Fading Multiple Access Channels: A Game Theoretic Perspective

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***Abstract*— We adopt a game theoretic approach for the design and analysis of distributed resource allocation algorithms in fading multiple access channels, where the users are assumed to be selfish, rational and limited by average power constraints. We show that the sum-rate optimal point on the boundary of the multiple-access channel capacity region is the unique Nash Equilibrium of the corresponding water-filling game. The base-station is then introduced as a player interested in maximizing a weighted sum of the individual rates. We propose a Stackelberg formulation in which the base-station is the designated game leader. We show that this formulation allows for achieving all the corner points of the capacity region, in addition to the sum-rate optimal point. On the negative side, we prove the non-existence of a base-station strategy in this formulation that achieves the rest of the boundary points. To overcome this limitation, we present a repeated game approach which achieves the capacity region of the fading multiple access channel. Finally, we extend our study to vector channels highlighting interesting differences between this scenario and the scalar channel case.**

I. Introduction

The design and analysis of efficient resource allocation algorithms for wireless channels has received significant research interest for many years. In a pioneering work, Tse and Hanly have characterized the capacity region of the fading multiple access channel and the corresponding optimal power and rate allocation policies [2]. The centralized nature of these policies motivates our work here on the design and analysis of distributed allocation strategies that approach the optimal performance.

In this paper, we adopt a game theoretic framework where the users are typically modelled as rational and selfish players interested in maximizing the utilities they obtain from the network. The selfish behavior implies that individual users do not care about the overall system performance. Over the last ten years, game theoretic tools have been used to design distributed resource allocation strategies in a variety of contexts. For example, Yu *et al.* focus on the digital subscriber line setup [4], Etkin *et al.* investigate the power allocation game in the Gaussian interference channel [5]. Probably the scenario closest to our work is the design of distributed power control algorithms for the up-link of Code Division Multiple Access (CDMA) systems considered in e.g., [3]. These papers focus on **time-invariant** channels and construct utility functions that allow the users to reach a socially optimal equilibrium. These works, however, reach the **negative** conclusion that the selfish behavior entails a fundamental performance loss in the sense that the achievable utilities at the equilibria points, if they exist, are usually inefficient as compared with the centralized policy [3]. The central contribution of this paper is showing how to overcome this negative conclusion in fading channels by exploiting the time varying nature of fading, modelling the base-station as an additional player with the appropriate decoding strategy, and resorting to a repeated game formulation if needed.

Due to the space limitation, we only give the sketch of the proofs in this paper, interested readers can refer to [1] for details.

II. Background

We consider a discrete-time flat fading multiple access channel with N users and one base-station. The signal received by the base-station at time n is

$$y(n) = \sum_{i=1}^{N} \sqrt{h_i(n)} x_i(n) + z(n), \qquad (1)$$

where $x_i(n)$ and $h_i(n)$ are the transmitted signal and fading channel gain of the ith user at time n. Similar to [2], we assume the fading process to be jointly stationary and ergodic. We further assume that the stationary distribution has a continuous density and is bounded. User i has an average power constraint $\bar{P}_i$ and $z(n)$ is a sample of a zero-mean white Gaussian noise process with variance σ^2. In this paper, we consider time-varying channels where the CSI is available *a-priori* at all the transmitters and the receiver. This scenario was considered by Tse and Hanly [2] where they characterized the capacity region $\mathcal{G}_c$ along with the corresponding centralized power and rate allocation policies $(\mathcal{P}_c, \mathcal{R}_c)$. It was also shown in [2] that the power and rate allocation policies are unique and each boundary point corresponds to the maximization of a weighted sum of the individual rates. All the boundary points are achieved by successive decoding, where the decoding order is determined by the rate award vector $\boldsymbol{\mu}$ [2].

The capacity region for the two user case is shown in Figure 1. The corner point CR_1 is achieved by using the following policy: user 1 water-fills over the background noise level and user 2 water-fills over the sum of the interference from user 1 and the background noise. At the base-station user 2 is decoded first followed by user 1. At point CR_2, the roles of users 1 and 2 are reversed. Another boundary point of particular interest is the maximum sum-rate point SP. Unlike the AWGN Multiple Access Channel (MAC), this point is unique in our case and is achieved by a time-sharing policy where only one user is allowed to transmit at any fading state [2], [6]. This observation will prove instrumental to the development of the main result in Section III-A.

