Numerical Integration

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Very often in engineering work you may have some experimental data, and you want to find the integral of it. A recent example is the Tensile Test lab, where you had a computer file of force vs. position, and you wanted to find total work done.

If you have a list of numbers like this in an Excel file, integration is **trivial**. Excel doesn't have an "integrate" function, but you can do it "by hand" in about one minute.

Theory: Finding an integral is like measuring an area. If you can find an easy way to approximate the area, then it is easy to approximate the integral.

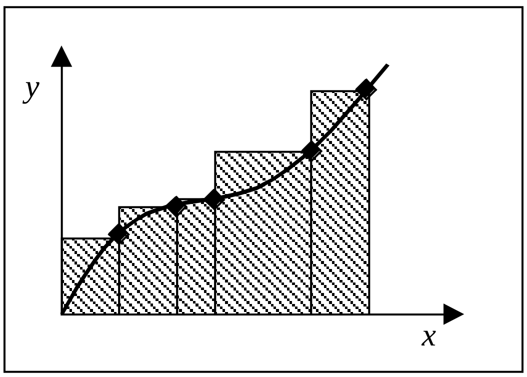

Figure 1: Integration of data points

Suppose you have the five data points shown in Figure 1. One way to to the integral would be to find a function that followed the points, then solve the integral of that function. But there's a much easier way.

If you calculate the area of the rectangles and add them up, then you'll have a pretty good approximation of the area under the curve.

So if each point has values (x_n, y_n), then you can find the area of the rectangles:

$$A = \sum_n y_n (x_n - x_{n-1})$$

Note that this method will slightly overestimate the value of the integral. If you have more data points, this error gets smaller. There are other ways to improve the accuracy, such as centering the rectangles on the data points.

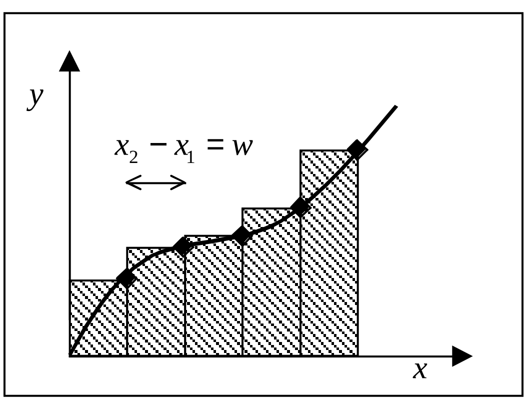

Figure 2: Evenly spaced data

If your data points are evenly spaced, then the integration gets even easier. As shown in Figure 2, the interval between all the x values is w. Now the area can be computed by:

$$A = \sum_n y_n w = w \sum_n y_n$$

So all you have to do is add up the y values and then multiply by the interval between them!

Integration in Excel: Excel doesn't have an integration function, but it does have a sum function. Suppose you have data like that below:

	A	B
1	Position (mm)	Force (kN)
2	0.01	5.1
3	0.02	6.2
4	0.03	6.8
5	0.04	7.5
6	0.04	9.5
8		=sum(b2:b6)

The interval between each x value is the same. $x_2 - x_1 = 0.01$. So the integral is just the sum of the second column times 0.01.

(To sum column B in the example, in the cell you want the sum to appear in, type "=sum(b2:b6)". You can also type "=sum(" use the mouse to highlight the range you want to sum, then type ")".

Even if the data is not evenly distributed, it is only slightly more difficult to integrate.

	A	B	C
1	Position (mm)	Force (kN)	Work (J)
2	0.01	5.1	=a2*b2
3	0.02	6.2	=(a3-a2)*b3
4	0.03	6.8	=(a4-a3)*b4
5	0.04	7.5	=(a5-a4)*b5
6	0.04	9.5	=(a6-a5)*b5
8			=sum(c2:c6)

All you need to do is find the interval between **each** point and the one before it. (Note that the 0^{th} point will be zero in this example, but it isn't always.) Now you need a third column that will be the area of each rectangle in Figure 1, and then you add up those areas. After you put the formula in cell C3, all you have to do is copy it and paste it in the other cells, and the numbers will increment correctly.

It's relatively easy to make improvements in this method by drawing the rectangles in a way that they approximate the curve better. Figures 3 and 4 show examples of this, and the formulae you put into excel are only slightly more complicated.

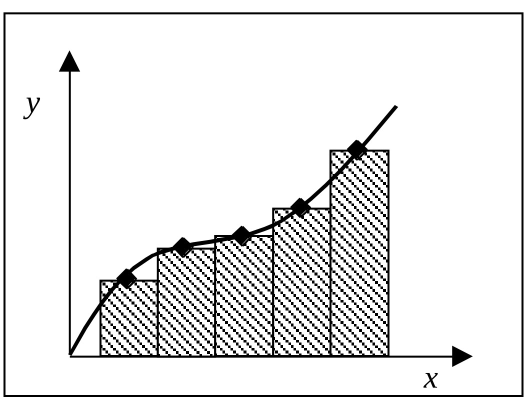

Figure 3: Centered Rectangles

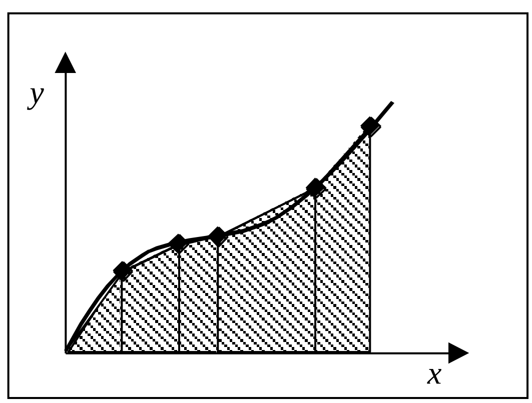

Figure 4: Triangular approximation