MAE 305
Engineering Mathematics I
Princeton University

Assignment # 6

October 17, 1997

Due on Friday, 2PM, October 24, 1997

1. **Mid-Term Next Friday!** It will be a one-hour (50 minutes, actually), closed book. No Matlab. So the homework this week is to prepare you for the Mid-Term. In addition, there is on the course web page a .pdf file of last year's first mid-term.

2. **The following are fair questions and problems:**

   - What is the definition of a linear ODE?
   - What is the order of an ODE?
   - What is a quasi-linear ODE?
   - What is a non-linear ODE?
   - How do you "prove" a solution is a solution?
   - What does a direction field graph do for you?
   - Under what (sufficient condition) can you conclude that a first order quasi-linear ODE has no existence and uniqueness problem in a box enclosing the initial point?
   - What did we learn after we studied the Clairnaut Equation?
   - Here is a list of first order ODE's that has known methods of solutions. You must know how to recognize them, and how to solve them.

     Linear First Order : must know method.

     Separable Equations : must know method.

Exact Equations : must know method for first order ODE. (For second order ODE, I will either give you the needed relation between the coefficients, or I will ask you to derive it.)

Homogeneous Equations : Remember, here the word homogeneous means the ODE is invariant when both dependent and independent variables changes scale by the same factor. Must know method.

Bernoulli's Equation : Must know method.

- There is a short list of second Order Linear Equations that has special tricks:

  Dependent variable missing : Reduce the order from two to one.

  Independent variable missing : Reduce the order from two to one.

  Euler Equation : Must know method.

- For a linear equation, if the term which does not depend on the dependent variable or its derivatives is missing (or removed), the resulting equation is called *homogeneous* (because the dependent variable being zero is a trivial solution to this equation). If a term which does not involve the dependent variable or its derivatives is present, the resulting equation is called nonhomogeneous (because the dependent variable equals to zero is *not* a solution).
- For a second order linear ODE, how is the general solution expressed? How many integration constants do we have?
- If some one gave me two homogeneous solutions $y_1(t)$ and $y_2(t)$ to a second order linear ODE, how do I find out if they are linearly independent? (Wronskian)
- Given a constant coefficient second order linear ODE. Can you interpret the "physical" meaning and implications of the coefficient of the first derivative term, and the meaning and implications of the coefficient of the dependent variable term?
- Given a second order constant coefficient ODE with positive damping and spring constants, how does it respond to a sinusoidal forcing function for various driving frequency (when does resonance occur)?

- What is the method of undetermined coefficients and what is the method of variation of parameters. They are useful for what kind of problems?
- How to deal with complex numbers? In particular, you are expected to know the relation between complex exponentials and sine/cosine by heart.
- What to do when the homogeneous solution to a constant coefficient second order ODE is of the form e^{rt} and the constant r is a repeated root of a quadratic equation?
- What are the straightforward generalizations of the second order linear case to the "higher order" linear case (one scalar unknown)?
- You are expected to be able to find the eigenvalues and eigenvectors of a 2×2 or 3×3 matrix long hand (If it is a 3×3, I will make sure the characteristic polynomial can be factored by inspection!).
- If the n eigenvectors of the (constant) matrix of a system of n constant coefficient first order homogeneous ODE's are linearly independent, you should know what to do—including getting the nonhomgeneous solutions.
- You should know what to do if an eigenvalue is repeated k times and there are less than k linearly independent eigenvectors associated with the repeated eigenvalue. Study Example 1 on page 391.
- You should know (from your linear algebra course) that if a (real) matrix is not symmetric, its (right) eigenvectors are not orthogonal to each other. (Do you know what "orthogonal" means?) You may recall from your linear algebra course that the right eigenvectors are "orthonormal" to the left eigenvectors.

3. **New Readings**. No readings assigned this week. I plan to do as much review and examples in class as you may wish in preparation of the Mid-Term. If time is available, I will begin lecturing on Chapter 9, *Nonlinear Differential Equations and Stability*, a fascinating chapter. You can take a peep.

4. **Practice Problems for the Mid-Term**

- Page 25, problem 33.
- Page 30, problem 3.
- Page 33, problem 40.
- Page 39, problem 21.
- Page 45, problem 11.
- Page 89, problem 15.
- Page 93, problem 2.
- Page 95, problem 21, 23.
- Page 128, problem 5.
- Page 129, problem 33.
- Page 130, problem 38.
- Page 159, problem 6.
- Page 171, problem 17.
- Page 224, problem 3.
- Page 387, problem 9
- Page 411, problem 4.

There are 17 problems above. The Mid-term will be very similar. In general, you are expected to be able to recognize a solvable problem, and solve it by the any one of the applicable methods. I won't give you any problem on "exact integrating factors" as described on page 87-88, because you may find (26) on page 87 too much to memorize. But you are expected to recongize all the others.

I plan not to include problem which requires Jordon's Normal Form in this mid-term, saving it for next time!

Since Matlab is not available, I will give only easy-enough problems so that you can find eigen stuff by hand.