Bubbles and Crashes

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Story of a typical technology stock

- Company X introduced a revolutionary wireless communication technology.
- It not only provided support for such a technology but also provided the informational content itself.
- It’s IPO price was $1.50 per share. Six years later it was traded at $85.50 and in the seventh year it hit $114.00.
- The P/E ratio got as high as 73.
- The company never paid dividends.
Story of RCA - 1920’s

- Company: Radio Corporation of America (RCA)
- Technology: Radio
- Year: 1920’s

It peaked at $397 in Feb. 1929, down to $2.62 in May 1932,
Internet bubble? - 1990’s

**NASDAQ Combined Composite Index**

- Chart (Jan. 98 - Dec. 00)
- 38 day average

**NEMAX All Share Index (German Neuer Markt)**

- Chart (Jan. 98 - Dec. 00) in Euro
- 38 day average

Loss of ca. 60 %
from high of $ 5,132

Loss of ca. 85 %
from high of Euro 8,583

- **Are bubbles *recurrent***?
- **What happened in March 2000***?
- **Further evidence of bubble**
  - crash was not accompanied by fundamental news.
  - excess volatility
Do (rational) professional ride the bubble?

- South Sea Bubble (1710 - 1720)
  - *Isaac Newton*
    - 04/20/1720 sold shares at £7,000 profiting £3,500
    - re-entered the market later - ended up losing £20,000
    - “I can calculate the motions of the heavenly bodies, but not the madness of people”

- Internet Bubble (1992 - 2000)
  - *Druckenmiller* of Soros’ Quantum Fund didn’t think that the party would end so quickly.
    - “We thought it was the eighth inning, and it was the ninth”
  - *Julian Robertson* of Tiger Fund refused to invest in internet stocks
Pros’ dilemma

➢ “The moral of this story is that irrational market can kill you …

➢ Julian said ‘This is irrational and I won’t play’ and they carried him out feet first.

➢ Druckenmiller said ‘This is irrational and I will play’ and they carried him out feet first.”

Quote of a financial analyst, New York Times

April, 29 2000
Classical Question

- Suppose behavioral trading leads to mispricing.

- Can mispricings or bubbles persist in the presence of rational arbitrageurs?

- What type of information can lead to the bursting of bubbles?
Main Literature

- Keynes (1936) ⇒ bubble can emerge
  - “It might have been supposed that competition between expert professionals, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself.”

- Friedman (1953), Fama (1965)
  Efficient Market Hypothesis ⇒ no bubbles emerge
  - “If there are many sophisticated traders in the market, they may cause these “bubbles” to burst before they really get under way.”

- Limits to Arbitrage
  - arbitrageurs are myopic/short-lived and risk averse (DeLong et al. [DSSW], 1990a)
  - fund managers (arbitrageurs) face liquidation risk due to principal-agent problem (Shleifer & Vishny, 1997)
  - arbitrageurs exploit delayed reaction of feedback traders (DeLong et al. [DSSW], 1990b)
Timing Game - Synchronization

- (When) will behavioral traders be overwhelmed by rational arbitrageurs?
- Collective selling pressure of arbitrageurs more than suffices to burst the bubble.
- Rational arbitrageurs understand that an eventual collapse is inevitable. But when?
- Delicate, difficult, dangerous TIMING GAME!
Elements of the Timing Game

1. **Coordination** at least $\kappa > 0$ arbs have to be ‘out of the market’
2. **Competition** only \( \text{first } \kappa < 1 \) arbs receive pre-crash price.
3. **Profitable ride** ride bubble (stay in the market) as long as possible.
4. **Sequential Awareness**
   arbs understand that for a variety of reasons (dispersion of ‘viewpoints’, risk exposure, etc.) they will individually come up with different solutions when to exit the market.

   **A Synchronization Problem arises!**
   - Absent of sequential awareness
     competitive element dominates $\Rightarrow$ and bubble burst immediately.
   - With sequential awareness
     incentive to TIME THE MARKET leads to $\Rightarrow$ “delayed arbitrage” and persistence of bubble.
Model setup

- common action of \( \kappa \) arbitrageurs
- sequential awareness
  (random \( t_0 \) with \( F(t_0) = 1 - \exp(-\lambda t_0) \)).

\[
pt = e^{gt}
\]

paradigm shift
- internet 90’s
- railways
- etc.

maximum life-span of the bubble \( \tau \)

bubble bursts for exogenous reasons
Payoff structure

- Cash Payoffs (difference)
  - Sell ‘one share’ at $t-\Delta$ instead of at $t$.
    \[ p_{t-\Delta} e^{r\Delta} - p_t \]

  where
  \[
  p_t = \begin{cases} 
  e^{gt} & \text{prior to the crash} \\
  (1 - \beta(t - t_0))e^{gt} & \text{after the crash}
  \end{cases}
  \]

