On the Achievable Rates and Cooperation Strategies of Three-Node Wireless Network

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Abstract

We consider a wireless network composed of three nodes and limited by the half-duplex and total power constraints. This formulation encompasses many of the special cases studied in the literature and allows for capturing the common features shared by them. Here, we focus on three special cases, namely 1) Relay Channel, 2) Multicast Channel, and 3) Three Way Channel. These special cases are judicially chosen to reflect varying degrees of complexity while highlighting the common ground shared by the different variants of the three node wireless network. For the relay channel, we propose a new cooperation scheme that exploits the wireless feedback gain. This scheme combines the benefits of decode-and-forward and compress-and-forward strategies and avoids the noiseless feedback assumption adopted in earlier works. Inspired by the proposed feedback strategy, we identify a greedy cooperation framework applicable to both the multicast and three way channels. Our performance analysis shows the asymptotic optimality of the proposed greedy approach and the central role of list source-channel decoding in exploiting the receiver side information in the wireless network setting.

1 Introduction

We are in the midst of a new wireless revolution, brought on by the adoption of wireless networks for consumer, military, scientific, and wireless applications. These applications have sparked a renewed interest in network information theory. Despite the recent progress (e.g., [1]), developing a unified theory for network information flow remains an elusive task.

In our work, we consider, perhaps, the most simplified scenario of wireless networks. Our network is composed of only three nodes and limited by the half-duplex and total power constrains. Despite its simplicity, this model encompasses many of the special cases that have been extensively studied in the literature. These special channels are induced by the traffic generated at the nodes and the requirements imposed on the network. More importantly, this model exposes the common features shared by these special cases and allows for constructing universal cooperation strategies that yield significant performance gains. In particular, we focus here on three special cases, namely 1) Relay Channel, 2) Multicast Channel, and 3) Three Way Channel. We adopt a greedy framework for designing cooperation strategies and characterize the achievable rates of the proposed schemes. Our analysis reveals the structural similarities of the proposed strategies in these three special cases, and establishes the asymptotic optimality of such strategies in several cases. In the relay channel, we propose a novel cooperation strategy with *noisy* feedback. Our strategy combines the benefits of both decode-and-forward (DF) and compress-and-forward (CF) and avoids the idealistic assumptions adopted in earlier works. Inspired by the feedback strategy for the relay channel, we construct a greedy cooperation strategy for the multicast scenario. Motivated by a greedy approach, we show that the *weak* receiver is led to help the *strong* receiver first. Based on the same greedy motivation, the strong user starts to assist the weak receiver after successfully decoding the transmitted codeword. We compute the corresponding achievable rate achieved by this scheme and use it to establish the significant gains offered by this strategy, as compared with the non-cooperative scenario. Motivated by the sensor networks application, we identify the three way channel model as a special case of our general formulation, under which these three nodes observe correlated data streams and every node wishes to communicate its observations to the other two nodes. Our proposed cooperation strategy in this scenario consists of three stages of *multicast with side information*, where the multicasting order is determined by a low complexity greedy scheduler. In every stage, we identify the central role of list source-channel decoding in exploiting the side information available at the receivers. By contrasting the minimum energy required by the proposed strategy with the genie-aided and non-cooperative schemes, we establish its superior performance. Due to the space limitation, we omit the proofs in this paper, interested readers can refer to [7] for details.

2 Model

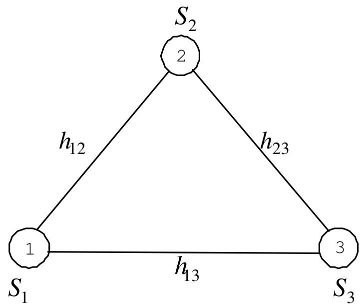

Figure 1: An illustration of the three-node (half-duplex) wireless networks.

Fig. 1 illustrates a network consisting of three nodes each observing a source random variable S_i drawn from a finite set $\mathcal{S}_i$. The distributed source variables S_i's are correlated. Denoted by S_i^K the length-K source sequence $S_i(1), \ldots, S_i(K)$ at the i-th node. Nodes are interested in getting a subset or all the source variables at the other nodes. To achieve this goal, nodes are allowed to coordinate and exchange information over the wireless channel. In this paper, we consider the discrete-time *additive white Gaussian noise* (AWGN) channel. At time instant n, node j receives

