

A Robust Universal Controller¹

S. H. Lam

Princeton University

Department of Mechanical and Aerospace Engineering

Princeton, NJ 08544

lam@princeton.edu

Abstract

The present paper advocates the use of ODE-based (instead of algebra-based) constraints on the output variables of a controlled system, and the use of dynamic control laws (instead of static control laws)—when finite but small errors in honoring the constraints are considered acceptable. It is assumed that the sensor signals are reliable and have good signal-to-noise ratios, that the controller has ample computational power, and that the hardware/software “sampling time” t_s is much smaller than the system characteristic time t_g . A universal *dynamic* control law is proposed which does not require detailed knowledge of the system itself—and is therefore completely robust with respect to uncertainties of the system—provided the control problem to be solved is reasonable. The meaning of the equivocal word “reasonable” will be explained.

1 Introduction

The approach advocated by this paper is based on the concatenation of two new ideas:

- Instead of imposing *algebra-based* constraints on the sensor signals, a set of *ODE-based* constraints are specified.
- Instead of looking for *static control laws*, the new approach looks for *dynamic control laws* which *approximately* honor the ODE-based constraints.

With this new approach, the only information about a dynamic system needed by the controller is some “surrogate” of the *extended pulse-response matrix* $B_i^m(t)$ (defined in §2.3 later by Eq. (7c)) which is closely

related—but is not identical—to the so-called “decoupling matrix” (Isidori, 1995). Most importantly, this $B_i^m(t)$ matrix can be determined on-the-fly—in principle—directly by the controller itself (see §2.3). Unlike most current adaptive control theories (Ioannou and Sun, 1996; Marino and Tomei, 1995; Krstić *et al.*, 1995; Kokotović *et al.*, 1991; Kokotović *et al.*, 1986; Ioannou and Kokotović, 1983), no detailed knowledge of the system is needed—not even a parametrized form. The present theory exploits $t_s/t_g \ll 1$, the virtually unlimited computing power available to the controller, and the mathematical insights provided by the so-called *quasi-steady approximation* on stable “stiff” ODE’s (O’Malley, 1991; Lam, 1993; Lam and Goussis, 1994). The proposed *universal dynamic control law* is in the form of a stable stiff ODE for the control inputs, and its numerical solution (using Euler’s method) automatically generates an *approximation* to the exact static control law—using only the current and past sensor signals, the past control inputs, and a “surrogate” of the real $B_i^m(t)$. The theory of the “regular” case—when the true B_i^i is non-singular and not nearly singular—is presented, and a simple example is provided in §3.1.1. The generalization to the “irregular” case—when B_i^i is singular or nearly singular—is non-trivial. Only a brief summary of the major results of the irregular case is presented here; the details are presented in separate papers (Lam, 1997a,b).

2 Formulation

A system is to be controlled by a black box with I actuator control inputs (to the system) and I primary sensor outputs (from the system) which are assumed to be reliable and have good signal-to-noise ratios. The job of the black box is to compute and issue the appropriate control input signals to the system so that the resulting sensor signals outputted from

¹Supported by NSF Grant #MSS-9302294 and AFOSR URI Grant F49620-93-1-0427. Presented at the 1997 ASME IMECE, Dallas, Texas.

the system indeed honor the user-specified constraints satisfactorily—*i.e.* the measured constraint errors are kept below some user-specified, finite *accuracy threshold*. We assume that the black box has no detailed knowledge of the system except for the following:

1. The maximum allowable amplitudes of the actuator control inputs are known.
2. The characteristic timescale of the system is known to be of the order of t_g seconds. A pragmatic definition of t_g is the following: if any of the control inputs were suddenly set to a “wrong” (but allowable) value, the measured constraint errors would evolve continuously and become significant in no less than t_g seconds. In other words, t_g is the minimum “grace period” allowed by the problem. Hence, t_g can in principle be directly measured by the black box itself.
3. The *sampling period* t_s of the system, defined as the time required by the black box hardware/software to compute and update the control inputs based on the current and past sensor outputs and control inputs, is much, much smaller than t_g .

The black box has full detailed knowledge of the desired ODE-based constraints and the acceptable threshold of constraint errors. It is allowed to use only the present and past sensor signals (output feedback) in computing the control inputs to the system. Obviously, it must also have the ability to identify “unreasonable” tasks which cannot be performed by the system and so inform the user.

