

MAE 305
Engineering Mathematics I
Mid-Term, Fall 1996
50 minutes, closed book.

1. Given an ODE for a single dependent variable:
 - (a) What is the “order” of the ODE? (5 points)
 - (b) How to identify it as “linear,” “nonlinear,” or “quasi-linear?” (5 points)
 - (c) For a first order quasi-linear ODE, how can you be certain that you do not have existence and uniqueness issues to worry about for an initial value problem? (5 points)
 - (d) How would you convert a 3rd order quasi-linear ODE (which may be an nonlinear ODE) into a system of three first order (simultaneous) ODE’s? (5 points)
2. Solve the following ODEs. Give the name of the method you are using (or the name of the equation). Comment as appropriate.

- (a) (10 points)

$$\frac{dy}{dx} + xy = 1$$

- (b) (10 points)

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = \log(x)$$

- (c) (10 points)

$$\frac{dy}{dx} = \frac{y-1}{x-2}$$

- (d) (10 points) (This one is somewhat tricky because it is not on the short list in B&D—do this problem last if you don’t see how to do it immediately).

$$\frac{dy}{dx} = \frac{y^3}{1-2xy^2}$$

3. Solve the following system of two simultaneous ODEs:

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} \exp(t) \\ \sqrt{3} \exp(-t) \end{pmatrix}$$

- (a) Find the eigenvalues (using pencil and paper! No software!) (10 points).
- (b) Find the associated eigenvectors (put an overbar on top of your vectors to distinguish it from scalars). How many linearly independent eigenvectors do you get? (10 points)
- (c) Write down the homogeneous solution in terms of what you found in (a) and (b). (10 points)
- (d) Proceed to find the inhomogeneous solution. Pick a method of your choice, outline the strategy, and implement it as far as you have time. (10 points)