$$Y_j(n) = \sum_{i \neq j} h_{ij} X_i(n) + Z_j(n), \tag{1}$$

where $X_i(n)$ is the transmitted signal at node-i and h_{ij} is the channel coefficient from node i to j. To simplify the discussion, we assume the channel coefficients are symmetric, i.e., $h_{ij} = h_{ji}$. We also assume that the additive zero-mean Gaussian noise is spatially and temporally white and has the same unit variance ($\sigma^2 = 1$). The nodes in this network are half-duplex nodes that cannot transmit and receive *simultaneously* using the same degree of freedom. Without loss of generality, we split the degrees of freedom available to each node in the temporal domain, so that, at each time instant n, a node-i can either transmit (*T-mode*, $Y_i(n) = 0$) or receive (*R-mode*, $X_i(n) = 0$), but never both. Due to the half-duplex constraint, at any time instant, the network nodes are divided into two groups: the T-mode nodes (denoted by $\mathcal{T}$) and the R-mode nodes ($\mathcal{R}$). A partition $(\mathcal{T}, \mathcal{R})$ is called a network state. Let $P_i^{(l)}$ denote the average transmit power at the i-th node during the m_l network state. We assume a power normalization such that the *total* power of all the T-mode nodes at any network state is limited to P, that is,

$$\sum_{i \in \mathcal{T}_l} P_i^{(l)} \leq P, \quad \forall m_l. \tag{2}$$

Node-i is associated with an index set I_i, such that $j \in I_i$ indicates that node-i is interested in getting S_j from node-j ($j \neq i$). An efficient cooperation strategy should strive to maximize the achievable rate given by $\frac{KH(S_1,S_2,S_3)}{N}$, where N is the minimum number of channel uses necessary to satisfy the network requirements. For a fixed $H(S_1, S_2, S_3)$, this optimization is equivalent to minimizing the bandwidth expansion factor $\tau = \frac{N}{K}$. Due to a certain additive property, using the bandwidth expansion factor will be more convenient in the three way channel scenario. A bandwidth expansion factor τ is said to be achievable if there exists a series of source-channel codes with $N, K \to \infty$ but $\frac{N}{K} \to \tau$, such that every node can decode the intended messages with vanishing error probability. In the feedback-relay and multicast channel, minimizing the bandwidth expansion factor reduces to the more conventional concept of maximizing the rate given by $R = \frac{\log_2(M)}{N}$, where M is the size of message set at the source node.

The three-node network model encompasses many important network communication scenarios with a wide range of complexity, controlled by various configurations of the index sets I_i's and the distributed source S_i's. From this perspective, the relay channel represents the simplest situation where one node serves as the relay for the other source-destination pair, e.g., $\mathcal{S}_2 = \mathcal{S}_3 = \phi$, $I_1 = I_2 = \phi$ and $I_3 = \{1\}$. If we enlarge the index set $I_2 = \{1\}$, meaning node-2 now is also interested in getting source message, then the problem becomes the multicast channel. The most complex situation can be intuitively referred to as the three way channel in which every node tries to get source from all the other nodes, that is, $I_i = \{1, 2, 3\} - \{i\}$. While it is easy to envision other variants of the three node network, we decide to limit ourselves to these special cases. This choice stems from our belief that other scenarios do not add further insights to our framework.

3 Relay Channel With Noisy Feedback

Our formulation for the three node network allows for a more realistic investigation of the relay channel with feedback. In this scenario, node-1 is designated as the source node, node-3 the destination, and node-2 the relay. Since there is only one source in this case, one can easily see that maximizing the achievable rate R from source to destination is equivalent to minimizing the bandwidth expansion factor.

In a recent work [3], Kramer *et al.* present a comprehensive overview of the existing cooperation strategies and the corresponding achievable rates for relay channels. One class of relay schemes is referred to as *decode-and-forward* (DF), in which the relay node first completely decodes the source message and then aids the destination node decoding. When the source-relay link is very noisy, one can argue that requiring the relay node to decode the message before starting to help the destination may, in fact, adversely affect performance. The *compress-and-forward* (CF) strategy avoids this drawback by asking the relay to "compress" its observations and send it to the destination. In this approach, Wyner-Ziv source compression is employed by the relay to allow the destination node to obtain a noisy copy of the relay observations. [4, 5] have applied these strategies to the half-duplex relay channel , and got the corresponding achievable rates.

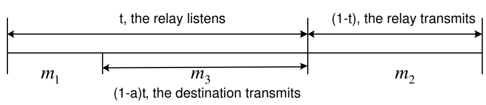

Figure 2: Half-duplex relay channel with noisy feedback.

