

MAE 224 Notes #6a Elements of Thermodynamics and Fluid Mechanics

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1 Notice of Midterm

Mid Term this Friday, March 12, in class. It will be a closed-book, 50 minutes test. No calculator is needed. Please arrive promptly at or before 10AM.

2 Study Hints for Mid-Term

Look over the hints below, and ask me questions on Wednesday, March 10.

Dimensional Analysis and Pipe Flows :

- Once you make your list of relevant variables and parameters, how many dimensionless parameters are there? You need to work out the *matrix of dimensions*, and to know how to determine the rank of that matrix. (Rank of a rectangular matrix is the “order” of the largest square matrix it contains with a nonzero determinant).
- Remember, once you know how many you should be looking for, there is no unique “correct” answer. Any answer that provides the correct number of “independent” dimensionless parameters is a correct answer, as far as dimensional analysis is concerned. How useful is the set you chose depends on whether you can interpret the physical meanings of each of the parameters.

- In practice, an experienced engineer uses well known dimensionless parameters whenever possible, *e.g.* Reynolds number, Froude number, Mach number, Moody’s friction factor f , etc.
- If a chosen dimensionless parameter is either very large or very small in the applications you have in mind, you should take advantage of it. For example, except for the chapter on pressure drop in pipe flows, we have taken advantage of large Reynolds numbers and make the “inviscid” assumption—we pretend our Reynolds number is infinite or our viscosity is zero.
- Make sure you know how to use the Moody diagram. If there is a problem which needs the Moody diagram, it will be provided.

Hydrostatics :

- Know the hydrostatic equation (the z momentum balance) by heart:

$$\frac{\partial p}{\partial z} = -\rho g. \quad (1)$$

From this you should be able to work out the pressure variation with altitude in any liquid at rest, or in an isothermal atmosphere.

- You should know that given an element of surface $d\mathcal{A}$ with its own unit normal \mathbf{n} pointing into the fluid, a (vector) force $-\mathbf{n}pd\mathcal{A}$ is being exerted by the fluid pressure p on that surface element. Note: the force is perpendicular to the surface element!
- If you see a frictionless hinge, you need to know how to compute moments or torques. Remember, the moment of a force (vector) \mathbf{f} acting on a point with position (vector) \mathbf{r} about the origin is $|\mathbf{f}|$ times the moment arm (the perpendicular distance from the origin to the line of the force vector), and the direction of the moment vector is given by the right hand rule. Mathematically, we have

$$\text{moment vector} = \mathbf{r} \times \mathbf{f} \quad (2)$$

The net moment about a frictionless hinge is zero.

- Given a fancy (or not so fancy) surface, and you are asked to find either the total horizontal or the total vertical force it experiences as a consequence of hydrostatic pressure, remember the concept of control volume and Archimedes. They make life easier for you.

- Even though we did not talk about it in class, you are expected to understand and be able to deal with manometer readings.

Control Volume :

- You must understand the difference between a system and a control volume. All Laws of Physics are originally stated for systems. In fluid mechanics, we prefer to look at a user-chosen volume fixed in space—our *control volume*. The big deal here is Reynolds Transport Theorem. You do not need to be able to reproduce the derivation, but you must know the results by heart:

$$\frac{d}{dt} \iiint_{\text{system}} \Phi(x, y, z; t) d\mathcal{V} \quad (3)$$

$$= \iiint_{\text{Vol of CV}} \frac{\partial \Phi}{\partial t} d\mathcal{V} \quad (4)$$

$$+ \iint_{\text{Surf of CV}} \mathbf{n} \cdot \mathbf{q} \Phi d\mathcal{A} \quad (5)$$

where $\Phi(x, y, z; t)$ is a scalar. You are expected to be able to explain the meaning of each of the terms in English.

- You must know the meaning of *substantial derivative*. You are expected to know its Cartesian form by heart:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}. \quad (6)$$

You are expected to know its vector form by heart:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \quad (7)$$

- Remember, you are free to pick any volume you like to be your control volume. A smart choice is a control volume for which you have the best information on its boundary so that the “flux” term can readily be computed.

Bernoulli’s Equation :

- You must know, by heart, the four required conditions for the validity of the Bernoulli’s equation.

- When fluid is emitted from a tube into the atmosphere, we assume (from experience) that it comes out as a jet with parallel streamlines. Thus, the pressure in the jet at the exit plane is the atmospheric pressure. When fluid is sucked into a tube from the atmosphere, we assume (from experience) that the fluid comes from all around. The pressure at the entrance plane must be below atmospheric.
- There is most likely a mid-term question which requires the use of Bernoulli's equation and the control volume, asking for the external force required to hold the whole mess in place.

Open Channel Flow :

- Froude number; you must know its definition and its significance.
- How much Lake Carnegie water flows over the bump in Kingston?
- Hydraulic jumps. I will give you the hydraulic jump formula if you should need it. Why is hydraulic drops forbidden?
- Know how to handle hydraulic jump problems posed in an unsteady observation frame. Change your coordinates so that it looks steady before applying the hydraulic jump formula.

Vorticity :

- You must know by heart the definition of vorticity ($\Omega = \nabla \times \mathbf{q}$), and its physical meaning (twice the average angular velocity of an element of fluid).
- You must know why

$$\nabla \cdot \Omega = 0, \tag{8}$$

and its physical meaning in English. This equation is a mathematical identity—the divergence of a curl is always zero. Physically, it says a vortex line (a line whose tangent everywhere is parallel to the local vortex line) can never end.

- You must know by heart the required conditions for a flow to be irrotational: (1) the inviscid assumption is valid (because the Reynolds number is large), (2) the density is a constant (for air, this is an approximation when the flow Mach number is low), and (3) the fluid elements in the flow field had no vorticity prior to entering into your domain of interest.

- You must know by heart the definition of velocity potential ϕ , and how to obtain velocity from it once you know it. You must know how to do it in both the Cartesian coordinates and in cylindrical polar coordinates.
- You are expected to know the derivation of the Laplace equation for ϕ —at least in Cartesian coordinates.
- You must understand why the principle of superposition works for the Laplace equation—because it is a linear equation!
- You are expected to be able to *sketch* the streamlines of simple flows obtained by superposition of our short list of elementary flows. Hint: sketch what the flow looks like where one member of the velocity potential is dominant locally. Then use your intuition on what is likely to happen inbetween.
- You are expected to know how to find pressure once you have found ϕ .
- What is D'Alembert's paradox?
- What is the Joukowski Lift Law?
- Why is there a vortex wake behind a lifting wing? What role does it play in explaining the D'Alembert's paradox?
- What is Kutta Condition?

Keep checking the course web page. I may update this before the mid term.

Good luck.