In the present paper, a function of time is said to be *smooth* if it is differentiable and its characteristic timescale is $O(t_g)$.

2.1 A Dynamic Model of the System

Consider the following nonlinear dynamical system for a N -dimensional column vector $\mathbf{x}(t)$:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f} + \sum_{i=1}^I \mathbf{b}_i u^i, \quad (1)$$

where $\mathbf{f}(\mathbf{x}, t)$ and the $\mathbf{b}_i(\mathbf{x}, t)$'s are differentiable and bounded (column) vector functions of \mathbf{x} and t and $I \leq N$. The first term on the right hand side represents the resultant *applied force* acting on the system, while the second term represents the resultant *control force* acting on the system, expressed as the sum of

I linearly independent *actuator forces*. We attribute no special significance to the point $\mathbf{x} = 0$, make no assumption on the value of $\mathbf{f}(0, t)$ or the functional form of $\mathbf{f}(\mathbf{x}, t)$. The column vector $\mathbf{b}_i(\mathbf{x}, t)$ and the scalars u^i 's represent the direction and the amplitude—the latter is called *control input* here—of the i -th actuator force, respectively. We assume that the $u^i(t)$'s have been non-dimensionalized by their (known) maximum allowable values so that $u^i = O(1)$ is required. The characteristic timescale t_g of the problem (as described previously) is, loosely speaking, determined by the order of magnitude of the \mathbf{b}_i 's. No loss of generality was incurred by assuming that the right hand side of Eq. (1) is linear with respect to the u^i 's and contains none of their time derivatives. If the right hand side of Eq. (1) is a general function $\mathbf{f}(\mathbf{x}, t; u^i, \dot{u}^i, \ddot{u}^i, \dots)$, it can always be transformed into the form of the current Eq. (1) by introducing new state variables and new control inputs (Lane and Stengel, 1988).

There are I sensor signals. The i -th sensor signal is denoted by y^i which is a measurement of the current state of the system:

$$y^i = \psi^i(\mathbf{x}, t), \quad i = 1, \dots, I, \quad (2)$$

where the $\psi^i(\mathbf{x}, t)$'s are I linearly independent, smooth and differentiable functions of \mathbf{x} and t . We assume the y^i 's have been appropriately non-dimensionalized, and call them the *primary outputs* of the system. The primary ψ^i 's are strictly forbidden to depend on the u^i 's. Under certain well-defined conditions, the problem may spawn additional $J \geq 0$ *derived outputs* y^{I+j} 's (Lam, 1997a,b). By including the derived outputs, the total number of outputs becomes $M = I + J \leq N$ (M can be interpreted as the sum of “relative degrees” of the primary outputs). Together the M outputs can be succinctly represented by:

$$y^m = \psi^m(\mathbf{x}, t; \dots), \quad m = 1, \dots, M. \quad (3)$$

We assume that $\psi^m(\mathbf{x}, t) = 0$ is a non-empty region in \mathbf{x} space. Unlike the primary ψ^i 's which have no dependence at all on the u^i 's, the derived ψ^{I+j} 's may have a “weak” dependence.

The mathematical problem is to find control algorithms to compute the $u^i(t)$'s—without using detailed knowledge of the system—such that not only the resulting $y^m(t)$'s satisfactorily honor the user-specified constraint equations, but also the $u^i(t)$'s are $O(1)$. In the present paper, we shall confine our attention mostly to the regular case when B_i^i is non-singular (and therefore $J = 0$ and $M = I$); only a brief summary of results of the irregular case is included.

2.2 The Constraint Equations

We divide the N -dimensional \mathbf{x} space into two complementary subspaces: a M -dimensional *output subspace* $\{y^1, \dots, y^M\}$ (which includes both primary and derived output variables), and its complementary K -dimensional *residual subspace*, $\{y^{M+1}, \dots, y^N\}$ where $K = N - M \geq 0$. The output subspace is defined by Eq. (3). The residual subspace is defined by

$$y^{M+k} = \psi^{M+k}(\mathbf{x}, t), \quad k = 1, \dots, K, \quad (4)$$

where the $\psi^{M+k}(\mathbf{x}, t)$'s are any differentiable functions subject to the requirement that the transformation Jacobian between \mathbf{x} and \mathbf{y} is non-singular. In the dynamics community, the residual variables y^{M+k} 's are called *generalized coordinates*.