In this paper we introduce a novel cooperation strategy for the relay channel with noisy feedback. In a nutshell, the proposed strategy combines the DF and CF strategies to overcome the bottleneck of a noisy source-relay channel. In this FeedBack (FB) approach, the destination first assists the relay in decoding via CF cooperation. After decoding, the relay starts helping the destination via a DF configuration. Due to the half-duplex constraint, every cycle

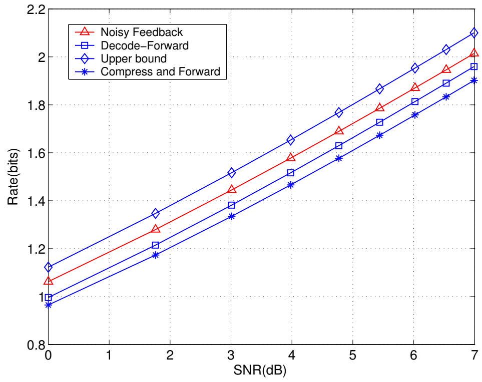

Figure 3: The achievable rate of various scheme in the relay channel.

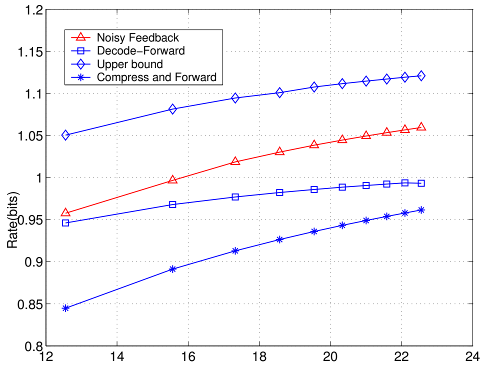

Figure 4: The achievable rate of various scheme in the relay channel.

of transmission is divided into the following three stages, as Fig. 2 shows. In the first state m_1, which lasts for a fraction αt of the cycle ($0 \leq t, \alpha \leq 1$), both the relay and the destination listen to the source. In the feedback stage m_3, which lasts for a fraction $(1-\alpha)t$ of the cycle, the relay listens to both the destination and the source. Since the destination is not yet able to completely decode the source message, it sends to the relay node a Wyner-Ziv compressed version of its observations. The final stage m_2 lasts for a fraction $(1-t)$ of the cycle. Having obtained source information, the relay is now able to helps the destination node in decoding the source message. Here, we stress that this formulation for a relay channel with feedback represents a "realistic" view that attempts to capture the constraints imposed by the wireless scenario (as opposed to the *noiseless* feedback assumed in existing works, e.g., [2]). The feedback considered here simply refers to transmission from the destination to relay over the same (noisy) wireless channel. Using random coding arguments we obtain the following achievable rate for the proposed feedback scheme, in which max is taken over the total power constraint.

Lemma 3.1 *The achievable rate of the proposed noisy feedback scheme is given by*

$$\begin{aligned} R_{FB} = \max_{\alpha,t,r_{12},P_i^{(j)}} \quad \min\Bigg\{ & \alpha t C\Big((\frac{h_{13}^2}{1+\sigma_3^2} + h_{12}^2) P_1^{(1)} \Big) + (1-\alpha) t C\Big(h_{12}^2 P_1^{(3)} \Big) \\ & + (1-t) C\Big((1-r_{12}^2) h_{13}^2 P_1^{(2)} \Big); \\ & \alpha t C\Big(h_{13}^2 P_1^{(1)} \Big) + (1-t) C\Big(h_{13}^2 P_1^{(2)} + 2 r_{12} h_{13} h_{23} \sqrt{P_1^{(2)} P_2^{(2)}} + h_{23}^2 P_2^{(2)} \Big) \Bigg\} \end{aligned} \tag{3}$$

where $\sigma_3^2 = \frac{\left(h_{12}^2 + h_{13}^2\right) P_1^{(1)} + 1}{\left(h_{12}^2 P_1^{(1)} + 1\right)\left(\left(1 + \frac{h_{23}^2 P_3^{(3)}}{h_{12}^2 P_1^{(3)} + 1}\right)^{\frac{1-\alpha}{\alpha}} - 1\right)}$, *and* r_{12} *is the correlation between* X_1, X_2 *during state* m_2.

Fig. 3 reports the achievable rate of various schemes, when $h_{12} = 2.55$ dB, $h_{13} = 0$ dB, $h_{23} =$ 23 dB. This corresponds to the case when the source-relay channel is a little better than the source-destination channel, and the relay-destination channel is quite good. This is the typical scenario where the feedback results in a significant gain, as demonstrated in the figure. Fig. 4 reports the achievable rates of various schemes, when $h_{12} = 2.55$ dB, $h_{13} = 0$ dB, $P = 0$ dB, as we vary the channel coefficient between relay-destination h_{23}. We can see that as the relay-destination channel becomes better, the advantage of feedback scheme increases.