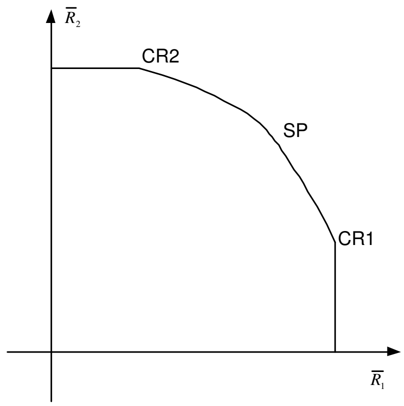

Fig. 1. The capacity region of the two user fading multiple access channel.

III. The Water-Filling Game

For simplicity of presentation, we only consider in details the two user scenario. We also discuss the extensions to the N users channel and vector channel briefly.

A. Nash Formulation

Here, we consider a static non-cooperative game where the players are the multiple-access users. In this game, the strategy of user i is the power control policy $\mathcal{P}_i$ and rate control policy $\mathcal{R}_i$. The corresponding payoff function is defined as the average achievable rate $\bar{R}_i = E_{\mathbf{h}}[\mathcal{R}_i]$ with $\mathbf{h} = [h_1, h_2]^T$. The goal of user i is to

$$\max_{\mathcal{P}_i} \bar{R}_i(\mathcal{P}_i, \mathcal{P}_{-i}) \text{ s.t. } \mathcal{P}_i \in \mathcal{F}_i, \tag{2}$$

where $\mathcal{F}_i = \{\mathcal{P}_i : E_{\mathbf{h}}[\mathcal{P}_i] \leq \bar{P}_i, \mathcal{P}_i(\mathbf{h}) \geq 0\}$ is the set of all feasible power control policies of user i, and $\mathcal{P}_{-i}$ represents the power control policy of the other user (in the more general $\mathcal{P}_{-i}$ refers to the strategies of all users except user i). Since the base-station is not a player of the game, we assume that each user will treat the signal of the other user as interference. Given the power control policy $\mathcal{P}_2(h_1, h_2)$ of user 2, the payoff of user 1 is given by

$$\bar{R}_1 = \int\int \frac{1}{2}\log\left(1 + \frac{\mathcal{P}_1(h_1,h_2)h_1}{\sigma^2 + \mathcal{P}_2(h_1,h_2)h_2}\right) f(h_1,h_2)dh_1dh_2.$$

Here $f(h_1, h_2)$ is the joint probability density function of the two fading coefficients. The payoff function of user 2 is defined similarly. As we can see the payoff function of each user depends on the two power control policies ($\mathcal{P}_1$, $\mathcal{P}_2$). In this paper, all the $\log$ are based on 2.

Definition 1: A Nash equilibrium is a policy pair $(\mathcal{P}_1^*, \mathcal{P}_2^*)$ such that

$$\begin{aligned} \bar{R}_1(\mathcal{P}_1^*, \mathcal{P}_2^*) &\geq \bar{R}_1(\mathcal{P}_1^{'}, \mathcal{P}_2^*), \quad \forall \mathcal{P}_1^{'} \in \mathcal{F}_1, \\ \bar{R}_2(\mathcal{P}_1^*, \mathcal{P}_2^*) &\geq \bar{R}_2(\mathcal{P}_1^*, \mathcal{P}_2^{'}), \quad \forall \mathcal{P}_2^{'} \in \mathcal{F}_2. \end{aligned} \tag{3}$$

Given a fixed power control policy of user 2, the optimal strategy $\mathcal{P}_1(h_1, h_2)$ of user 1 is the solution to the following optimization problem

$$\begin{aligned} &\bar{R}_1 = \max_{\mathcal{P}_1} E_{\mathbf{h}}\left[\frac{1}{2}\log\left(1 + \frac{\mathcal{P}_1(h_1,h_2)h_1}{\sigma^2 + \mathcal{P}_2(h_1,h_2)h_2}\right)\right], \\ &\text{s.t.} \int\int \mathcal{P}_1(h_1,h_2)f(h_1,h_2)dh_1dh_2 \leq \bar{P}_1, \\ &\mathcal{P}_1(h_1,h_2) \geq 0. \end{aligned} \tag{4}$$

