Read Chapters 6 and 8 of Li and Lam.

I won't spend much time on stream functions, except to say here that it is a very useful thing to know but only in two-dimensional flows. Streamfunctions do exist for three-dimensional flows, but few people ever use them except for axi-symmetric flow which is 'really' two-dimensional.

You must know what Stokes' Theorem is.

Do the following homework problems from Li and Lam:

8-1, 8-2, 8-3 on page 133,
8-4, 8-5 on page 135
8-6, 8-7, 8-8 on page 140
and 8-11 on page 145 (see below).

Problem 8-11 asked you to "show" a displayed result which has a typo in it. So here is Problem 8-11 restated below:

8-11 Consider a steady, frictionless, two-dimensional flow in the x-y plane of an incompressible fluid. The flow is generally in the x-direction; the upstream velocity $U_1$ is solely in the x-direction and is a constant (independent of y), the downstream velocity $U_2(y)$ is also solely in the x-direction but is not a constant. The flow is in a channel and its upstream cross-section is different from the downstream cross-section, with a smooth transition in between. There is no gravity.

Far upstream, the density is a function of y, given by

$$\rho = \rho_0 \exp(-y/L) \quad (L = \text{characteristic length called \textit{lapse-rate}),}$$

and $\rho_0$ is a constant.

- Find $\rho(\psi)$ where $\psi$ is the streamfunction.
- Find the typo in the ordinary differential equation for $U_2(y)$ as shown in Li and Lam.