Next we give a summary of the asymptotic behavior of the proposed feedback scheme, which has been reported in [8]. Interested readers can refer to [7, 8] for details. The FB scheme can achieve the multi-transmitter upper-bound when $h_{12} \to \infty$. It can also achieve the multi-receiver upper-bound when $h_{23} \to \infty$. But the gain of FB diminishes at low or high SNR region. In low SNR, the channel output is dominated by the noise, and hence, the compression (note that in the feedback state, the destination uses CF scheme) inevitably operates on the noise resulting in a diminishing gain. For high SNR, if $\alpha \neq 1$, the destination spends a fraction $(1-\alpha)t$ of time in transmitting to the relay, which cuts off the time in which it would have been listening to the source in non-feedback schemes. Such a time loss reduces the pre-log constant, which cannot be compensated by the cooperative gains when SNR is large.

4 Multicast Channel

In this section we study another example of this three-node model by requiring node-2 to decode the message. This corresponds to the multicast scenario. Similar to the relay scenario, we can focus on maximizing the achievable rate from node-1 to both node-2 and 3 without any loss of generality. The half-duplex and total power constraints, adopted here, introduce an interesting design challenge. For example, if node-2 decides to help node-3 in decoding, then not only does node-2 compete with the source node for transmission power, but it also sacrifices its listening time for the sake of helping node-3. It is, therefore, not clear *a-priori* if the network would benefit from cooperation. In the following, we address this question in the affirmative and further propose a greedy cooperation strategy that is asymptotically optimal.

In the non-cooperative scenario, both node-2, 3 will listen all the time, and hence, the achievable rate is given by $C_{non-coop} = C(\min\{h_{12}^2, h_{13}^2\}P)$. Due to the half-duplex constraint, time is valuable to both nodes, which makes them selfish and unwilling to help each other. Careful consideration, however, reveals that such *greedy* approach will lead the nodes to cooperate. The enabling observation stems from the feedback strategy proposed for the relay channel in which the destination was found to get higher achievable rate if it sacrifices some of its receiving time to help relay. Motivated by this observation, our strategy decomposes into three stages, without loss of generality we assume $h_{12}^2 > h_{13}^2$, 1) m_1 lasting for a fraction αt of the frame during which both receivers listen to node-1; 2) m_3 occupying $(1-\alpha)t$ fraction of the frame during which node-3 sends its compressed signal to node-2; and 3) m_2 (the rest $1-t$ fraction) during which node-1 and 2 help node-3 finish decoding. We observe that the last stage of cooperation, in which node-2 is helping node-3, is still motivated by the greedy approach. The idea is that node-1 will continue transmitting the same codeword until both receivers can successfully decode. It is, therefore, beneficial for node-2 to help node-3 in decoding faster to allow the source to move on to the next packet.

Lemma 4.1 *The achievable rate of the greedy strategy based multicast scheme is given by*

$$R_g = \max_{\alpha,t,P_i^{(j)}} \min\Bigg\{ \quad \alpha t C\Big((\frac{h_{13}^2}{1+\sigma_4^2} + h_{12}^2)P \Big) + (1-\alpha)tC\Big(h_{12}^2 P_1^{(3)}\Big); \qquad (4)$$
$$\alpha t C\Big(h_{13}^2 P\Big) + (1-t)C\Big(h_{13}^2 P_1^{(2)} + 2r_{12}h_{13}h_{23}\sqrt{P_1^{(2)}P_2^{(2)}} + h_{23}^2 P_2^{(2)}\Big)\Bigg\}$$

Here $\sigma_4^2 = \frac{\left(h_{12}^2+h_{13}^2\right)P+1}{\left(h_{12}^2P+1\right)\left(\left(1+\frac{h_{23}^2P_3^{(3)}}{h_{12}^2P_1^{(3)}+1}\right)^{\frac{1-\alpha}{\alpha}}-1\right)}$, and r_{12} denotes the correlation between X_1, X_2 during state m_2. The max operator is taken over the total power constraint. Note that if we set $\alpha = 1$, this greedy strategy becomes DF scheme, in which only the node with better channel condition helps the node with worse channel. Next, we examine the asymptotic behaviors of the greedy strategy as a function of the channel coefficients and available power. In low SNR