It is easy to verify that the objective function in (4) is concave, the constraint set is convex, the Slater's condition is satisfied, and hence, the solution to this problem is the well-known water-filling power allocation, i.e.,

$$\mathcal{P}_1(h_1,h_2) = \left(\lambda_1 - \frac{\sigma^2}{h_1} - \frac{\mathcal{P}_2(h_1,h_2)h_2}{h_1}\right)^+, \tag{5}$$

in which $(x)^+ = \max\{x, 0\}$ and λ_1 is the power level that satisfies

$$\int\int \left(\lambda_1 - \frac{\sigma^2}{h_1} - \frac{\mathcal{P}_2(h_1,h_2)h_2}{h_1}\right)^+ f(h_1,h_2)dh_1dh_2 = \bar{P}_1.$$

Similarly the optimal policy of user 2, given a fixed policy for user 1, is given by

$$\mathcal{P}_2(h_1,h_2) = \left(\lambda_2 - \frac{\sigma^2}{h_2} - \frac{\mathcal{P}_1(h_1,h_2)h_1}{h_2}\right)^+. \tag{6}$$

From these expressions, one can see that the optimal policy of each user depends largely on its *guess* of the other user policy. Based on this guess, each user will determine its policy and adjusts its water-filling level to maximize its own average rate. At the Nash equilibrium, the water-filling pair (λ_1, λ_2) satisfies the two average power constraints with equality. Now we are ready to present our first result.

Theorem 1: The maximum sum-rate point SP of the capacity region $\mathcal{G}_c$ is the unique Nash equilibrium of our water-filling game.

Proof: (Sketch) At first, we show that there only exist time-sharing Nash equilibria. Then we show that at the time sharing equilibrium, the power control policy of each user is the same at the optimal centralized control policy corresponding to the SP point, which is also shown to be unique in [2]. ■

Two comments are now in order.

1) Theorem 1 establishes the remarkable fact that the selfish behavior of the users will lead them to **jointly** optimize the sum-rate of the channel. In fact, this result provides a new interpretation of the opportunistic communication principle [6]. At any particular instance, the user with the strongest channel sees a relatively weak interference from the other user, and hence, decides to transmit with a high power level. On the other hand, the other user sees a strong interferer in addition to a weak channel, and hence, decides to conserve the power for later usage. This way, they reach the *opportunistic* time sharing equilibrium distributively. This result also establishes a certain game-theoretic fairness of the point SP. The underlying idea is that the selfishness of the different users will *balance-out* at the sum-rate optimal

point. To impose other fairness criteria, the base-station must be involved in the game as argued in the next section.

2) Theorem 1 contrasts the negative conclusions drawn in earlier works on the efficiency of game theoretic approaches in CDMA up-link power control (e.g., [3]). The enabling vehicle behind this result is the time varying nature of the fading channel. With this temporal variations, the CSI (available at all transmitter) acts like a common randomness that allows the users to reach a more efficient equilibrium based on a selfish rationale. This is yet another manifestation of the positive impact that fading, if properly exploited, can have on certain aspects of wireless systems.

B. Stackelberg Formulation

In the previous section, we have shown that the only boundary point achievable by our Nash game is the optimal sum-rate point. One can attribute this limitation to the assumption that every user (player) will treat the other user's signal as noise. While this assumption does not entail a loss at the *time sharing* point SP, it does not allow for achieving other boundary points. Such points require the base-station to employ a more sophisticated decoding rule. In [2], it was shown that successive decoding, with the appropriate ordering, is sufficient to achieve all the boundary points. This observation motivates a game theoretic formulation where the base-station is introduced as an additional player. The base-station strategy corresponds to a particular choice of the decoding order, as detailed next. We wish to stress that, unlike the centralized scenario [2], the base-station in our formulation does not dictate the power level and rate of the individual users. Still, it is reasonable to assume that the roles of the base-station and multiple-access users are not totally symmetric. Therefore, we do not model the base-station as an *ordinary* player in our game but rather appeal to the bi-level programming notion [7]. In our context, bi-level programming corresponds to a Stackelberg game [7] [9], where the leader announces its strategy first and then the remaining players react according to a specific equilibrium concept among them. Here, we designate the base-station as the game leader, and hence, it will announce its decoding strategy in the first level of the game. This way, the base-station can rely on the rational and selfish nature of the multiple access players to *influence* their behavior in the second stage (i.e., low level game).

