Notes #5
MAE 533, Fluid Mechanics

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1 Take Home MidTerm #1

To be submitted after Fall Break

Consider a jet engine in steady subsonic flight in an uniform atmosphere otherwise at rest. We use a coordinate system fixed on the jet engine so that the flows comes from the left with velocity U_∞. We consider a streamtube coming from left infinity (station '∞'), and exits the jet engine in a straight and constant area exhaust streamtube (station 'e'). We label the station just in front of the compressor station #1, just after the compressor and just before the combustor station #2, just after the combustor and just before the turbine station #3, and just after the turbine station #4. The gas obeys the Ideal Gas Law with constant specific heats, etc.

The compressor and turbine are both purchased from a reputable company. The performance of the compressor is characterize by a compressor efficiency, η_c, defined by:

$$\frac{p_2^\circ}{p_1^\circ} = \eta_c \left(\frac{T_2^\circ}{T_1^\circ} \right)^{\frac{\gamma}{\gamma-1}} . \tag{1}$$

Similarly, the turbine efficiency, η_t, is defined by:

$$\frac{p_4^\circ}{p_3^\circ} = \eta_t \left(\frac{T_4^\circ}{T_3^\circ} \right)^{\frac{\gamma}{\gamma-1}} . \tag{2}$$

The company indicated that the values of η_c and η_t are approximately constant for a wide range of operating conditions.

1.1 Problems

1. Find the entropy rise across the compressor as a function of η_c.
2. Find the entropy rise across the turbine as a function of η_t.
3. Assume the incoming Mach Number M_∞ is moderately low. Show that the Mach Number M_2 is expected to be lower than M_1.
4. Assume that the Mach Number M_3 remains small even though heat is being added in the combustion chamber. Find the entropy rise across the combustion chamber (as a function of $\dot{Q}/\dot{m}$ and *something else*) when terms of order $O(M_2^2)$ and $O(M_3^2)$ are neglected.
5. Find the entropy of the exhaust gas at station 'e'.
6. Convince yourself that $p_e \approx p_\infty$. Now go find T_e (assume $h = C_pT$, etc.).
7. Follow Notes 5 and derive a formula for the thrust per unit mass flow rate $F/\dot{m}$ of this jet engine as a function of heat addition per unit mass flow rate $\dot{Q}/\dot{m}$, η_c, η_t and U_∞. What happens to your formula when $U_\infty \to 0$?
8. Find the density ρ_e in the exhaust streamtube.
9. Find the velocity u_e in the exhaust streamtube.
10. Find the mass flow rate per unit exhaust streamtube area, $\rho_e u_e$.
11. How does the thrust F of this engine scale with A_e? (what happens to F if A_e is doubled?)
12. At which station is temperature the highest in the jet engine? From the materials point of view, there is a maximum temperature which must not be exceeded. Given this as a constraint, what would you like your compressor to do for you (besides having as high a value of η_c as possible)?