- Execution price at the time of bursting.

\[
 p_t^{\text{burst}} = \begin{cases} 
 e^{gt} & \text{for first random orders up to } \kappa \\
 (1 - \beta(t - t_0))e^{gt} & \text{all other orders}
  \end{cases}
  \]
Payoff structure (ctd.), Trading

- Reputational penalty $z p_t$ for attacking if bubble does not burst
  - relative performance evaluation
  - draw downs
- Small transactions costs $c e^{r t}$
- Risk-neutrality but max/min stock position
  - max long position
  - max short position
  - due to capital constraints, margin requirements etc.

Definition 1: trading equilibrium

- Perfect Bayesian Nash Equilibrium
- Belief restriction: trader who attacks at time $t$ believes that all traders who became aware of the bubble prior to her also attack at $t$. 

Trigger Strategies

- **Bursting date** \( T^*(t_0) = \min \{ T(t_0 + \eta \kappa), t_0 + \overline{\tau} \} \)

- **Role of Preemption Motive**
  - Rules out coordinated sell out on Friday July 13th.
  - Bubble never bursts with strictly positive prob. at some \( t^{13} \).
    - Suppose it would, then selling pressure would exceed \( \kappa \) with prob>0.
    - Hence, price would drop already at \( t^{13} \) \( \Rightarrow \) incentive to sell out earlier
  - well defined density of bursting date \( \pi(t|t_i) \) for each arb.

**Proposition 1**: Trigger strategies.

- Given \( c > 0 \), arb \( t_i \) never sells out only for an instant. He stays out of the market at least until \( t_i + \epsilon \) sells out.
- Arb \( t_i + \epsilon \) stays out until \( t_i + 2\epsilon \) exits and so on.
- By trading equilibrium, arb \( t_i \) stays out until \( t_i + \eta \kappa \) exits.
Sell out condition for $\Delta \to 0$ periods

- sell out at $t$ if

$$\Delta h(t|t_i) E_t[\text{bubble}|\cdot] \geq (1 - \Delta h(t|t_i))[(g-r) + z] p_t \Delta$$

- appreciation rate
- reputational penalty
- cost of attacking

$$h(t|t_i) \geq \frac{(g-r) + z}{\beta(t - T* - 1(t))}$$

RHS converges to $\Rightarrow [(g-r) + z]$ as $t \to \infty$
introduction

model setup

preliminary analysis

persistence of bubbles
  exogenous crashes
  endogenous crashes
  lack of common knowledge

public events

price cascades and rebounds

conclusion
Persistence of Bubbles

**Proposition 2:** Suppose \( \frac{\lambda}{1 - e^{-\lambda \eta \kappa}} \leq \frac{q-r}{\beta} \).

- existence of a unique trading equilibrium
- traders begin attacking after a delay of \( \tau^1 < \bar{\tau} \) periods.
- bubble does **not** burst due to endogenous selling prior to \( t_0 + \tau \).
Sequential awareness

Distribution of $t_0$
(bursting of bubble if nobody attacks)

- **trader $t_i$**
  - Since $t_i \leq t_0 + \eta$
  - $t_i - \eta$

- **trader $t_j$**
  - Since $t_j \geq t_0$
  - $t_j - \eta$

- **trader $t_k$**
  - $t_0$

Distribution of $t_0 + \tau$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$
when $\kappa$ traders are aware of the bubble

If $t_0 < t_i - \eta \kappa$, the bubble would have burst already.
Conj. 1 (ctd.): Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

hazard rate of the bubble
$h = \frac{\lambda}{1 - \exp(-\lambda(t_i + \eta \kappa - t))}$

Distribution of $t_0$  
$\frac{\lambda}{1 - e^{-\lambda \eta \kappa}}$  

Distribution of $t_0 + \eta \kappa$

$t_i - \eta$  
$t_i - \eta \kappa$  
$t_i$  
$t_i + \eta \kappa$

Bubble bursts for sure!
Conj. 1 (ctd.): Immediate attack

⇒ Bubble bursts at $t_0 + \eta \kappa$

hazard rate of the bubble $h = \lambda / (1 - \exp\{-\lambda (t_i + \eta \kappa - t)\})$

Recall the sell out condition:

\[
h(t \mid t_i) \geq \frac{(g-r) + z}{\beta(t-T^*-\frac{1}{t}}(t))
\]