In this work, we consider a class of successive decoding strategies parameterized by the decoding order as a function of the fading gains (h_1, h_2). More precisely, the base-station divides the whole possible space of (h_1, h_2) into two subsets D_1, D_1^c. When $(h_1, h_2) \in D_1$, the base-station will decode user 1's information first whereas $(h_1, h_2) \in D_1^c$ implies decoding user 2's signal first. After the base-station announces its strategy, i.e., D_1, the multiple access users play the low level game using the Nash equilibrium concept. The strategy space of user i is still $\mathcal{F}_i$, and the payoff function of user i is defined as the supremum of the achievable rate. Here supremum refers to the fact that in the rate expressions to follow we always assume the users to be decoded successfully (which is a critical assumption in the successive decoding approach). We will show later that, at the Nash equilibrium this condition indeed holds. Hence, the supremum corresponds exactly to the achieved payoff. With a slight abuse of notation, the payoff function of each user is written as

$$\begin{aligned}
&\bar{R}_1(D_1, \mathcal{P}_1, \mathcal{P}_2) = \\
&E_{\mathbf{h}}\left[\frac{1}{2}\log\left(1 + \frac{\mathcal{P}_1(h_1,h_2)h_1}{\sigma^2 + \mathcal{P}_2(h_1,h_2)h_2 I_{\{(h_1,h_2)\in D_1\}}}\right)\right], \\
&\bar{R}_2(D_1, \mathcal{P}_1, \mathcal{P}_2) = \\
&E_{\mathbf{h}}\left[\frac{1}{2}\log\left(1 + \frac{\mathcal{P}_2(h_1,h_2)h_2}{\sigma^2 + \mathcal{P}_1(h_1,h_2)h_1 I_{\{(h_1,h_2)\in D_1^c\}}}\right)\right].
\end{aligned}$$

Here $I_{\{\cdot\}}$ is the indication function. The payoff function of the base-station is defined as

$$\mu_1 \bar{R}_1(D_1, \mathcal{P}_1, \mathcal{P}_2) + \mu_2 \bar{R}_2(D_1, \mathcal{P}_1, \mathcal{P}_2). \tag{7}$$

This payoff function has a natural economical interpretation as the revenue of the base-station where μ_i can be viewed as the payment that user i owes per unit rate. The value of μ_i can be decided using an auction process [10], where each user submits its proposed payment μ_i to the base-station in order to maximize its own utility. In this work, we do not consider this auction process and assume that $\boldsymbol{\mu} = [\mu_1, \mu_2]^T$ is given.

We first study the properties of the low level game. The Nash equilibrium under a fixed base-station strategy D_1 is a power control pair $(\mathcal{P}_1^*, \mathcal{P}_2^*)$ that satisfies

$$\begin{aligned}
\bar{R}_1(D_1, \mathcal{P}_1^*, \mathcal{P}_2^*) &\geq \bar{R}_1(D_1, \mathcal{P}_1', \mathcal{P}_2^*),\ \forall \mathcal{P}_1' \in \mathcal{F}_1, \\
\bar{R}_2(D_1, \mathcal{P}_1^*, \mathcal{P}_2^*) &\geq \bar{R}_2(D_1, \mathcal{P}_1^*, \mathcal{P}_2'),\ \forall \mathcal{P}_2' \in \mathcal{F}_2.
\end{aligned}$$

For any given power control policy $\mathcal{P}_2$, the optimal power control policy of user 1 is the solution to the following optimization problem

$$\begin{aligned}
&\max_{\mathcal{P}_1} \quad \bar{R}_1(D_1, \mathcal{P}_1, \mathcal{P}_2) = \\
&E_{\mathbf{h}}\left[\frac{1}{2}\log\left(1 + \frac{\mathcal{P}_1(h_1,h_2)h_1}{\sigma^2 + \mathcal{P}_2(h_1,h_2)h_2 I_{\{(h_1,h_2)\in D_1\}}}\right)\right], \\
&s.t. \int\int \mathcal{P}_1(h_1,h_2) f(h_1,h_2) dh_1 dh_2 \leq \bar{P}_1, \\
&\mathcal{P}_1(h_1,h_2) \geq 0.
\end{aligned} \tag{8}$$

The optimal power control policy of user 2 is also the solution to a similar optimization problem for any power control policy of user 1. For a given D_1, the solution set for this low level game is written as $S(D_1) = \{(\mathcal{P}_1, \mathcal{P}_2) : (\mathcal{P}_1, \mathcal{P}_2)$ is a Nash equilibrium of the low level game$\}$. The following result characterizes the pure-strategy Nash equilibria of our low level game.

