# Notes #5
# MAE 533, Fluid Mechanics

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October 14, 1998

## 1 Air Breathing Jet Engines

Consider a vehicle in steady high speed flight in an otherwise quiescent atmosphere. The vehicle either has an air-breathing engine. Far downstream where the static pressure is the same as the undisturbed upstream value, a certain "exhaust" streamtube with higher stagnation enthalpy $H$ and entropy $S$ than the upstream flow are found.

We shall use subscripts $\infty$ and $e$ to denote conditions far upstream and in the exhaust streamtube (I used subscript '5' in my first lecture on this problem). We have:

$$H_\infty = h^*_\infty + h_\infty + \frac{1}{2}U^2_\infty, \quad (1)$$

$$H_e = h^*_e + h_e + \frac{1}{2}U^2_e, \quad (2)$$

$$\Delta H \equiv H_e - H_\infty \geq 0, \quad (3)$$

$$\Delta S \equiv S_e - S_\infty \geq 0. \quad (4)$$

where $h^*$ represents "non-thermal internal energy"[1] and $h$ "sensible thermal energy."

[1]The typical use of $h^*$ is to represent the chemical energy. We will later include kinetic energy of trailing vortices in $h^*$.

Now consider a control volume which encloses the entire streamtube, starting at some station far upstream and becomes at some station far downstream the exhaust streamtube.

- The continuity equation gives (neglecting the small amount of fuel which may have been added to the streamtube):

$$\dot{m} = \dot{m}_\infty = \dot{m}_e. \tag{5}$$

- The linear momentum balance gives:

$$F = \dot{m}\Delta U, \quad \Delta U \equiv U_e - U_\infty, \tag{6}$$

  where $F$ is the "thrust" generated by this streamtube. The downstream station is assumed sufficiently far away that the static pressure there is $p_\infty$.

- The energy equation gives:

$$\dot{Q} = \dot{m}(H_e - H_\infty) \tag{7}$$

  where $\dot{Q}$ is the net power (energy per unit time; either heat or shaft work) begin added to the streamtube.[2] Using (1) and (2), we have:

$$\dot{Q} = \dot{m}(\Delta h + U_\infty \Delta U + \frac{1}{2}(\Delta U)^2) \tag{8}$$

  where

$$\Delta h \equiv \Delta h^* + (h_e - h_\infty) \tag{9}$$

where $\Delta h_* = h_e^* - h_\infty^*$ is energy "stored" in the exhaust streamtube in the form of (transverse) kinetic energy of trailing vortices, ionization, dissociation or other non-combustion related chemical energy reservoirs. Assuming local thermodynamic equilibrium prevails at both stations, then $h_e - h_\infty$ can be computed since the pressures of the two stations are the same and the entropy

[2]Release of chemical energy is by the burning of fuel is included in $\dot{Q}$. Remember, it is the NET power input. So if some mechanical energy was extracted (*e.g.* by a turbine) and all the extracted energy was added back to the stream via the compressor, then these two processes as a whole contributes nothing $\dot{Q}$. However, these processes can affect the entropy rise of the exhuast stream (unless the machines are from the mythical Isentropic Inc.) and therefore can affect the overall performance of the device.

increase $\Delta S$ between the two stations is known. Assuming perfect gas with gas constant $R$, we have:

$$\Delta h = \Delta h^* + h_\infty(\exp(\frac{\Delta S}{C_p}) - 1) \geq 0. \tag{10}$$

Note that the device would be ideal if $\Delta h = 0$. This can be achieved only if $\Delta h^*$ were zero and the energy were "isentropically" added to the streamtube—with an ideal compressor driven by external torque (this ideal propulsion engine is then not a *heat engine* since heat energy is not involved). If heat energy is added, then there is a minimum amount $\Delta S > 0$ even if the heat were added "reversibly." For example, in a constant pressure low speed combustion chamber, we have:

$$\left(\exp(\frac{\Delta S}{C_p}) - 1\right)_{\text{heat addition}} \approx \frac{h_2 - h_1}{h_1} = \frac{\dot{Q}}{\dot{m}h_1} \tag{11}$$

where $h_1$ and $h_1$ are the absolute static enthalpy before and after the combustion, and $C_p$ the specific heat at constant pressure. In essence, $h_1$ is the static enthalpy where the energy addition *commences*. Obviously, the total $\Delta S$ is the sum of all the individual components of entropy increases.

Manipulating the equations from the three conservation laws, we can readily obtain:

$$F = \dot{m}U_\infty \left(\sqrt{1 + \frac{2}{\dot{m}U_\infty^2}(\dot{Q} - \dot{m}\Delta h)} - 1\right) \tag{12}$$

If $\dot{Q}$ is pure heat addition at constant pressure and the contribution to $\Delta h^*$ by the heat addition process is approximated by (11) is used to approximate the associated entropy rise, we have:

$$F \approx \dot{m}U_\infty \left(\sqrt{1 + \frac{2}{\dot{m}U_\infty^2}\left(\dot{Q}(1 - \frac{h_\infty}{h_1}) - \dot{m}\Delta h^{**}\right)} - 1\right) \tag{13}$$

where $\Delta h^{**}$ is $\Delta h^*$ minus the (inescapable) contribution from the heat addition process. Note that $\dot{Q}$ is multiplied by the factor $(1 - h_\infty/h_1)$ where $h_1$, we repeat, is the static enthalpy when the heat addition process commences. It is clear that a "good" engine should have as high a value of $h_1$ as possible, and as low a value of $\Delta h_*$ as possible.

Equations (12) and (13) are valid for any propulsive device. If my compressor and my turbines are isentropic machines and there are no other sources, then for this ideal jet engine we would have $\Delta h^{**} = 0$. A good engine, therefore, strives to have $h_1$ (or $T_1$) as high as possible, and $\Delta h^{**}$ as small as possible.

## 1.1 Exercise

1. Assume that you have an compressor purchased from Isentropic Inc. Compute $h_\infty/h_1$ when $p_1/p_\infty$ is 8.0—station #1 is at the end of the compressor and at the start of the combustor. Assume perfect gas with constant specific heats.

2. Recently (1998), some people suggests that if the air in front of a supersonic aircraft is somehow ionized by microwave or laser energy beamed from the aircraft, the drag of the aircraft can be reduced. What do you think about this idea?