

MAE 224 Notes #2a

Elements of Thermodynamics and Fluid Mechanics

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1 Reading and Homework Assignments

The problems are due on Wednesday, February 17, 1999, 5PM. Please submit your homework to the MAE 224 homework **IN** tray outside D-302, E.Q.

- Read Chapter 9, pp. 305-338 of Professor Smits' *A Physical Introduction to Fluid Mechanics*.

Supplementary comments to help your reading, and problem assignments.

1. §9.1, scan the introductory remarks.
2. §9.2. We will derive the Navier Stokes Equation, displayed on page 306 as eq.(9.1), in the coming weeks. For the moment, we take (9.1) as granted. If you non-dimensionalize \mathbf{V} by U , distance by ℓ , and time by ℓ/U (where U and ℓ are “characteristic velocity” and “characteristic length” of the problem of interest, then Reynolds number, a dimensionless parameter ususally denoted by R_e and is defined by:

$$R_e = \frac{U\ell}{\nu} \quad (1)$$

where ν is the so-called kinematic viscosity (related to viscosity μ by $\nu = \mu/\rho$) will appear in the non-dimensionalized version of (9.1). The dimension of ν is length²/time or velocity/time. See Table C.1 and C.3

or C.2 and C.4 for the values for air and water in SI or Engineering units, respectively. If $U = 1$ m/s and $\ell = 0.1$ m (SI units), the Reynolds number at 20°C would be 6,600 in air and about 100,000 in water. Physically, R_e has the physical meaning of *dynamics* effects divided by *viscous* effects. So large R_e means the effects of viscosity is estimated to be “generally” small.

3. §9.3. If R_e is large, it seems appropriate to “neglect” viscosity as a first approximation. Indeed this is “generally” a good idea. But there are places when viscosity always play a dominant role, no matter how large R_e is. The two places are: (1) inside boundary layers and (2) in “fully developed” flows.
4. §9.4. Have you even watch the smoke from a lighted cigarette rises smoothly for a few inches, then bursts into a mess of turbulent eddies? This is another example of transition and turbulence.
5. §9.5. This section introduces the *friction factor*, f , defined by the first equal sign of the equation on page 313 between (9.8) and (9.9). The punch line of all the mathematics is: if the flow is laminar, f can be found analytically: for a two-dimensional channel, the answer is (9.9), for a circular pipe, the answer is (9.16). For now, you can ignore the details of the mathematical derivations that led to (9.9) and (9.16).
6. §9.6. What happens in real life? Well, the flow is not always laminar. What happens if the flow is turbulent? When do we expect the flow to become turbulent? Let us do a dimensional analysis. The variables and parameters of interest are: Δp (pressure change between two stations), \bar{V} , (average fluid velocity), D , (diameter of the circular pipe, L , (length of pipe between the two stations), ρ , (density), μ , (viscosity), and k (“height” of roughness pipe surface). We have $N = 7$. Checking the rank of the matrix of dimensions, we have $R = 3$. Hence there are 4 dimensional parameters. Here comes the Moody diagram, which is one of the major triumphs of dimensional analysis! Fig. 9.7 gives the dimensionless output parameter f (call it Π_1) as a function of the dimensionless input parameters R_e (call it Π_2) and relative roughness k/D (call it Π_3). How come Professor Moody ignored to find and present data for Π_4 ? Well, an intelligent choice of Π_4 would be L/D (you can verify that it is an independent dimensionless parameter). In

principle, the output f should depend on R_e , k/D **and** L/D . A little bit of thinking will enable a smart engineer, like Professor Moody, to suspect that f is most likely to be independent of L/D for large L/D . Indeed Professor Moody confirmed this speculation in the laboratory, and advised all to use his diagram only when the $L/D \geq 150$ (when the flow is laminar, considerably shorter when the flow is turbulent) so that the flow is “fully developed.”

The physical meaning of the Π_i 's are clear: f represents the output, it tells us the pressure drop; R_e represent it effects of viscosity; k/D represents roughness, and L/D distinguishes short pipes from long pipes. The fact that f is independent of L/D means that, for long pipes, Δp is proportional to L when all other dimensional variables and parameters are held constant.

