Final Examination
Engineering Mathematics II

Professor S. H. Lam

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Closed-book; 3 hours

Look at all the problems, and budget your time intelligently. Taylor Series of $f(z)$ about z_o is:

$$f(z) = f(z_o) + f'(z)(z - z_o) + \frac{f''(z)}{2!}(z - z_o)^2 + \frac{f'''(z)}{3!}(z - z_o)^3 + \dots \quad (1)$$

Problems

1. (30 points) The governing PDE for $u(x, t)$ is:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0. \quad (2)$$

The domain of interest is:

$$|x| < \ell; \quad t \geq 0. \quad (3)$$

The initial conditions are:

$$u = \frac{\partial u}{\partial t} = 0 \quad \text{at} \quad t = 0. \quad (4)$$

The boundary conditions are

$$u(-\ell, t) = \exp(-at), \quad (5)$$
$$u(\ell, t) = -\exp(-at), \quad (6)$$

where $a \geq 0$ is a constant.

(i) Find u(x,t) for the special case of $a = 0$. (15 points)

(ii) Outline and discuss your strategy to deal with the $a \neq 0$ case; identify the issues and explain how they can be handled. (15 points)

2. (15 points) The Joukowski mapping relation between z and w is:

$$w = w(z) = z + \frac{a^2}{z} \tag{7}$$

where a is a real constant, $z = x + iy$ and $w = \xi + i\eta$. Find all the non-conformal points of this mapping and show that the domain in the z plane outside a circle of radius a about the origin maps into the whole w plane. Hint: find out what happens to *any* circle (with radius equal to or bigger than a) centered at the origin of the z plane.

3. (15 points) Give succinct answers to the follwoing:

i What is an analytic function of a complex variable? (5 points)

ii Why is analytic continuation an important concept? (5 points)

iii What is the Residue Theorem? (5 points)

4. (40 points) More questions on Theory of Complex Variable.

i What is the Cauchy-Riemann Condition? (5 points)

ii Show that the real and the imaginary parts of a complex function $F(z) = U(x, y) + iV(x, y)$ satify the two-dimensional Laplace equation. (5 points)

iii Compute $A(z_o)$ defined by the following contour integral:

$$\oint_C \frac{\sin(z)dz}{(z - z_o)^3} \tag{8}$$

where C is a closed contour enclosing z_o in the complex z plane. (15 points)

iv Compute B defined by the following real integral:

$$B = \int_0^\infty \frac{dx}{\sqrt{x}(x^2 + 1)^2} \tag{9}$$

(15 points)