Conventionally, *algebra-based constraints* are imposed on the output variables:

$$y^m(t) = O(\epsilon), \quad (5a)$$

where ϵ is a user-specified dimensionless accuracy threshold. Such problems are called *tracking* or *regulation* problems. In the present paper, we shall impose, instead of Eq. (5a), the following *ODE-based constraints*:

$$\frac{dy^m}{dt} + \phi^m(\mathbf{y}, t; \varpi^{m'}) = O(\epsilon/t_g), \quad (5b)$$

where the $\phi^m(\mathbf{y}, t; \varpi^{m'})$'s are user-specified functions (and the $\varpi^{m'}$'s are *residual controls* to be exploited later). It is easy to be convinced that Eq. (5b) can include Eq. (5a) as a special case.

When $N > M$, the stability of the residual variables is sometimes also of interest. It is possible to exert some influence on the residual variables of the system *provided* some additional information is available to the controller. The *residual control problem* will be briefly discussed in §3.2.2.

2.2.1 A Special Choice of $\phi^m(\mathbf{y}, t; \varpi^{m'})$: In order for the universal controller to be able to compute $u^i(t)$ from Eq. (9), the function $\phi^m(\mathbf{y}, t; \varpi^{m'})$'s should depend only on the y^m 's (output feedback). The following special choice is of particular interest:

$$\phi^m(\mathbf{y}, t; \varpi^{m'}) = \frac{1}{\tau} \sum_{m'=1}^M \Omega_{m'}^m \left(y^{m'} - \varpi^{m'} \right) \quad (6)$$

where $\tau > 0$ is a user-chosen characteristic controller timescale (the “settling time”), $\Omega_{m'}^m$ is a $M \times M$ dimensionless positive-resolute matrix (see §3.1 later), and

the ϖ^m 's are residual controls which are not allowed to depend on the y^m 's but may depend smoothly on t (it will later be allowed to depend on the y^{M+k} 's in dealing with the residual control problem). When this special choice Eq. (6) is used, it is easy to show (using the quasi-steady approximation) that $y^m \approx \varpi^m$ is a good approximate solution of Eq. (5b) when $\tau \ll t_g$.

2.3 Dynamics of Output and Residual Variables

Differentiating Eq. (3) with respect to time and using Eq. (1), we obtain the ODE's for the output variables:

$$\frac{dy^m}{dt} = g^m + \sum_{i=1}^I B_i^m u^i, \quad (7a)$$

where

$$g^m(\mathbf{x}, t) = \frac{\partial \psi^m}{\partial t} + \mathbf{c}^m \cdot \mathbf{f}, \quad (7b)$$

$$B_i^m(\mathbf{x}, t) \equiv \mathbf{c}^m \cdot \mathbf{b}_i, \quad (7c)$$

$$\mathbf{c}^m(\mathbf{x}, t) \equiv \frac{\partial \psi^m}{\partial \mathbf{x}}. \quad (7d)$$

The M row vectors \mathbf{c}^m 's are assumed linearly independent. The order of magnitude of B_i^m is $O(1/t_g)$ (based the pragmatic definition of t_g given previously). If B_i^i is non-singular and *not* nearly singular, the problem is *regular* ($M = I$), otherwise, it is *irregular* ($M > I$). For regular problems, the g^i 's have no dependence at all on the u^i 's. We shall see later in §4 that for irregular problems the same cannot be said for the derived g^m 's.

Geometrically, the row vector \mathbf{c}^m is normal to the $\psi^m = 0$ surface. The column vector \mathbf{b}_i represents the direction of the i -th actuator force vector. The $M \times I$ rectangular matrix $B_i^m(\mathbf{x}, t)$, defined as the dot product of $\mathbf{c}^m(\mathbf{x}, t)$ with $\mathbf{b}_i(\mathbf{x}, t)$, plays a most important role in the present theory. It is *not* the same as the $I \times I$ square *decoupling matrix* in standard nonlinear control textbooks (Isidori, 1995; Marino, 1995)—even though they are closely related. Its matrix elements are called *first Markov parameters* in linear system textbooks (Rugh, 1996; Brogan, 1985; Kailath, 1980; Chen, 1970). To avoid confusion, we shall call it the *pulse-response matrix* of the system since in principle it can be directly determined on-the-fly by the controller by pulsing the u^i 's and measuring the immediate response of the y^m 's.