Theorem 2: For any strategy D_1 of the base-station, and any channel distribution, there exist Nash equilibria for the low level distributed power/rate control game.

Proof: (Sketch) The solution to the optimization problem (8) is water-filling with water-filling level λ_i. For any (h_1, h_2), we can write down the power of each user as a function of λ_i. Using the average power constraints of both users, we get two functions that the water-filling level pair (λ_1, λ_2) has to satisfy. Following the same line as [4], we prove that there exist solutions to these functions. Hence there exist Nash equilibria. ■

Theorem 2 only establishes the existence of a Nash equilibrium, but it tells nothing about the uniqueness of this equilibrium. To prove uniqueness, one is typically forced to find a contraction mapping whose fixed point is the Nash equilibrium. In [4], [5], the authors apply this method to the interference game and find that uniqueness requires very restrictive conditions. Fortunately, we are able to prove uniqueness in our setup by using the concept of **admissible** Nash equilibrium [9].

Definition 2: A Nash equilibrium strategy pair $(\mathcal{P}_1^*, \mathcal{P}_2^*)$ is said to be admissible if there exists no other Nash equilibrium strategy pair $(\mathcal{P}_1^{'}, \mathcal{P}_2^{'})$ such that $\bar{R}_1(D_1, \mathcal{P}_1^{'}, \mathcal{P}_2^{'}) \geq \bar{R}_1(D_1, \mathcal{P}_1^*, \mathcal{P}_2^*)$, $\bar{R}_2(D_1, \mathcal{P}_1^{'}, \mathcal{P}_2^{'}) \geq \bar{R}_2(D_1, \mathcal{P}_1^*, \mathcal{P}_2^*)$ and at least one of these equalities is strict.

This notion allows for eliminating Nash equilibria which are dominated by other equilibrium points [9]. This approach allows for modifying the solution set for our low level game to only include admissible Nash equilibria $S^*(D_1) = \{(\mathcal{P}_1, \mathcal{P}_2) :$ $(\mathcal{P}_1, \mathcal{P}_2)$ is an admissible Nash equilibrium of low level game$\}$.

Theorem 3: For any strategy D_1 of the base-station, and any channel distribution function, there exists a **single** admissible Nash equilibrium for the low level power/rate allocation game (i.e., for any D_1, $S^*(D_1)$ is a singleton).

Proof: (Sketch) We first prove that the water-filling level pairs at the equilibria have strict order. That is if $(\lambda_1^*, \lambda_2^*)$ and $(\lambda_1^{'}, \lambda_2^{'})$ are two water-filling pairs at the equilibrium, we have $\lambda_1^* > \lambda_1^{'}$ if $\lambda_2^* > \lambda_2^{'}$, $\lambda_1^* = \lambda_1^{'}$ if $\lambda_2^* = \lambda_2^{'}$, $\lambda_1^* < \lambda_1^{'}$ if $\lambda_2^* < \lambda_2^{'}$. Then we show that the achievable rate at the equilibria also have strict order. The only admissible Nash equilibrium is the equilibrium with the smallest water-filling level. ■

Now, we turn our attention to characterizing efficient base-station strategies. In the following we use $\mathcal{P}_{iD_1}$ to refer to the unique power control policy of each user, under strategy D_1, at the admissible Nash equilibrium.