A major observation: if $R_e < 2300$, the flow is “always” laminar, the experimental data agree fully with the experiments and roughness makes no difference! Another major observation: the f of “rough” pipes for very large R_e becomes very insensitive to the actual value of R_e —in other words, f becomes independent of R_e when R_e is sufficiently large. Another major observation: the magnitude of f , for moderately large R_e (say $R_e > 1000$), are all “small” numbers. Hence, for large R_e and short constant area pipes, a good advice is that f is a “negligibly small” number—*i.e.* the viscous losses for short constant area pipes at large Reynolds number is “small,” regardless of whether the flow is laminar or turbulent. If R_e is actually a small number, however, the story would be different (*e.g.* sucking honey up a straw).

What happens if, instead of \bar{V} , we had chosen $\dot{\mathcal{V}}$ (the volume flow rate; $\dot{\mathcal{V}} = \bar{V}\pi D^2/4$; dimension=length³/time) as one of our original 7 dimensional variables? We can get the answer from our new dimensional analysis problem by substituting \bar{V} by $4\dot{\mathcal{V}}/(\pi D^2)$. We have:

$$f \equiv \frac{\Delta p D}{\frac{1}{2}\rho \bar{V}^2 L} = \frac{\Delta p \pi^2 D^5}{8\rho \dot{\mathcal{V}}^2 L} \quad (2)$$

Or, the dimensional Δp is given by:

$$\Delta p = \frac{8f}{\pi^2} \frac{\rho \dot{\mathcal{V}}^2 L}{D^5}. \quad (3)$$

Equation (3) is extremely useful for design engineers when the expected R_e of the pipe under design is sufficiently large so the f is expected to be moderately insensitive to R_e ! What happens if your client wants to pump double the volume flow rate specified in the original contract? You tell him/her that Δp would be quadrupled! What happens if your client wants to use a pipe with half your recommended D to save money? The required Δp will be 32 times your recommended value!

What happens if you are pretty certain that your R_e is expected to be below 2300 so that your flow is going to be always laminar? For the laminar case, (3) is not the useful formula (because $f \approx \text{constant}$ is never a good approximation). A good candidate for a mid-term question is: what is the most useful formula (similar to (3)) when the pipe flow is expected to be laminar?

7. §9.7. Just scan it.
8. §9.8. This section, *Energy Equation for Pipe Flow*, needs some clarifications.

Without going into the details, the steady energy equation is given in Smits' notes on page 322, the first equation in §9.8.1:

$$\dot{Q} + \dot{W}_{\text{shaft}} = \int (\mathbf{n} \cdot \rho \mathbf{V}) \left(u + \frac{p}{\rho} + \frac{1}{2} V^2 + gz \right) dA \quad (4)$$

where \dot{Q} and \dot{W}_{shaft} are rate of addition of heat and shaft energy to the system of interest. If \dot{Q} is not neglected (as was done in Smits), the second equation and (9.21) in §9.8.1 can be written as follows:

$$\dot{m}(\Delta \mathcal{H} + \Delta u) = \dot{Q} + \dot{W}_{\text{shaft}} \quad (5)$$

where

$$\mathcal{H} \equiv \frac{p}{\rho} + gz + \frac{\alpha}{2} V^2, \quad (6)$$

$$\Delta \mathcal{H} = \mathcal{H}_2 - \mathcal{H}_1, \quad (7)$$

$$\Delta u = u_2 - u_1, \quad (\text{denoted as } h_{eT} \text{ in Smits}), \quad (8)$$

and α is the *kinetic energy coefficient* as defined in Smits. The entity \mathcal{H} is often called the “head” of the fluid in pipe flow problems, and

its dimension is velocity-square. If α is set to 1, then \mathcal{H} is called the *Bernoulli's constant* for incompressible flows. In many books, \mathcal{H}/g is called the head, with dimension length. We shall follow Smits here.

The question being discussed here is: when and how do you use this energy equation for pipe flow problems?