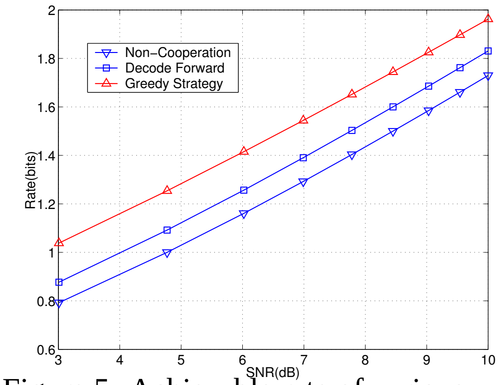

Figure 5: Achievable rate of various multicast schemes

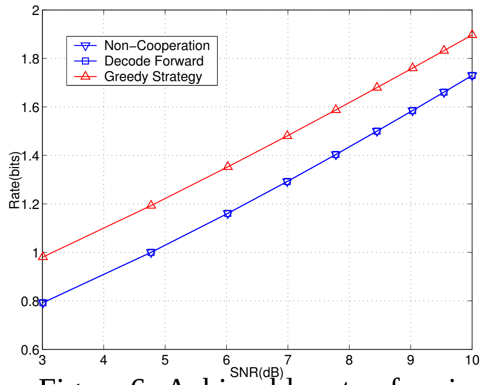

Figure 6: Achievable rate of various multicast schemes.

region, we study the slope S of the achievable rate with respect to P($R \sim \frac{1}{2}(\log e)SP$). In high SNR region, we characterize $R \sim \frac{1}{2}\log P + \frac{1}{2}G$ as $P \to \infty$.

Theorem 4.2 *1) The greedy cooperative multicast scheme strictly increases the multicast achievable rate (as compared to the non-cooperative scenario). 2) The greedy strategy approaches the beam-forming benchmark as h_{12} increases, it also approaches the multi-receiver benchmark as h_{23} increases. 3) As $P \to 0$, the slope of rate decay is given by $S_g = h_{12}^2(h_{23}^2 + h_{13}^2)/(h_{12}^2 + h_{23}^2)$. As $P \to \infty$, the SNR gain $G_g = G_{non-coop} = \log h_{13}^2$ with $t_{opt} \to 1$.*

Part 2) of this theorem demonstrates the asymptotic optimality of the greedy multicast as the channel gains increase. On the other hand, the gain of receiver cooperation in the multicast channel disappears as P increases. This is because, due to the half-duplex constraint, at least one receiver must cut its listening time in any cooperative multicast scheme . Such a reduction induces a pre-log penalty in the rate, which results in substantial loss that cannot be compensated by the cooperation gain as $P \to \infty$, and hence, the greedy strategy reduces to the non-cooperative mode automatically.

Fig. 5 compares the achievable rates of the various multicast schemes, when $h_{12} = 0.4$ dB, $h_{13} = 0$ dB, and $h_{23} = 23$ dB. We can see that the DF cooperation strategy outperforms the non-cooperation scheme. It is also shown that optimizing the parameter α provides an additional gain . Fig. 6 reports the achievable rate of the three schemes when $h_{12} = h_{13} = 0$ dB, $h_{23} = 23$ dB. In this case, it is easy to see that DF strategy yields **exactly** the same performance as the non-cooperative strategy. On the other hand, as illustrated in the figure, the proposed greedy strategy is still able to offer a sizable gain.

5 Three Way Channel

Arguably the most challenging instantiation of the three-node network is the three way channel. Here, the three nodes are assumed to observe correlated data streams and every node is interested in communicating its observations to the other two nodes. In a first step to understand this channel, one is naturally led to applying cut-set arguments to obtain a lower bound on the necessary bandwidth expansion factor. To exchange the information with the other two nodes, every node needs to transmit its message to the other two nodes and receive messages from them. Due to the half duplex constraint, these two tasks can not be completed simultaneously. Take node-1 as an example, we assume that node-2 and node-3 can fully cooperate, from a joint source-channel coding perspective, which converts the problem into point to point situation. Suppose node-1 transmits using N_t channel uses and receives using N_r channel uses, then in order to decode S_1^K at node-$2, 3$, and decode S_2^K, S_3^K at node-1 with a vanishingly small error probability, the following conditions should be satisfied,

$$KH(S_1|S_2,S_3)/N_t \le C((h_{12}^2 + h_{13}^2)P), \; KH(S_2,S_3|S_1)/N_r \le C((h_{12}^2 + h_{13}^2)P).$$

These two genie-aided bounds at node-1 imply that the minimum bandwidth expansion factor required for node-1 is $\tau_{1,gen} = \frac{H(S_1|S_2,S_3)+H(S_2,S_3|S_1)}{C((h_{12}^2+h_{13}^2)P)}$. Similarly, we can obtain the corresponding genie-aided bounds for node-2 and node-3. To satisfy the requirement for all these three nodes, the minimum bandwidth expansion factor for this three-node half-duplex network is

$$\tau_{gen} \geq \max_{i=1,2,3} \tau_{i,gen}. \tag{5}$$

Here, we remark that it is not clear whether the genie-aided bound in (5) is achievable. Moreover, finding the optimal cooperation strategy for the three way channel remains an elusive task. However, inspired by our greedy multicast strategy, we propose in the following a modular cooperation approach composed of three *cooperative multicast with side information* stages. In this scheme, each node takes a turn to multicast its information to the other two nodes. The multicast problem here is more challenging than the scenario has considered in Section 4 due to the presence of correlated, and different, side information at the two receive nodes. As argued in the following section, in order to fully exploit this side information, one must adopt a list source-channel decoding approach in every multicast stage. Furthermore, from one stage to the next, the side-information available at the different nodes changes accordingly. The overall performance depends on the efficiency of the scheduling algorithm.

