MAE 224 Notes #1a
Elements of Thermodynamics and Fluid Mechanics

S. H. Lam

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1 Reading and Homework Assignments

- Read Chapter 8, pp. 269-304 of Professor Smits' *A Physical Introduction to Fluid Mechanics.*
- Problems **8.10**, **8.11**, **8.22** and **8.26**.

The problems are due on Wednesday, February 10, 1999, 3PM. Please submit your homework to the MAE 224 homework **IN** tray outside D-302, E.Q.

2 Supplemental Notes

The following are step-by-step instructions for doing dimensional analysis:

1. List the dimensional parameters and variables that you believe may be involved in the problem. The list should include your "output" variables and your "input" variables. The total number of parameters and variables on your list is denoted by N. This is the hardest part of dimensional analysis.

2. List the "basic" dimensions involved in your problem. In mechanics, the basic dimensions are *mass* (M), *length* (L) and *time* (T). When thermodynamics is involved, then *temperature* (θ) is often considered an additional basic dimension. For the sake of simplicity, we shall ignore

temperature in the introductory presentations, and commit ourselves to the proposition that there are **three** basic dimensions—M, L and T. Later on, I shall show that temperature can always be "converted" to "belong" to the classical basic dimensions of mechanics.

3. Find out how many *independent* dimensionless parameters there are for your problem. In most problems in mechanics, the answer is $N - 3$. In general, the answer is $N - R$ where R is the rank of the matrix of dimensions (*i.e.* sometimes there are more than $N - 3$ independent dimensionless parameters!). To sort out whether you have $N - 3$ or more, you go through the analysis of the *Buckingham's* Π*'s Theorem* to be presented in class. This is the easy part of dimensional analysis.

4. Let the number of indepedent dimensionless parameters be denoted by I (where $I = N - R$). The next task is to find the independent Π_i's (The I Π_i's are independent if it is impossible to express any one in terms of the others). Remember, there is no unique answer to this search! Once you have found (somehow) I independent Π's by whatever means, you have obtained an answer to the dimensional analysis problem.

5. In general, the Π_i's are related by physical laws that govern the problem. If $I = 1$ (lucky you!), then Π_1 is a constant. The value of the constant can be determined either by theory or by experiment. If $I = 2$, the $\Pi_1 = \phi_1(\Pi_2)$, and the function $\phi_1(x)$ can be determined either by theory or by experiments. For higher values of I, the conclusions are similar—the selected Π_i's representing the desired output variables are some functions of the remaining Π_i's representing the input variables and parameters.

6. The *quality* of how good a job you did depends on how convincingly you can explain (to yourself and to all who are interested) the physical meaning of each Π_i. Remember, since each Π_i is dimensionless, it can always be interpreted as the *ratio* of two dimensional and physically meaningful quantities. So the meaning of a large or small value of a Π_i should be obvious. Here comes the **fun** part of dimensional analysis: If some of your input Π_i are expected to be either very, very large or very, very small for your problems, can you intelligently take advantage of this observation? If you can, then something significant has been accomplished (once your hypothesis is confirmed by experiments).