The ODE's for the remaining K residual variables y^{M+k} 's (*i.e.* generalized coordinates) are simply given by (7a,b,c,d) with the superscript index m replaced by $M+k$.

3 The Regular Case

We confine our attention to regular problems for which no additional derived outputs are needed ($M = I$). The completely straightforward treatment below serves as a guide for the more involved treatment of singular and nearly singular problems—which must deal with the complications of derived outputs.

Since $[B_{i'}^i]^{-1}$ exists, we can analytically eliminate dy^i/dt between Eq. (7a) and Eq. (5b) and solve for the u^i 's (with ϵ set to zero). The resulting *exact* static control law, denoted by u_∞^i , is:

$$u_\infty^i(\mathbf{x}, t) \equiv u_o^i(\mathbf{x}, t) - \sum_{i'=1}^I [B_{i'}^i]^{-1} \phi^{i'}(\mathbf{y}, t; \varpi^i). \quad (8a)$$

where $u_o^i(\mathbf{x}, t)$ is:

$$u_o^i(\mathbf{x}, t) \equiv - \sum_{i'=1}^I [B_{i'}^i]^{-1} g^{i'}, \quad (8b)$$

Note that u_o^i has no dependence on the u^i 's because the primary $g^i(\mathbf{x}, t)$'s are strictly forbidden to have any dependence at all on the u^i 's.

This exact static control law u_∞^i cannot function as an universal controller because the detailed knowledge of the system (and full state feedback) needed for its evaluation is assumed unavailable.

3.1 Regular Universal Dynamic Control Law

We now assume that, in addition to good quality $y^i(t)$'s and $\dot{y}^i(t)$'s, a certain non-singular “surrogate matrix” $\bar{B}_{i'}^i(t)$ is also somehow made available to the controller. The proposed *regular universal dynamic control law* is:

$$\frac{du^i}{dt} = -\frac{1}{\Delta t} \sum_{i'=1}^I [\bar{B}_{i'}^i]^{-1} \left(\dot{y}^{i'} + \phi^{i'}(\mathbf{y}, t, \varpi^{i'}) \right), \quad (9)$$

where Δt is a user-chosen “small” positive timescale such that $\Delta t/t_g = O(\epsilon)$. What properties must $\bar{B}_{i'}^i$ have in order to ensure that Eq. (9) is asymptotically stable? Using Eq. (7a) to eliminate \dot{y}^i from Eq. (9), factoring out $B_{i'}^i$, and using Eq. (8b), we obtain:

$$\frac{du^i}{dt} = -\frac{1}{\Delta t} \sum_{i'=1}^I \bar{W}_{i'}^i \left(u^{i'} - u_\infty^{i'} \right), \quad (10a)$$

where $u_\infty^{i'}$ was given previously by Eq. (8a) and

$$\bar{W}_{i'}^i \equiv \sum_{i''=1}^I [\bar{B}_{i''}^i]^{-1} B_{i'}^{i''}. \quad (10b)$$

We are reminded that u_∞^i is known to have no dependence at all either on the u^i 's or their higher time derivatives. Hence Eq. (10a), and therefore Eq. (9), is simply a system of first order ODE's for the u^i 's when the problem is regular.

We shall call a matrix a *resolute* matrix if all the real part of its eigenvalues are of the same sign and are all $O(1)$. A resolute matrix is said to be *positive-resolute* if the real part of all its eigenvalues are positive. By inspection of Eq. (10a), we require $\bar{W}_{i'}^i$ to be positive-resolute to ensure the asymptotic stability of Eq. (10a) and therefore Eq. (9), and to allow Δt to be interpreted as the characteristic time needed by the dynamic control law to allow u^i to converge to an approximation of u_∞^i , regardless of its initial condition. Whenever $u^i(t)$ computed from Eq. (9) is found to be bounded and smooth using $\Delta t/t_g = O(\epsilon) \ll 1$, we can conclude—using insights from the methodology of quasi-steady approximation—that the $u^i(t)$'s obtained are good approximations to the $u_\infty^i(\mathbf{x}(t), t)$'s. Formally, we have $u^i = u_\infty^i + O(\Delta t/t_g)$ for $t \gg \Delta t$. For best performance, the smallest allowable value should be used for Δt (e.g. $\Delta t = t_s$).

