

MAE 305
Engineering Mathematics I
Princeton University

Second Mid-Term, Closed-book, 50 minutes

November 27, 1996

State your strategy of attack in English whenever appropriate.

1. Give succinct answers.
 - a. What is meant by “an ODE system is autonomous?” (5 points)
 - b. Given an “almost linear system.” You performed an eigen-analysis on the linear system (with the ... terms neglected) on its equilibrium point, and obtained stability information. Can you use this information on the “almost linear system?” Tell me the case where you may not. (5 points)
2. Here is a two variable ODE system:

$$\frac{dx}{dt} = (2 + x)(y - x), \quad \frac{dy}{dt} = (4 - x)(y + x) \quad (1)$$

- a. Draw the phase portrait, and identify the singular points (how many, and where). I want arrows on the trajectories indicating the movement for increasing time. For each singular point, identify what kind it is. (20 points)
 - b. Pick one singular point (your choice), and derived the appropriate linear equations (under a “microscope”) valid in the neighborhood of that singular point. Do an eigen-analysis there to confirm your conclusions obtained by inspection of the phase portrait. (10 points)
3. Find the Laplace Transform of the following, long hand (you are welcome to check your answer with the Table):
 - a. $\delta(t - t_o)$, the Dirac Delta function, also known as the *impulse function*. (5 points)

- b.** $u_c(t)$, the Step function, which is zero for $t < c$, and is unity for $t \geq c$. (5 points)

4. Solve the following ODE by Laplace Transform:

$$y'' + y = \delta(t - \pi/4), \quad y(0) = 0, \quad y'(0) = 0. \quad (2)$$

Use the Table provided. (20 points)

5. Here comes series solutions for second order linear ODE's.

- a.** When is a point an “ordinary point?” (5 points)
b. When is a point a “regular singular point?” (5 points)
c. We are interested in the series solution about $x = 0$ of the following ODE:

$$2xy'' + y' + xy = 0. \quad (3)$$

Determine the two “exponents” r_1 and r_2 from the indicial equation, and derive the recurrence relation for the coefficients in the series. (20 points)

Good luck!