Definition 3: A strategy D_1^* is called a Stackelberg equilibrium strategy for a given (μ_1, μ_2), if

$$\begin{aligned} R^* &= \mu_1 \bar{R}_1(D_1^*, \mathcal{P}_{1D_1^*}, \mathcal{P}_{2D_1^*}) + \mu_2 \bar{R}_2(D_1^*, \mathcal{P}_{1D_1^*}, \mathcal{P}_{2D_1^*}) \\ &\geq \mu_1 \bar{R}_1(D_1, \mathcal{P}_{1D_1}, \mathcal{P}_{2D_1}) + \mu_2 \bar{R}_2(D_1, \mathcal{P}_{1D_1}, \mathcal{P}_{2D_1}), \end{aligned}$$

for all D_1. Moreover, for any $\epsilon > 0$, a strategy $D_{1,\epsilon}^*$ is called an ϵ-Stackelberg strategy if

$$\mu_1 \bar{R}_1(D_{1,\epsilon}^*, \mathcal{P}_{1D_{1,\epsilon}^*}, \mathcal{P}_{2D_{1,\epsilon}^*}) + \mu_2 \bar{R}_2(D_{1,\epsilon}^*, \mathcal{P}_{1D_{1,\epsilon}^*}, \mathcal{P}_{2D_{1,\epsilon}^*}) \geq R^* - \epsilon.$$

Corollary 1: For every pair $(\mu_1, \mu_2), 0 \leq \mu_1 < \infty, 0 \leq \mu_2 < \infty$, an ϵ-Stackelberg strategy exists.

Proof: (Sketch) We prove that R^* is bounded. Based on Property 4.2 of [9], the claim is proved. ■

Combining Theorem 3 and Corollary 1, we see that the proposed Stackelberg game setup has a very desirable structure. For any given vector $\boldsymbol{\mu}$, the existence of equilibrium is guaranteed and the optimal policy for every rational multiple access user in the low level game is unique. Therefore, the users will have no difficulty in deciding the power and rate levels in a distributed way.

Theorem 4: Let

$$\mathcal{G}_s = \bigcup_{D_1} \{(\bar{R}_1(D_1, \mathcal{P}_{1D_1}, \mathcal{P}_{2D_1}), \bar{R}_2(D_1, \mathcal{P}_{1D_1}, \mathcal{P}_{2D_1}))\}.$$

Then, $\mathcal{G}_s$ includes the three boundary points CR_1, CR_2, SP of the capacity region $\mathcal{G}_c$. However, $\mathcal{G}_s$ does not include any other boundary points of $\mathcal{G}_c$.

Proof: (Sketch) CR_1 can be achieved by setting $D_1 = \phi$, where the base-station will always decode user 2's signal first. In this case, the optimal power control policy of user 1 is to do water-filling over noise and the optimal power control policy of user 2 is to do water-filling over the sum of noise and the interference from user 1. This policy is the same as the centralized power control policy that achieves CR_1. Similarly, CR_2 can be achieved by setting $D_1^c = \phi$. SP can be achieved by setting D_1 as the region given in Theorem 2.

Suppose the region partition D_b with corresponding admissible power control pair $(\mathcal{P}_{1D_b}, \mathcal{P}_{2D_b})$ achieves another boundary point. Since the optimal power control policy that achieves any boundary point is unique, this admissible power control pair is the same as the centralized power control pair. Then we can show that the rate of user 1 at the equilibrium is less than the rate of user 1 with centralized control, if $\lambda_1 > \lambda_2$. If $\lambda_2 > \lambda_1$, the rate of user 2 in the equilibrium is less than the rate of user 2 with centralized control. Hence we have a contradiction. The theorem is proved. ■

The set $\mathcal{G}_s$ is shown on Figure 2. We see that introducing the base-station into the game allows us to achieve all the corner points and the maximum sum-rate point of the capacity region.

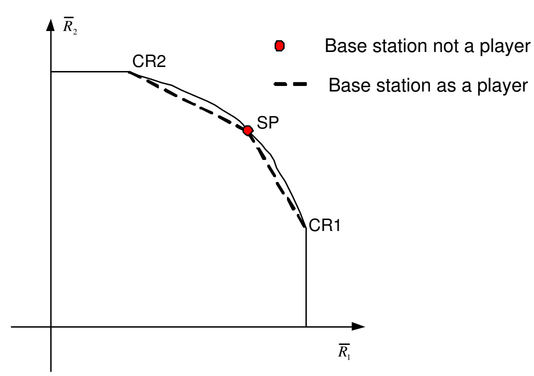

Fig. 2. The equilibria points of the Stackelberg power/rate allocation game.