While the most simple-minded choice for $\bar{B}_{i'}^i$, for a regular problem is obviously $B_{i'}^i$ (so that $\bar{W}_{i'}^i$ is the identity matrix), we see that any non-ideal surrogate $\bar{B}_{i'}^i$ can be used so long as the resulting $\bar{W}_{i'}^i$ is positive-resolute. The details of a positive-resolute $\bar{W}_{i'}^i$ impact the u^i 's only in a brief $O(\Delta t)$ transient period. In other words, the condition on $\bar{B}_{i'}^i$ is quite weak for regular problems.

3.1.1 A Simple Example: In this subsection only, superscript of column vectors will appear as a subscript to avoid confusion with exponents. Consider a problem with $I = 2$ and its impulse-response matrix $B_{i'}^i(t)$ is assumed known (it is non-singular and its smallest singular value is $O(1/t_g)$), and the measured $y_i(t)$'s and $\dot{y}_i(t)$'s are assumed reliable and have good signal-to-noise ratios. We do not know N (except that $N \geq 2$), $\mathbf{f}(\mathbf{x}, t)$, $\mathbf{b}_i(\mathbf{x}, t)$, $\psi_i(\mathbf{x}, t)$ or $\mathbf{c}_i(\mathbf{x}, t)$. It is desired for the y_i 's to emulate a Van der Pol's oscillator:

$$\dot{\phi}_1 = -y_2, \quad (11a)$$

$$\dot{\phi}_2 = -\mu(1 - y_1^2)y_2 + \kappa^2 y_1, \quad (11b)$$

where μ and κ are user-chosen positive constants with $\kappa t_g = O(1)$.

Using $\bar{B}_{i'}^i \approx B_{i'}^i(t)$ and choosing Δt such that $\kappa \Delta t = O(\epsilon) \ll 1$, one can easily verify—using either asymptotic analysis or direct numerical simulations on desktop computers—that the universal dynamic control law Eq. (9) will indeed honor the ODE-based constraints

Eq. (5b) with accuracy $O(\epsilon)$ for any reasonable $\mathbf{f}(\mathbf{x}, t)$ (after a brief $O(\Delta t)$ transient). If the resulting $u_i(t)$'s—which depend on $\mathbf{f}(\mathbf{x}, t)$ —are $O(1)$, then all is well.

If instead it is desired for the y_i 's to remain $O(\epsilon)$ (*i.e.* a regulation problem), the universal controller can use the special ϕ_i given by Eq. (6) with $\varpi_i = 0$. One can also easily verify that the desired algebra-based constraints are honored (after a brief $O(\tau)$ transient).

3.2 Dynamics of Residual Variables

If $K = N - M > 0$, what happens to the K residual variables y^{M+k} 's? Since the universal controller is unaware of even their existence, this question is meaningful only if some supplementary information such as additional sensor measurements and/or indeed a detailed mathematical model are made available to the controller. If no such information are provided, the universal controller is incapable of accepting any responsibility for influencing the unobserved y^{M+k} 's. To continue the discussion, we assume that all needed supplementary information are indeed available.

3.2.1 Zero Dynamics: When algebra-based constraints $y^m = O(\epsilon)$ are imposed, the dynamics problem of the residual variables y^{M+k} 's obtained by honoring the algebra-based constraints precisely (*i.e.* $\epsilon = 0$) is classically called the *zero dynamics problem*. We shall generalize the definition of zero dynamics by allowing the y^m 's to honor *either* the algebra-based constraints *or* the ODE-based constraints precisely.

If the zero dynamics solutions are deemed acceptable with all residual controls set to zero, then all is well. Generally speaking, the usefulness of the proposed universal controller is limited to such problems. Classically, linear problems with stable zero dynamics are called minimum-phase problems. Otherwise, they are called non-minimum phase problems. Note that whether the problem is linear or nonlinear plays no role in the present theory.

3.2.2 The Residual Control Problems: Non-zero residual controls ϖ^i 's may be used to exert some influence on the residual variables.

