Wigner Crystals
Confined in Micrometer-wide Channels

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1. Introduction

2. Device

3. Nonlinear transports of a Wigner crystal
   - Bragg-Cherenkov scattering of ripplons
   - decoupling of a Wigner crystal from the dimple lattice

4. Melting of a Wigner crystal in quasi-1D geometry
   - 1D-2D crossover

5. 1.6 μm channel

6. Summary
**Wigner Crystal**

**Electrons on helium surface**

very low density  
density: $10^{11} \sim 10^{13}$ m$^{-2}$  
electron spacing: 0.1 ~ 1 μm

Coulomb repulsion $>>$ kinetic energy  
($\sim 93$ K @ $1 \times 10^{13}$ m$^{-2}$)

Wigner Crystal coupled with Dimple Lattice

Unique transport properties of the Wigner crystal

- **Nonlinear transports of a Wigner crystal on superfluid $^4$He**  
  - Bragg-Cherenkov scattering of ripplons  
  - decoupling of a Wigner crystal from the dimple lattice

- **Transports of a Wigner crystal on superfluid $^3$He**  
  - resistance caused by the quasiparticle scattering  
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Electrode

- $W = 5, 8, 15 \mu m$
- Upper electrode (5 \mu m)
- Lower electrode
- Source
- Drain
- Center channel
- Reservoirs
Electrode

Equivalent Circuit

Reservoir
Source gate drain

SiO₂
Guard
Drain gate source
Simulation of Electrode

96% of the resistive component comes from the center-channel.
Simulation of Electrode

**distribution of $I$ and $E_{||}$ along the channel**

$I/I_{\text{max}}$, $E_{||}/E_{||\text{max}}$

$\sim 5\%$

8 $\mu$m channel

$I$ and $E_{||}$ are homogeneous along the channel
Previous Experiments with Corbino electrode

Current distribution in Corbino geometry

In nonlinear regime, distribution of $E$ smears the features.

Bragg-Cherenkov scattering
A. Kirstensen et al, PRL 77, 1350 (‘96).

Decoupling transition
K. Shirahama and K. Kono, PRL 74, 781 (‘95).
Outline

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Temperature Dependence of Resistance

8 \mu m channel

Solid line: Theory, M. Saitoh JPSJ 42, 201 (‘77).

Wigner crystal transition

Strong nonlinear behavior in the Wigner crystal phase
Results: Nonlinear Behavior in Wigner Crystal Phase

- Electron velocity shows the saturation at low excitations
- Velocity jumps to a high value at large excitations

H. Ikegami, et al., PRL 102, 046807 ('09)
**Bragg-Cherenkov Scattering**

**Experiment:** Kristensen et al, PRL 77, 1350 (‘96).
**Theory:** Dykman and Rubo, PRL 78, 4813 (‘97).

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**Cherenkov emission of ripplon**

\[ \mathbf{v} \cdot \mathbf{k} = \omega(\mathbf{k}) \]

**Bragg condition**

\[ \mathbf{k} = \mathbf{G} \]

**constructive interference**

\[ \mathbf{v} \cdot \mathbf{G} = \omega(\mathbf{G}) \]

- **v:** velocity of electron
- **k:** wave number of ripplon
- **\( \omega \):** energy of ripplon
- **G:** reciprocal lattice vector of Wigner crystal

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**Ripplon phase velocity**

\[ v_1 = \frac{\omega(\mathbf{G}_1)}{|\mathbf{G}_1|} \approx 6.9 \text{ m/s} \]

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**Velocity of electrons is limited by the phase-velocity of ripplon**
**Dimple becomes deeper**
Decoupling of a Wigner Crystal from Dimple Lattice

Decoupling occurs from the Bragg-Cherenkov state.
Decoupling of a Wigner Crystal from Dimple Lattice

Decoupling from rigid dimple lattice

K. Shirahama and K. Kono, PRL 1995

Decoupling occurs when the local minimum disappears.