At first, let's examine the multicast channel with side-information. Without sacrificing any generality, we assume that node-1 is the source and node-2 and 3 are provided with the side information S_2^K and S_3^K, respectively. An efficient non-cooperative solution to this problem utilizes a *nested binning* approach that combines the information required by the two receive nodes into a single hierarchical binning scheme. This scheme provides a benchmark for this problem. For the convenience of exposition, we assume that $H(S_1|S_2) > H(S_1|S_3)$. A source sequence s_1^K is randomly assigned to one of $2^{KH(S_1|S_2)}$ bins. These bin indices are then (randomly) divided into $2^{KH(S_1|S_3)}$ equal-sized groups. We index the group number with b, which is sufficient for node-3 to decode S_1^K with the aid of side-information S_3. We also identify the bin indices within each group with c. With the aid of side-information S_2, node-2 needs to know (b, c) in order to decode S_1^K successfully. This nested binning scheme permits the source node to send (b, c) to node-2 while only b to node-3. Such a structured message is called the *degraded information set* in [6], where b is the "common" information for both receivers and c is the "private" information required by node-2. Combining the capacity of broadcast channel with a degraded message set, which is given in [6], with our nested binning approach, we obtain the following result.

Lemma 5.1 *In multicast with side-information, the achievable bandwidth expansion factor $\tau = N/K$ based on nested binning and degraded information set broadcasting is given by*

$$\text{if } h_{12}^2 < h_{13}^2,\ H(S_1|S_2) \leq \tau C(h_{12}^2 P). \tag{6}$$

$$\text{if } h_{12}^2 > h_{13}^2,\ H(S_1|S_2) - H(S_1|S_3) \leq \tau C\left(\gamma h_{12}^2 P\right),\ \text{and } H(S_1|S_3) \leq \tau C\left(\frac{(1-\gamma)h_{13}^2 P}{1+\gamma h_{13}^2 P}\right),$$
$$\text{for some } \gamma. \tag{7}$$

Now, we describe our greedy list source-channel decoding approach. Similar to the multicast scenario, the receiver nodes follow a greedy strategy to determine the order of decoding. Due to the presence of side information, a more careful approach must be employed in choosing the *strong* receiver. In our scheme, each node calculates the expected bandwidth expansion factor assuming no receiver cooperation, $\tau_{ex,i} = N^i/K = H(S_1|S_i)/C(h_{1i}^2 P)$. The receiver node with the smaller τ_{ex} will decode first and is deemed as the *strong* node. Our definition of strong and weak highlights the list source-channel decoding approach proposed in this paper. Without loss of generality, we assume $\tau_{ex,2} < \tau_{ex,3}$. We randomly bin all the sequence S_1^K into

$2^{KH(S_1|S_2)}$ bins and denote the bin index $w \in [1, 2^{KH(S_1|S_2)}]$. We then independently generate another bin index b for every sequence s_1^K by picking b uniformly from $[1, 2^{KR}]$, where R is to be determined later. Let $B(b)$ be the set of all sequences S_1^K allocated to bin b. Thus, every source sequence has two bin indices $\{w, b\}$ associated with it. A full cooperation cycle is divided into three network stages. In the first two stages, node-1 sends the message w to node-2, while node-3 acts as relay for node-2 using CF strategy. More specifically, in first state m_1, which occupies αN_1 channel uses, both receiver nodes listen to the source node; then in state m_3, which occupies $(1-\alpha)N_1$ channel uses, node-3 sends a compressed version of received signal to node-2. At the end of this state, node-2 can get a reliable estimate $\hat{w} = w$ if $KH(S_1|S_2) \leq N_1 R_{CF2}(\alpha)$. Here $R_{CF2}(\alpha)$ is the achievable rate of the following relay channel: node-1 acts as the source, node-3 is the relay that spends $1-\alpha$ part of the time in helping destination using CF scheme, and node-2 is the destination [7, 8]. Next, node-2 searches in the bin specified by $\hat{w}$ for the one and only one $\hat{s}_{21}^K$ that is typical with s_2^K. If none exists, decoding error is declared, otherwise, $\hat{s}_{21}^K$ is the decoding sequence. During this stage, node-3 computes a list $\ell(\mathbf{y}_{3,m_1})$ such that if $w' \in \ell(\mathbf{y}_{3,m_1})$ then $\{\mathbf{x}_{1,m_1}(w'), \mathbf{y}_{3,m_1}\}$ are jointly typical. A key point of our scheme is that node-3 does not attempt to decode w, but rather proceeds to decoding s_1^K directly. After node-2 decodes S_1^K correctly, it knows the pair $\{w, b\}$, then in the state m_2, node-2 and node-1 cooperate to send the message b to node-3. At the end this stage, if N_0, R are appropriately chosen, node-3 can decode b correctly. Node-3 then searches in the bin $B(b)$ for the one and only one $\hat{s}_{31}^K$ that is jointly typical with s_3^K and that $f_{s1}(\hat{s}_{31}^K) \in \ell(\mathbf{y}_{3,m_1})$, and declares $\hat{s}_{31}^K$ as the source sequence.