If Eq. (6) is adopted, then the following simple-minded strategy is theoretically available: substitute u_∞^i as given by Eq. (8a) for u^i in Eq. (1) to obtain a new control problem. The residual controls ϖ^i 's now play the role originally played by the u^i 's—except that they are not allowed to depend on the primary output variables y^i 's. Up to I offending residual variables

may be influenced in some way by the I available residual controls—provided the new residual control problem poses a reasonable problem. An example is worked out in Lam (1997b) to show what can be done.

4 The Irregular Case

When B_i^j is singular or nearly singular, the problem is said to be *irregular*. Irregular problems are quite commonplace in practical systems (Godbole and Sastri, 1995). Hence, the generalization of the present approach is of fundamental interest. Since space limitation here does not allow a full presentation, we shall limit ourselves to a brief summary of the major results presented in Lam (1997b).

For irregular problems, we must search for J additional *derived outputs* y^{I+j} 's such that the traditional rank of the $M \times I$ *extended impulse-response* matrix B_i^m is I . If a full-rank B_i^m can be found, the irregular problem is said to be *regularizable*; otherwise, the control problem as posed is said to be unreasonable.

An outline of the searching procedure is given below. The singular value decomposition of a full-rank B_i^m is represented by (Golub, 1989):

$$B_i^m = \sum_{i'=1}^I [U_m^{i'}]^T \omega(i') V_i^{i'}, \quad (12)$$

where U_m^m and V_i^i are $M \times M$ and $I \times I$ dimensionless orthogonal matrices and the $\omega(i)$'s are the singular values (always positive by convention) which satisfy the following inequality:

$$\omega(i) > |\omega_*| \quad (13)$$

where ω_* is a user-chosen signed (small) parameter with the dimension of frequency. The bottom $J = M - I$ rows of U_m^m , which do not appear in Eq. (12), are called *left null-vectors* of B_i^m (for obvious reasons). Any row vector $\tilde{U}_m^{(\cdot)}$ which satisfies

$$\sum_{m=1}^M \tilde{U}_m^{(\cdot)} B_i^m = O(\omega_*) \quad (14)$$

is called a *left epsilon-vector* of B_i^m —it can be approximately represented by linear combinations of the J left null-vectors. A bootstrapping procedure is used to recursively find J linearly independent left epsilon-vectors of B_i^m which, in addition to satisfying Eq. (14), also satisfy:

$$\tilde{U}_m^{I+j} = 0, \quad m \geq I + j, \quad j = 1, \dots, J, \quad (15)$$

one after the other. A special generalization of Eq. (6) for $\phi^m(\mathbf{y}, t; \varpi^{m'})$ is adopted in which all elements of $\Omega_{m'}^m$ are user-chosen except $\Omega_{I+j}^m = [\tilde{U}_m^{I+j}]^T$ for $m \geq I+j$. In addition, the M residual controls $\varpi^{m'}$'s are required to satisfy:

$$\sum_{m,m'=1}^M \tilde{U}_m^{I+j} \Omega_{m'}^m \varpi^{m'} = 0. \quad (16)$$

Hence, only I degrees of freedom are available to the M residual controls.

The J derived output variables y^{I+j} 's are defined by:

$$\sum_{m=1}^M \tilde{U}_m^{I+j} (\dot{y}^m + \phi^m) = O(\omega_*), \quad j = 1, \dots, J. \quad (17)$$

The special of \tilde{U}_m^{I+j} , $\Omega_{m'}^m$, and $\varpi^{m'}$ mentioned above allowed y^{I+j} to be recursively solved for in terms of the y^m 's and the \dot{y}^m 's where $m < I+j$. Assuming a full-rank B_i^m is found, the *universal dynamic control law* is:

$$\frac{du^i}{dt} = -\frac{1}{\Delta t} \sum_{m=1}^M [B_i^m]_*^+ (\dot{y}^m + \phi^m) \quad (18)$$

where $[B_i^m]_*^+$, called the *epsilon-inverse* of B_i^m , is given by (Golub, 1989):

$$[B_i^m]_*^+ \equiv \sum_{i'=1}^I [V_i^{i'}]^T \frac{1}{\omega(i')} U_m^{i'}. \quad (19)$$