Decoupling from deepened dimple

$E_{\parallel}^{th}$
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Melting of a Wigner Crystal

In 2D system:

Kosterlitz-Thouless- Halperin-Nelson-Young (KTHNY) mechanism

Melting occurs at plasma parameter $\Gamma = U/K \sim 130$

$U$: Coulomb interaction
$K = k_B T$: kinetic energy

unbinding of dislocation pairs

Motivation

How the melting proceeds in the quasi-1D geometry?

Nonlinear transport measurements of electrons in quasi-1D channels

In 1D system:

short range order at finite temperature
Temperature Dependence of Mobility

For a narrower channel, $\mu^{-1}$ starts to increase at a higher temperature.
Excitation Dependence of $\mu^{-1}$ and $v$

The BC scattering disappears:

$T_0 = 0.62 \pm 0.02$ K

periodic structure

Bragg-Cherenkov scattering
Density Dependence of $T_0$

Deviation from the 2D behavior is more significant for a narrower channel and at a lower density.

$T_{m}^{2D}: \Gamma = U/K \approx 130$

on bulk liquid helium
the plasma parameter at $T_0$ is smaller for a fewer electrons across the channel
Melting of a Wigner Crystal in 2D
Kosterlitz-Thouless-Halperin-Nelson-Young (KTHNY) mechanism

\( T<T_m^{2D} \)

- dislocation-pair
- three dislocations

- positional-correlation function:
  \( \Psi \sim r^{-\eta} \) due to phonons
- quasi-long range order

\( T>T_m^{2D} \)

- free dislocation

- positional-correlation function:
  \( \Psi \sim \exp \left( -r/\xi_+ \right) \) due to free dislocations
- short range order

Correlation length
(mean distance between dislocations)

\[
\xi_+ = A a_0 \exp \left[ \frac{b}{(T/T_m^{2D} - 1)^\nu} \right]
\]

- \( a_0 \): core size of a dislocation
- \( \nu = 0.36963 \)
Melting of a Wigner Crystal in quasi-1D Geometry

Positional Correlation in quasi-1D geometry

\[ r > W : \Psi \sim \exp (-r/\xi) \quad (1D\text{-like}) \]

Short range order

correlation length: \( \xi \sim T^{-1} \)

\[ r < W : 2D\text{-like} \]

\[ \xi \sim W: 1D-2D \text{ crossover} \]
$\Gamma_0$ as a Function of the Number of Electron across Channel

plasma parameter at $T_0$: $\Gamma_0 = U/k_B T_0$

solid line: $\xi_+ = W$

$$\xi_+ = Aa_0 \exp \left( \frac{b}{(T/T_m^{2D} - 1)^\nu} \right)$$

$$= Aa_0 \exp \left( \frac{b}{(\Gamma_m^{2D} / \Gamma - 1)^\nu} \right)$$

$a_0$: lattice constant ( ),
$\nu = 0.36963$

$b = 1.8$ for $E_c/k_B T_m = 4.9$
(numerical calculation by Fisher, et al, PRB 20, 4692 (‘79))

$A = 1.0$, $\Gamma_m^{2D} = 130$

Emergence of free dislocations in the channel causes the disappearance of the Bragg-Cherenkov scattering
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1.6 μm Channel

Channel dimensions:
- Width (W) = 1.6 μm
- Length (L) = 200 μm
Nonlinear Transport in 1.6 μm Channel

Current as a function of excitation

Guard voltage
- -0.200 V
- -0.215 V
- -0.220 V
- -0.230 V

Source=Drain =+0.25 V
Gate=+0.45 V
f=10.03 kHz
T=0.44 K

I=enWv

For three rows of electrons,

I=5.3 pA

the Bragg-Cherenkov scattering occurs in the 1.6 μm-channel
Summary

We have investigated nonlinear transports of Wigner crystals in the quasi-1D geometry.

**Nonlinear behaviors**
- Electron velocity is limited by the Bragg-Cherenkov scattering of ripplons.
- Mobility jumps at a large excitation due to the decoupling of the Wigner crystal from the dimple lattice.
- The decoupling takes place from the deepened dimple lattice.

**Melting of a Wigner crystal in quasi-1D geometry**
The nonlinear behaviors disappear at a higher temperature for a fewer electrons across the channel.
- disappearance of the lattice structure
- caused by the emergence of free dislocations in the channel