MAE 305

Engineering Mathematics I

Princeton University

Assignment # 1

September 12, 1997

Due on Friday, 2PM, September 19, 1997

1. **Teaching Staff.** Professor Harvey Lam is the lecturer, supported by six TA's. Outside my office at D302c Equad, there are three MAE 305 trays: one for *handouts*, one for *homework in*, and one for *graded homework*.

2. **Course Web Page**. MAE 305 maintains a course web page (accessible only to Princeton students on campus). Its URL is:

   http://www.princeton.edu/~lam /course/mae305.html

   You will find relevant and useful information about the course, the instructor, the TA's, the assignments, and supplementary materials there. This assignment, for example, can be downloaded (and printed) from the web page. You will need "Acrobat Reader" (the latest version is 3.0) in order to read and print the so-called .pdf files. If your computer does not already have Acrobat Reader, you can download it directly from Adobe. Instruction is given on the course web page. There will be no more hard copy assignment handouts—all will be accessed through the course web page.

3. **Email for questions** My email address, as well as the email address of all the TA's, are given on the course web page. We, and myself in particular, welcome questions via email. From time to time, I will send email to the whole class when the need arises.

4. **TA Office Hours** The office hours of the TA's will be posted on the course web page. If you want to see me, give me a call (8-5133) before coming over.

- **Reading Assignment**. Boyce and DiPrima, Chapters 1 and 2. See Supplementary Notes on the next pages.

- **Home Work Problems.** The answers of homework problems are given in the back of Boyce and DiPrima. So the idea of doing the homework problems is not merely to get the right answers, but to practice so that you know how to get the right answers when you don't have the back of the book available.

  Chapter 1 .

  Page 10. Classifications & Direction Field. Problems (2, 4, 6) and Problem 39. For Direction Filed, use the method I will show you in class.

  Chapter 2 .

  Page 23. Linear 1st Order and Integrating Factor. Problems (13 and 16).

  Page 33. Bernoulli Equations. Problem 38.

  Page 38. Separable Equations. Problem 23.

  Page 45. Theorem 2.4.1. Problem 2.

  Does it satisfy the Theorem?. It is not required but it would be fun for you to look at Problem 18 which is actually the Clairnaut Equation I plan to talk about in class.

  Page 54. Applications. Problem 24.

  Draw a picture of your tank, and enclose it with a dotted line which represents a *control volume*. What is the total amount of dye we have in the control volume? We have $y * V$ grams, where y is the concentration grams/liter and V is the volume (liter) which is a constant. What is the rate of change of the total amount of dye (grams/sec) in the control volume with respect to time? Well, the incomeing water is fresh, so it contributes nothing. The outgoing stuff is going out at 2 liters/sec, and its concentration is y grams/liter. You now have enough information to write down the ODE for y.

  Page 88. Exact Equations. Problems (5 and 25).

Page 93. Homogeneous Equations. Problem 3.

There are a total of 13 problems, and I believe it represents a reasonable load. The problems bracketed together are very similar. If you are pressed for time, you may select one in the bracket and do it. I would appreciate feedbacks on the workload via email.

Supplementary Notes

This course studies ODEs (ordinary differential equations) and PDEs (partial differential equations). In the first segment of the course, we focus on ODEs. Our goal is to make sure you would know what to expect and what to do when you encounter an ODE problem in your engineering endeavors. If the problem has a known analytic method of solution, you should know the method. If the problem does not have a known analytical method of solution, you should know this fact, and have the ability to tackle it numerically. Some problems are more difficult than others, and you should know why it is so.

page 1-6. Definition of terms. "System" of ODEs. "Order" of an ODE, "Solution" of an ODE. "Linear" and "Non-Linear" equations. I will introduce the term "quasi-linear" ODEs: it is an ODE whose *highest* order term is linear, while the lower order terms may be non-linear.

page 6-10. The graphical method of "direction fields," sometimes also known as the "method of isoclines." It is most useful when you have only one dependent variable, and you just want to have a general *qualitative* idea of the variety of solutions.

First Order ODEs. There is a short list of first order ODEs that have known analytical method of solution. Boyce and Diprima provides you with the best known list: linear equations (use integrating factor), **pp.15-33** (skip *Variation of Parameters for now* on page 25); separable equations (separate them), **pp.33-38**; exact equations (use Theorem 2.8.1) and integrating factors (if you are lucky and μ depends only on one variable), **pp.83-88**; homogeneous equations (transform them into separable), **pp.90-93**. While this list is not complete, it is nearly complete. A good engineer knows all of them.

Two More Solvable Non-linear ODEs. Boyce and DiPrima included on **p.33** the special method for the (non-linear) *Bernoulli equation* and provides problems 37-41 for practice. I will talk a bit about the following non-linear (*Clairnaut*) equation (see Problem 18 on page 46):

$$y = x\frac{dy}{dx} + F(\frac{dy}{dx})$$

where $F(c)$ is any differentiable function of c. Note that this equation is fully non-linear (it is *not* quasi-linear) if $F(c)$ is a nonlinear function of c. We will have some fun talking about *existence* and *uniqueness* of solutions (why should we worry about such issues?), using this problem as the test case. We shall see that these are issues educated engineers do need to worry about.

The Picard Iteration and the Lipschitz Condition. For quasi-linear ODEs with bounded Jacobian (if you don't remember what a Jacobian is, I will remind you), the solution to an ODE initial-value problem exists and is unique in some finite interval about the starting point. This is dealt with in §2.11 on **p.98** which is considered advanced stuff. It is supported by problems 15-18 on **p.108**. I hope I will have time to talk about this.

Numerical Methods. What if none of the above works? Make your problem quasi-linear, and go to a computer. We won't deal with numerical methods this week. Skip §2.12 on p.108 on difference equations.

Applications. The rest of the reading in these two chapters are examples in "realistic" applications.

I plan to use Matlab and/or Mathematica in this course. Of course, they are available on university computers. If you can afford it, I encourage you to buy Students Edition of Matlab, version 4 or later, if you own a personal computer (PC or Mac). Mathematica is fine also, except that it is more expensive. Click on the Matlab and Mathematica buttons on the MAE 305 web page to get more information. Fool around with them—its lots of fun.