Operationally, the universal controller is responsible for the numerical integration of Eq. (18) (using Euler's method), while Nature is responsible for the analog integration of the systems of ODE's for the M output variables Eq. (7a) and the K residual variables. We can analytically eliminate \dot{y}^m from Eq. (18) using Eq. (7a) to obtain:

$$\frac{du^i}{dt} = -\frac{1}{\Delta t} (u^i - u_\infty^i) \quad (20)$$

where u_∞^i is now given by an expression similar to Eq. (8a) and Eq. (8b) except that the inverse of B_i^m is replaced by the epsilon-inverse of B_i^m . Most importantly, whenever $\omega_* \neq 0$ the irregular u_∞^i is now "weakly" dependent on the u^i 's (and perhaps even their higher time derivatives) because the derived $\psi^{I+j}(\mathbf{x}, t; \dots)$'s obtained by Eq. (17) are weakly dependent on them. These theoretically negligible "higher order" terms in u_∞^i have subtle and profound consequences on the stability of Eq. (18) or Eq. (20). The universal controller is not a theoretician, and it is incapable of "neglecting" theoretically negligible terms—it must use Eq. (18) as it is displayed above. We denote the order of the highest time derivative of u^i appearing in u_∞^i by q . Hence,

whenever $\omega_* \neq 0$ Eq. (18) or Eq. (20) is *not* a system of first order ODE's, but rather a system of q -th order ODE's for the u^i 's. The following recommendations are arrived at after careful scrutiny and analysis. To be assured of stability, Euler's method for numerical integration must be used (to render the small, higher order time derivative terms impotent), *and* Δt must respect as a lower bound τ_* defined by:

$$\tau_* \equiv |\epsilon_*|^{1/q} t_g, \quad \epsilon_* \equiv \omega_* t_g. \quad (21)$$

Because Δt now has a lower bound, the minimum constraint error for irregular problems is $O(|\epsilon_*|^{1/q})$.

5 Reasonable Problems

We are now ready to define what is a reasonable problem. A problem is said to be reasonable if

- a non-singular impulse-response matrix B_i^m ($M \geq I$) can be found, and
- the $u^i(t)$'s computed by the universal dynamic control laws are $O(1)$ and are smooth.

We take it for granted that the I sensor signals (and their time derivatives) have good signal-to-noise ratios, a good estimate of t_g is known, t_s/t_g is sufficiently small, and a good surrogate to the regularized $B_i^m(t)$ is somehow provided (directly measured or otherwise provided). The control of the residual variables is left to the residual controls when additional information is provided. The design of an universal controller which honors Eq. (6) then simply involves the selection of τ and the user-specifiable elements of the positive-resolute $\Omega_{m'}^m$ matrix. The smallest possible Δt should be used to get the best performance.

The universal controller relies completely on the sensor signals to go about the task of controlling the system. All sensor measurements, including measurement noises, are being interpreted by the universal controller as externally applied forces acting on the system. It blindly strives to make the measured $y^m(t)$'s honor the ODE-based constraints within the user-specified constraint error threshold. Hence, the measured $y^m(t)$'s must be reliable and have very good signal-to-noise ratio in order for the universal controller to be usable.

Theoretically, u_∞^i as given by Eq. (8a) can be rewritten for any general ϕ^i as:

$$u_\infty^i = - \sum_{m=1}^M [B_i^m]_*^+ (\phi^m + g^m). \quad (22)$$

Since $u^i \approx u_\infty^i$, Eq. (22) can be used to estimate the magnitude of the u^i 's. Since $[B_i^m]_*^+ = O(t_g)$ (by definition of t_g), we require $\phi^m + g^m = O(1/t_g)$ for reasonable problems.

6 Concluding Remarks

The strength of the present approach is that no detailed knowledge of the system is needed. Whether the problem is linear or nonlinear is irrelevant, so long as it is reasonable in accordance with §5. The Achilles' heel is that it requires not only reliable sensor signals of the output variables, but also their time derivatives. This is the price paid for robustness with respect to the system. Obviously, no one should expect any controller, which is not only uninformed about the system to be controlled but is also provided with unreliable sensor signals, to be able to perform its assigned control task successfully. The universal controller simply expects all sensor signals to be reliable and to have "sufficiently good" signal-to-noise ratios. Ideally, the highest time derivatives of the primary output variables should be reliably measured by the sensors, and their lower time derivatives and the primary output variables themselves should be obtained by numerical integration. Obviously, if enough detailed knowledge about the system is provided, the universal controller can certainly take advantage of the additional information to construct observers and use them to reduce its present total reliance on the sensor signals.