Lemma 5.2 *Using proposed cooperative source-channel coding, if τ_0, τ_1 satisfy the following conditions, both node-$2,3$ will decode S_1^K with vanishingly small probability of error.*

$$\begin{aligned} H(S_1|S_2) &\leq \tau_1 R_{CF2}(\alpha), \\ H(S_1|S_3) - \frac{\alpha \min\{I(X_1;Y_3|m_1); I(X_1;\hat{Y}_3,Y_2|m_1)\} H(S_1|S_2)}{R_{CF2}(\alpha)} &\leq \tau_0 C_{(1,2)-3}. \end{aligned} \tag{8}$$

The symbol $\hat{Y}_3$ stands for the compressed receiver signal Y_3 at node-3 in the compress-forward scheme. In order to shed more light on the relative performance of the different schemes, we introduce the *minimum energy per source observation* metric. Given the total transmission power P, the bandwidth expansion τ translates to the energy requirement per source observation as $E(P) = \tau(P)P = \frac{N(P)P}{K}$. Let $E_1(P)$ denotes the energy per source symbol for the benchmark based on broadcast with degraded information set and $E_2(P)$ for the proposed cooperative multicast scheme. It is easy to see that both $E_1(P)$ and $E_2(P)$ are non-increasing function of P, and hence, approach their minimal values as $P \to 0$, that is

$$E_{i,m} = \lim_{P \to 0} E_i(P) \text{ for } i \in \{1,2\}. \tag{9}$$

Under the assumption that $\tau_{ex,2} < \tau_{ex,3}$, we have the following theorem.

Theorem 5.3 *1. Broadcast with degraded information set:*

$$\text{When } H(S_1|S_2) > H(S_1|S_3),\ E_{1,m} = \frac{2}{\log_2 e}\left(\frac{H(S_1|S_2)}{h_{12}^2} + \left(\frac{1}{h_{13}^2} - \frac{1}{h_{12}^2}\right)^+ H(S_1|S_3)\right).$$

$$\text{When } H(S_1|S_3) > H(S_1|S_2),\ E_{1,m} = \frac{2}{\log_2 e}\left(\frac{H(S_1|S_3)}{h_{13}^2} + \left(\frac{1}{h_{12}^2} - \frac{1}{h_{13}^2}\right)^+ H(S_1|S_2)\right).$$

2. Greedy strategy:

$$E_{2,m} = \frac{2}{\log_2 e}\left(\frac{H(S_1|S_2)}{h_{12}^2} + \frac{h_{12}^2 H(S_1|S_3) - \min\{h_{13}^2, h_{12}^2\} H(S_1|S_2)}{(h_{13}^2 + h_{23}^2) h_{12}^2}\right). \tag{10}$$

3. $E_{2,m} < E_{1,m}$.

Thus, when combined with the results in Section 4, this result argues strongly for receiver cooperation in the multicast scenario even under the stringent half-duplex and total-power constraint.

The second step in the proposed solution for the three way channel is the design of the scheduler. Given a specific multicast order, one can compute the overall bandwidth expansion by adding up the required τ of every multicast stage. The *optimal* scheduler will choose the multicast order corresponding to the minimum bandwidth expansion among all the possible multicast orders. The following result argues for the efficiency of our proposed cooperation scheme for the three way channel.

