{m}_2 = \dot{m}_3$, $\dot{m}_5 = \dot{m}_6$, $\dot{m}_2 = \dot{m}_1$

$$\dot{m}_1 h_3 = \dot{m}_1 h_2 + \dot{m}_5(h_5 - h_6) \quad \text{or} \quad h_3 = h_2 + \frac{\dot{m}_5}{\dot{m}_1}(h_5 - h_6) = h_2 + \frac{\dot{m}_5(h_1 - h_2)}{\dot{W}_{t_1}}(h_5 - h_6)$$

$$h_3 = 1161\,\tfrac{kJ}{kg} + \frac{20\,\tfrac{kg}{s}\left(1515\,\tfrac{kJ}{kg} - 1161\,\tfrac{kJ}{kg}\right)}{10{,}000\text{ kW}}\left(1611\,\tfrac{kJ}{kg} - 1277\,\tfrac{kJ}{kg}\right)$$

$h_3 = 1390\,\tfrac{kJ}{kg}$ so from interpolation $\boxed{T_3 \approx 1295\text{ K}}$

for CV_3: $-\dot{W}_{t_2} + \dot{m}_3 h_3 - \dot{m}_4 h_4 = 0$ so $\dot{W}_{t_2} = \dot{m}_1(h_3 - h_4)$

$$\dot{W}_{t_2} = \frac{\dot{W}_{t_1}}{(h_1 - h_2)}(h_3 - h_4) = \frac{10{,}000\text{ kW}}{(1612 - 1277)\,\tfrac{kJ}{kg}}\left(1390\,\tfrac{kJ}{kg} - 1023\,\tfrac{kJ}{kg}\right) \approx \boxed{10{,}350\text{ kW} = \dot{W}_{t_2}}$$

(used table A22 for h values)