The controller can exert an influence on the unobserved residual variables y^{M+k} 's of the system by the use of the I degrees of freedom available in the residual controls—provided some additional information and knowledge are provided to the controller.

The universal dynamic control laws presented above are in the form of ODE's for the u^i 's. For irregular problems with $\epsilon_* \neq 0$, it is theoretically imperative that Euler's method be used when $q \geq 2$. The achievable accuracy of the universal controller degrades as q increases (because the lower bound for Δt increases). A discussion of this point is given in Lam (1997b).

It is possible for a system to be non-regularizable at certain isolated special system configurations, but can be literally "shaken" off such "top-dead-center" configurations by random pulsing of the actuators. It is also possible for a generally reasonable problem to tolerate brief unreasonable periods (*e.g.* its zero dynamics problem becomes temporarily non-minimum phase). In numerical simulations, the universal controller is quite capable of recovering from such episodes.

References

- [1] Brogan, W. L., 1985. *Modern Control Theory*, Prentice Hall, Inc.
- [2] Chen, C. T., 1970. *Introduction to Linear System Theory*, Holt, Reinhart and Winston, Inc.
- [3] Godbole, D. N. and Sastry, S. S., 1995. "Approximate Decoupling and Asymptotic Tracking for MIMO Systems," *IEEE Transactions on Automatic Control*, Vol. 40, **3**, pp. 441-450.
- [4] Golub, G. H. and Van Loan C. F., 1989. *Matrix Computations*, Second Edition, The Johns Hopkins University Press.
- [5] Ioannou, P. A. and Kokotović, P. V., 1983. *Adaptive Systems with Reduced Models*, Lecture Notes in Control and Information Sciences, #47, Springer-Verlag.
- [6] Ioannou, P. A. and Sun, J., 1996. *Robust Adaptive Control*, Prentice Hall.
- [7] Isidori, Alberto, 1995. *Nonlinear Control Systems*, 3rd Edition, Springer-Verlag.
- [8] Kailath, T., 1980. *Linear Systems*, Prentice Hall.
- [9] Kokotović, P. V., Kanellakopoulos, I. and Morse, A. S., 1991. "Adaptive Feedback Linearization of Nonlinear Systems," pp. 347-434, and Praly, L., Bastin, G., Pomet, J. B. and Jiang, Z. P., "Adaptive Stabilization of Nonlinear Systems," pp. 311-346, Kokotović, P. V. (Ed.), in *Foundations of Adaptive Control*, Lecture Notes in Control and Information Sciences, **160**, Thoma, M. and Wyner, A., Editors, Springer-Verlag.
- [10] Kokotović, P., Khalil, H. K. and O'Reilly, J., 1986. *Singular Perturbation Methods in Control: Analysis and Design*, Academic Press.
- [11] Krstić, M., Kanellakopoulos, I. and Kokotović, P., 1995. *Nonlinear and Adaptive Control Design*, John Wiley and Sons, Inc.
- [12] Lam, S. H., 1993. "Using CSP to Understand Complex Chemical Kinetics," *Combustion Science and Technology*, **89**, 5-6, pp. 375-404.
- [13] Lam, S. H. and Goussis, D. A., 1994. "The CSP Method for Simplifying Kinetics," *International Journal of Chemical Kinetics*, **26**, pp. 461-486.
- [14] Lam, S. H., 1997a. "Quasi-steady approximation and adaptive nonlinear control," presented in the *15th IMACS World Congress on Scientific Computation, Modeling and Applied Mathematics*, Berlin.

- [15] Lam, S. H., 1997b. "Universal Controller for Reasonable Nonlinear Systems," MAE Report 2089, Department of Mechanical and Aerospace Engineering, Princeton University.
- [16] Lane, S. H. and Stengel, R. F., 1988. "Flight Control Design Using Nonlinear Inverse Dynamics," *Automatica*, Vol. 24, 4, pp. 471-483.
- [17] Marino, R. and Tomei, P., 1995. *Nonlinear Control Design, Geometric, Adaptive and Robust*, Prentice Hall.
- [18] O'Malley, R. E. Jr., 1991. *Singular perturbation methods for ordinary differential equations*, Applied Mathematical Sciences **89**, Springer-Verlag.
- [19] Rugh, W. J., 1996. *Linear System Theory*, 2nd Edition, Prentice Hall.