Notes #1
MAE 533, Fluid Mechanics
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1 Introduction

Mechanics is the study of motion under the action of forces. Newton's Second Law is the primary basis of this course, with the First and Second Laws of thermodynamics playing equally significant roles.

This course is oriented toward applications in mechanical and aerospace engineering. It assumes a good background of undergraduate mathematics, and some undergraduate fluid mechanics. In addition to presenting the basic formulation of continuum fluid mechanics (including multi-component formulation), we will study compressible gasdynamics, dealing with nozzle flows and its various complications. Then we will study subsonic aerodynamics, dealing with the concepts of vorticity, and its relation to the concepts of lift and (induced) drag of flight vehicles.

1.1 Dimensional Analysis

A major feature of fluid mechanics is that its governing equations are *nonlinear* partial differential equations (PDE). The number of problems for which exact mathematical solutions can be found is very few. Hence, judicious simplifying approximations are almost always needed to render realistic problems tractable. All informed fluid mechanicists know the list of common simplifying approximations, and when each is applicable. In general, an approximation is applied when a term in the governing equation is "neglected." In other words, somehow a judgment is reached to "estimate" the "order of magnitude" of terms in the governing equations, so that their relative importance can be assessed. While this is done in all branches of engineering and science, fluid mechanicists do this in a very neat and (more or less) systematic way, using the the method of *dimensional analysis*. It is obvious that the "desired answer" (when found) can always be non-dimensionalized. The non-dimensionalized answer must then contain a set of *dimensionless parameters*. We shall go over the theoretical background of dimensional analysis in class, and show that for Newtonian fluid mechanics, the most relevant dimensionless parameters are Reynolds number and Mach number (plus others). We shall discuss how dimensionless parameters can be used to great advantage in presenting experimental (or theoretical) results.

1.2 Control Volume

When Newton said $\mathbf{F} = m\mathbf{a}$, we all understood what he said. If I am interesting in the problem of a falling apple, m is the mass of the apple, $\mathbf{a}$ is the acceleration of the apple, and $\mathbf{F}$ is the resultant of external forces acting on the apple. Tell me what $\mathbf{F}$ is, and I can tell you where the apple goes.

Now what happens when we are interested in the motion of every elements of a pool of apple juice?

The strategy is obvious. You mentally divide the pool of apple juice into a large collection of tiny volumes, and apply $\mathbf{F} = m\mathbf{a}$ to each tiny volume. This is indeed how the derivation of the governing PDE is done. But the idea of mentally choosing an arbitrary (finite) volume and applying conservation laws to it will be exploited over and over again in this course. To master this skill, you need to remember the so-called *divergence theorems* in your applied math courses.

1.3 Divergence Theorems

Here are the divergence theorems:

$$\int\int\int(\nabla\phi)dV = \int\int(\mathbf{n}\phi)dA, \quad (1)$$

$$\int\int\int(\nabla\cdot\mathbf{q})dV = \int\int(\mathbf{n}\cdot\mathbf{q})dA, \quad (2)$$

$$\int\int\int(\nabla\times\mathbf{q})dV = \int\int(\mathbf{n}\times\mathbf{q})dA, \quad (3)$$

where $\mathbf{n}$ is the unit outward normal of the surface element, ϕ is a differentiable scalar field, and $\mathbf{q}$ is a differentiable vector field. The physical world is 3-dimensional, and the control volume is "closed" (it has no edges).

You may also recall the so-called Stokes Theorem:

$$\int\int\mathbf{n}\cdot(\nabla\times\mathbf{q}dA = \int\mathbf{q}\cdot d\ell \quad (4)$$

where the right hand rule determines the direction of integration of the line integral along the "edge" of the "open" surface.