MAE 305
Engineering Mathematics I
Princeton University

Tips on the FINAL EXAM to be held on

January 20, 1998

CS 104, 1:30PM

Please arrive about 1:20PM so that you can do the course evaluations just before the exam. Thanks.

The best preparation for the final is practice. Go to Boyce and Diprima, and do problems similar to those assigned during the semester.

1. **Chapters 1 and 2**. You are expected to know how to do the list of problems that has standard methods: linear, separable, Bernoulli, Clairnaut, exact and integrating factors, and homogeneous. You need to know why the question of existence and uniqueness is useful and important. You should know Theorem 2.4.1 on page 41 by heart; it would be nice if you know what Lipschitz Condition is (page 107, problem 15).

2. **Chapter 8**. On numerical methods: I expect you to know that when a digital computer says it is solving an ODE, it cheats by replacing the derivative by a finite difference formula. The simplest-minded way is called the Euler method; the most popular method is Runge-Kutta, which is used by Matlab. In the calculation process, a 'step size' Δt must be chosen (Matlab did it for you). If the Runge-Kutta calculation 'needs' a very, very small Δt, the problem is said to be stiff. For stiff problems the Gear method (automatically used by Mathematica) is the method of choice.

3. **Chapter 3**. On second order linear equations: Here, the word *homogeneous* means something different and specific. Fundamental solutions of linear homogeneous second order ODEs has TWO integration constants! Know what a Wronskian is what does it do for you. Characteristic equation and its roots (can you see that the roots are eventually called eigenvalues?). Must know how to handle repeated roots, Method

of undetermined coefficients (inspired guess needed), and variation of parameters. Must know what to do with Euler's equation (page 53, problem 38).

4. **Chapter 4**. In retrospect, you must agree that this chapter is pretty straightforward. You should know that Euler's equation is not limited to second order.

5. **Chapter 7**. System of first order ODE's. This is where matrices come in! Once you learn this, you can handle BIG problems (1000 equations for 1000 unknowns!) on the computer. The big messy deal comes in when there is repeated eigenvalues. You need only to know that repeated eigenvalues need special handling (for those who knows Jordon's form, this is where it comes in), and where in Boyce and Diprima can you find help in doing it.

6. **Chapter 9**. Ah, phase plane and phase portrait! A problem similar to the second problem in the second midterm will be there! Please take advantage of the trick I taught you in sketching the phase portrait! You must know what conclusion you obtained from a linear system is applicable to its associated almost linear system (if the eigenvalues of the linear system are purely imaginary, you don't know if the almost linear system is stable or not).

7. **Chapter 6**. Laplace Transform table will be provided. Must be able to do routine problems. Know the convolution integral.

8. **Chapter 10**. Here comes PDE. What are the three kinds that we encounter? Heat conduction (parabolic), Wave (hyperbolic) and Laplace (elliptic). What kind of boundary conditions does each of them need? As far as separation of variables is concerned, it is totally straightforward. In the separation of variables process, you introduced separation constants which are determined by boundary conditions. The complete solution is the sum of all the individual solutions associated with each of the separation constants. The 'coefficient' of each of the terms is determined using the *orthogonal* properties of Fourier Series. Its totally straightforward, thought somewhat messy.

9. **Chapter 11**. Is Fourier Series the only game in town? NO! For second order linear homogeneous ODEs which left-hand side is real and self-adjoint, the left-hand side operator is called a Sturm-Liouville operator. You are expect to be able to find the *adjoint* operator from a given operator (and the needed boundary conditions so that the 'bilinear concomitant' term vanishes). When you find eigenvalues for a Sturm-Liouville (self-adjoint), you obtain a litter of eigenfunctions. *These eigenvalues are real, and these eigenfunctions are orthogonal to each other!* In other words, Fourier Series is NOT the only game in town.

There is a review session wednesday, January 7, 1998, at 11:00AM in our regular class room in the PMI Building, Bowen Hall. Bring questions.

Good luck!