Theorem 5.4 *The cooperative source-channel coding based multicast scheme with the optimal scheduler has the following properties: 1) It is asymptotically optimal, i.e., approaches the genie-aided bound, when any one of the channel coefficients is sufficiently large. 2) It outperforms the broadcast with degraded set based multicast scheme with any scheduler.*

One can argue, however, that the optimal scheduler suffers from a high computation complexity since every node is required to compute the overall bandwidth expansion factor for the six possible scheduling alternatives. To reduce the computational complexity, one can adopt the following greedy strategy. At the beginning of every multicast stage, all the nodes that have not finished multicast will calculate its expected bandwidth expansion factor based on the cooperative scheme of multicast with side-information. The greedy scheduler chooses the node with the least expected bandwidth expansion to multicast first. After this node finishes and the side-information is updated, the scheduler computes the expected bandwidth expansion for the rest of nodes and selects the one with the least expansion to multicast next. In general, this greedy strategy based scheduler constitutes a potential source for further sub-optimality. However, it can approach the genie-aided bound in the asymptotic limit when one of the channel is sufficiently large. Take $h_{23} \to \infty$ as an example, in this case if one of the following conditions is satisfied, list source channel decoding multicast scheme with greedy scheduler approaches the genie-aided bound: 1) $H(S_2|S_1) < \min\{H(S_1|S_2), H(S_1|S_3), H(S_3|S_1)\}$, and $H(S_3|S_1, S_2) < H(S_1|S_2, S_3)$. 2) $H(S_3|S_1) < \min\{H(S_1|S_2), H(S_1|S_3), H(S_2|S_1)\}$, and $H(S_2|S_1, S_3) < H(S_1|S_2, S_3)$. One can easily check that if 1) is satisfied, the greedy scheduler will choose the $2 \to 3 \to 1$ as the multicast order. If 2) is satisfied, the greedy scheduler will choose $3 \to 2 \to 1$ as the multicast order. Both orders approach the genie-aided bound.

The numerical results in Figures 7 and 8 validate our claims on the efficiency of the proposed cooperation strategy. For each randomly generated channel gains and correlation patterns, we use numerical methods to find minimum energy required per source observation using various schemes. The minimum energy required per source observation by the genie-aided bound E_{gen} is used as a benchmark. In particular, for each realization, we calculate the ratio of the minimum energy required by each scheme to the genie aided bound. We repeat the experiment 100000 times and report the histogram of the ratios in the figures. In Fig. 7, we see that 94 percent of the time, the proposed scheme with the greedy scheduler operates within 3 dB of the genie-aided bound. We also see that the performance of greedy scheduler is almost identical to the optimal scheduler. Fig. 8 shows that the non-cooperative scheme operates 3 dB away from the genie-aided bound for 90 percent of the time. Moreover, there is a non-negligible probability, i.e., 8 percent, that this scheme operates 100 dB away from genie aided bound. It is clear that receiver cooperation reduces this probability significantly.

6 Conclusions

We have adopted a formulation of the three-node wireless network based on the half-duplex and total power constraints. We have highlighted the structural similarities of the many special cases that have been considered in the literature. Furthermore, we have proposed a greedy

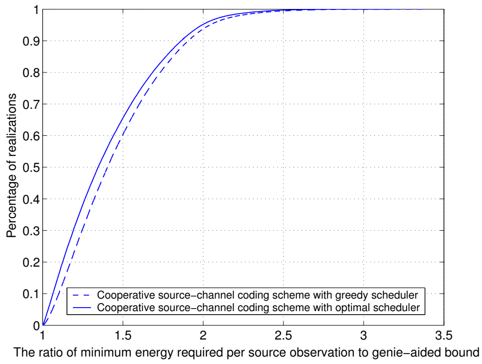

Figure 7: Cooperative source-channel based multicast with side-information.

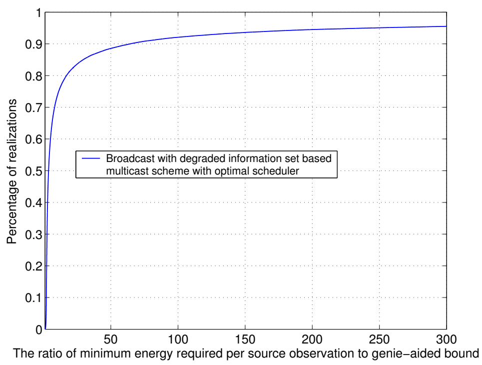

Figure 8: Broadcast with degraded information set based multicast with optimal scheduler.

cooperation strategy in which the *weak* receiver first helps the *strong* receiver to decode in a CF configuration. After successfully decoding, the strong user starts assisting the weak user in a DF configuration. We have shown that different instantiations of this strategy yield excellent performance in the relay channel with feedback, multicast channel, and three way channel. Our analysis for the achievable rates in such special cases sheds light on the value of feedback in relay channel and the need for a cooperative source-channel coding in the three way channel.

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