MAE 305
Engineering Mathematics I
Princeton University

First Mid-Term, Closed-book, 50 minutes

October 24, 1997

The approximate number of minutes you should spend on each problem should be about half the number of points allocated to that problem.

Do the easy problems first, the hard ones later. Please PRINT your name in block letters.

1. (5 points) Given the general quasi-linear first order ODE:

$$\frac{dy}{dx} = f(x, y)$$

and $f(x, y)$ is known to exist and is continuous with respect to both x and y in a finite box enclosing the origin. Under what condition (hint: give the condition in the form of an inequality) can you be absolutely sure that the solution which passes through an initial point in the box exists and is unique in the box?

2. Solve the following ODE's for y(x). Make sure you show the integration constant(s).

a. (5 points)

$$\frac{dy}{dt} + p(t)y = q(t)$$

b. (10 points)

$$\frac{dy}{dx} = \frac{y \cos x + 2x \exp(y)}{1 - \sin x - x^2 \exp(y)}$$

c. (10 points)

$$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} - 6y = 2$$

d. (10 points) Convert the following ODE into separable form (need not solve):

$$\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2}$$

e. (10 points) Convert the following second order ODE into a first order ODE (need not solve):

$$\frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^2 + y^2$$

3. Solve the following system of two simultaneous first order ODE's for the 2-dimensional column vector $\mathbf{x}$:

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{b}$$

where

$$\mathbf{A} = \begin{vmatrix} -5 & 3 \\ 3 & 5 \end{vmatrix}$$

and

$$\mathbf{b} = \begin{vmatrix} 1 \\ 2 \end{vmatrix} e^{at}$$

where 'a' is a constant.

a. (5 points) Find the eigenvalues of the matrix $\mathbf{A}$.

b. (10 points) Find the associated eigenvectors (put a bar either on top or on the bottom of your vector to indicate clearly that it is a vector).

c. (5 points) Write down the homogeneous solution.

d. (10 points) Write down a fundamental matrix for this problem.

d. (10 points) Find the inhomogeneous solution under the assumption that 'a' is distinct from the eigenvalues of $\mathbf{A}$.

e. (10 points) Discuss what you would do if 'a' is identical to one of the eigenvalues.

Good luck!