C. Repeated Game Formulation

The inability of our Stackelberg game to achieve all the boundary points of the capacity region can be attributed to the structural difference between our successive decoding strategy and the optimal decoding strategy characterized in [2]. In particular, the optimal decoding strategy will always decode user 1 first (i.e., for all channel states) if $\mu_1 < \mu_2$, whereas in our formulation the decoding order is a function of the channel state. Unfortunately, if we adopt any *static* decoding order, the game will always settle at one of the corner points of the capacity region as argued in the previous section. To solve this problem, we pursue our last resort of replacing the static game formulation with a dynamic one.

The static formulation assumes that the players interact with each other only once. This assumption models the case where the topology of the network changes quickly. In a more slowly varying environments, a dynamic game formulation seems more appropriate. Specifically, we call a game where the players interact for $T > 1$ instances a dynamic game (We note that every game stage is assumed long enough to justify invoking the ergodic assumption within every stage.). An example of a dynamic game is the repeated game where the same static game is played many times. Obviously, the users can play this game by repeating the same static strategy [8]. But, the advantage of the repeated game framework is that the players can do better than just repeating the same static strategy. Since the players will interact with each other many times, they can learn each other's strategies, which may allow them to cooperate to obtain higher payoffs. In this case, the players can start cooperating and if one player deviates from the cooperation phase, the other players will adjust their strategies to punish the deviating player. The punishment *threat* is credible only if the deviating player achieves a lower payoff under punishment as compared with the cooperating phase. Under these circumstances, the users will have no desire to deviate from the cooperation phase, thus all the users can achieve higher utilities as compared to the static scenario.

In the repeated game, the utility of each player can be defined as as a discounted sum of the payoff achieved in each stage. We refer to the discount factor by δ, where $0 < \delta < 1$. The larger δ is the more patient the player is. In the proof of the following theorem, we use a generalized version of a result due to Aumann and Shapley [8] and define the payoff of the repeated game as the time-average of payoff at each stage.

Theorem 5: As $T \to \infty$, all the boundary points of the capacity region are achievable under the repeated game setup with the base-station as the game leader. Moreover, the corresponding equilibria are subgame perfect.

Proof: (Sketch) For any boundary point, the base station first announces its rate award vector $\boldsymbol{\mu}$. The users start with optimal centralized control policy $\mathcal{P}_c$ that maximize $\sum \mu_i \bar{R}_i$. If user 1 deviates from this policy, the base-station then punishes it by putting it in CR_2 for T_1 period. Similarly, if user 2 deviates, the base-station can punish it by putting it in CR_1 for T_2 period. T_i is appropriately chosen to guarantee that the gain user i obtains by deviating the centralized policy is removed at the punishment phase. No sequence of a finite or infinite number of deviations can increase user i's policy. ■

D. Extensions

We can generalize our results to N transmitters case, except for the uniqueness of the admissible Nash equilibrium. However, if the multiple-access users choose the Nash equilibrium corresponding to the iterative algorithm used in the proof with $\boldsymbol{\lambda} = \mathbf{0}$, the rest of the results hold. For detail, please refer to [1].

We can also generalize our formulations to the case where the base-station is equipped with N_r receive antennas. We show that in the Nash formulation, the power control policy for each user at the equilibrium is the same as the central control policy that achieves the maximum sum-rate point SP. The achieved rates, however, are strictly smaller than the rates corresponding to SP. This contrasts with the scalar scenario, since in the vector case, we have $\min(N, N_r)$ degrees of freedom. Hence at the SP point, the optimal scheme that achieves this point is not time-sharing anymore. However, the results in the Stackelberg game formulation and the repeated game formulation carry over to vector channel. Please refer to [1] for detail.

IV. Conclusions

This paper has developed a game theoretic framework for distributed resource allocation in fading multiple access channels. In our first result, we showed that the opportunistic communications principle can be obtained as the unique Nash equilibrium of a water-filling game. By introducing the base-station as a player, we were able to achieve all the corner points of the capacity region, in addition to the sum-rate optimal point, distributively. In slow varying environments, where the multiple access users can be assumed to interact many times, the repeated game formulation was shown to achieve all the boundary points of the capacity region. Finally, we elucidated the limitations of our game theoretic framework in vector multiple access channels.

An interesting avenue for future work is to further investigate the practical aspects of our framework. For example, a natural extension is to consider the case with partial and/or distorted channel state information by borrowing tools from game theory with incomplete information.

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