1. You have a constant area tube filled with air at room temperature and pressure. The right end of the tube is closed. There is a piston at the left end which commences to move to the right with constant velocity \( u_p = 0.2a_0 \) where \( a_0 \) is the undisturbed speed of sound.

Approximate the expansion fan by four right running characteristics—the leading, the trailing, and two in the middle. Find the velocity and pressure at all the "nodes" of the characteristics "grid" or mesh, and the distinct regions in the flow field up to the time when the first "reflected" left running characteristic hit the piston again.

2. Do the same problem, except that this time the piston moves to the right, with the same amplitude of piston speed. Remember, there is now a shock, which happens to have constant strength. Solve this problem up to the time when the "reflected" shock hit the piston again.

3. For the second problem: it turns out that there is a small hole at the right end of this tube. Let us consider this small hole to be a converging nozzle (no diverging section). The radius of this small hole is much smaller than the radius of the tube. Do problem #2 again, up to the time the reflected shock hit the piston again. You must make approximations. Make them, and justify them. The ratio of hole radius to tube radius should be denoted by \( \varepsilon \).

If you have difficulty doing these problems, send me email.