If you want to find $\Delta\mathcal{H}$ between two pumping stations, use Moody's diagram to find Δp and then use the result onward to find $\Delta\mathcal{H}$. What happens if station #1 is the inlet to a pump, and station #2 is the outlet of the same pump? Some power (energy per unit time) is added to the system between these two stations (by the pump), so the right hand side of (5) is some positive number. What is the consequence of adding energy? Equation (5) tells us precisely how much $\dot{m}(\Delta\mathcal{H} + \Delta u)$ will go up.

From the point of view of pumping, the purpose of a pump is to raise the head of the fluid being pumped—the bigger share $\Delta\mathcal{H}$ gets, the better. If Δu is found to have been raised, the fluid has become hotter when it leaves the pump. For incompressible problems, the following definition of *efficiency*, denoted by η , is often adopted:

$$\eta \equiv \frac{1}{1 + \frac{\Delta u}{\Delta\mathcal{H}}} \quad (9)$$

so that (5) can be rewritten as:

$$\dot{m}\Delta\mathcal{H} = \eta(\dot{Q} + \dot{W}_{\text{shaft}}). \quad (10)$$

The value of η is a measure of the quality of the pump. The ideal pump (for incompressible fluids) has $\eta = 1$. A broken pump would have $\eta \ll 1$ —it just stirs to heat up the fluid without raising the fluid head. Assuming that the elevations z and the fluid kinetic energies at the inlet and the outlet of the pumps are approximately unchanged, we have $\Delta\mathcal{H} \approx \Delta p/\rho$. Using (3) for Δp , we have:

$$\dot{Q} + \dot{W}_{\text{shaft}} = \frac{\Delta p \dot{\mathcal{V}}}{\eta} = \frac{8f}{\eta\pi^2} \frac{\rho \dot{\mathcal{V}}^3 L}{D^5}. \quad (11)$$

Usually, \dot{Q} is zero for a pump (and, \dot{W}_{shaft} is zero for a heater), and \dot{W}_{shaft} is the non-thermal power consumed by the pump (if this is

specified in terms of horsepower, it must be converted into units of “mechanical energy/time”: one *watt* = one Newton-Meter/sec=one joule/sec; one kilowatt=1.34702 horsepower.). Obviously, the bigger \dot{W}_{shaft} is, the more expensive is the pump.

9. §9.8.2, major and minor losses. For a long straight pipe, the major loss is the head loss due to viscous friction as represented by the Moody data. But head loss also occurs due to entrances, fittings, (sudden) area changes, etc. Using dimensional analysis, we can ask the question: what can we say about the head loss of fittings? We get K , the dimensionless fitting loss coefficient. What can we say about bends, tees and valves? In this case the concept of “equivalent length” L_e is introduced, so that an engineer can look at a pipe bend, and say to himself/herself: the head loss of this bend in head drop is equivalent to L_e additional length of straight pipes!
10. §9.10. What happens if your conduit is not a circular pipe? The concept of “hydraulic diameter” is often the way to go.

Do the followin problems:

- page 331, **9.4**. Make sure you understand why f can be assumed to be a constant.
- page 332, **9.8**. Since f is recommended to be a constant, the flow is obviously turbulent. Assume the flow is fully developed. Use $\alpha = 1.05$ for all stations. Since in the middle of the pipe the pipe area decreases smoothly, there is no head loss incurred at the contraction “fitting.” Note: at the exit of the pump, the fluid (gage) pressure is at p_1 ; at the exit of the pipe system, the fluid gage pressure is zero. Do the whole problem using the volume flow rate \dot{V} instead of \bar{V} . You should find (3) useful for this problem.
- page 336, **9.29**. One Gallon= 0.13368 ft³. The pipe is horizontal.
- page 336, **9.30**. One Horsepower= 550ft-lb/sec. You need to work out \dot{m} which equals ρ (1.94 slug/ft³) times the volume flow rate (ft³/second) in the appropriate units. Use $g = 32.2$ ft/sec².

Remember, put in numerical values as late as possible!