$\Rightarrow$ “delayed attack is optimal”

no “immediate attack” equilibrium!
Conj. 2: Delayed attack by arbitrary $\tau'$

$\Rightarrow$ Bubble bursts at $t_0 + \eta \kappa + \tau' < t_0 + \bar{\tau}$

- hazard rate of the bubble
  \[ h = \lambda/(1-\exp\{-\lambda(t_i + \eta \kappa + \tau' - t)\}) \]

- bubble appreciation
- bubble size

lower bound: 
\[ \left( (g-r) + z \right)/\beta > \lambda/(1-e^{-\lambda\eta \kappa}) \]

$\Rightarrow$ attack is never successful
$\Rightarrow$ bubble bursts for exogenous reasons at $t_0 + \bar{\tau}$
Endogenous crashes

Proposition 3: Suppose \( \frac{\lambda}{1-e^{-\lambda \eta}} > \frac{g-r}{\beta} \).

- ‘unique’ trading equilibrium.
- Traders begin attacking after a delay of \( \tau^* \) periods.
- Bubble bursts due to endogenous selling pressure at a size of \( \rho_t \) times.

\[
\beta^* = \frac{1-e^{-\lambda \eta}}{\lambda} (g - r)
\]
Endogenous crashes - deriving $\tau^*$

- In equilibrium trader $t_i = t_0 + \eta \kappa$ bursts the bubble.
- When she sells his shares her support of $t_0$ is $[t_i - \eta \kappa, t_i]$, hence his hazard rate is $h = \lambda / (1 - \exp\{-\lambda \eta \kappa\})$ (1)
- The bubble bursts at $t_i = t_0 + \eta \kappa + \tau^*$, hence it bursts at a size of $e^{gt} \beta^*(\tau^*)$
bubble appreciation/ size $= (g-r+z) / \beta^*(\tau^*)$ (2)
Comparative statics

- Role of information dispersion $\lambda, \eta$
  - Prior distribution of $t_0$ \( F(t_0) = 1 - \exp\{-\lambda t_0\} \)
    - the smaller $\lambda$, the larger $\beta^*$, the size of bubble
    - $\lambda \to \infty \Rightarrow t_0 = 0$, no info dispersion $\Rightarrow$ no bubble
    - $\lambda \to 0 \Rightarrow$ distributions $\Rightarrow$ uniform \[ \text{[size is } \eta \kappa (g-r) \text{]} \]
  - Dispersion of opinion $\eta$
    - as $\eta \uparrow$ $\Rightarrow$ bubble’s life-span $\uparrow$
    - for $\eta > -\frac{1}{\lambda \kappa} \ln(1 - \lambda \frac{g-r}{\beta})$ $\Rightarrow$ exogenous crash

- Role of momentum traders $\kappa$ $\Rightarrow$ same as for $\eta$
Lack of common knowledge

⇒ standard backwards induction can’t be applied

\( t_0 \quad t_0 + \eta \kappa \quad t_0 + \eta \quad t_0 + 2\eta \quad t_0 + 3\eta \quad \ldots \quad \tau_0 + \tau \)

\( \kappa \) traders know of the bubble

everybody knows the bubble

everybody knows that everybody knows the bubble

everybody knows that everybody knows that everybody knows the bubble

(same reasoning applies for \( \kappa \) traders)
Related theoretical literature

- **Asynchronized clocks**
  - Halpern & Moses (1984) [computer science]
  - Morris (1995)
    - restricted strategy space: condition only on own clock
    - no conditioning on calendar time, past payoffs, etc.
    - no competitive element ($k = 1$ - case only)

- **Global Games**
  (uniqueness of equilibrium in static games with strategic complementarities)
  - Carlson & van Damme (1994)

- **Other timing games**
  - war of attrition - preemption games (private values)
  - herding models with endogenous sequencing (observable actions)
Pre-scheduled public news

- Pre-scheduled public events
  - news is unknown, but timing is fixed in advance. (macroeconomic news etc.)
  - $p_t = E_t[p_s]$ for all $s > t$.
  - ⇒ pre-scheduled news will only move price by its fundamental content, but not beyond.
    - Why? It cannot serve as a synchronization device.
    - If it would, then the bubble would burst with strictly positive probability on this date. In this case arbitrageurs have incentive to attack slightly earlier (same as Friday 13th of July)
Unanticipated public news

- Unanticipated public events
  - pre-emption argument does not apply!
  - can serve as synchronization device.
  - there are millions of public events (weather, etc.)
  - viewing something as a public event is also a coordination problem in itself.
- Extended setting
  - focus on news with *no* informational content (sunspots).
  - public event occurs with Poisson arrival rate $\theta$.
  - Arbitrageurs who are aware of the bubble become increasingly worried about it over time.
    - only traders who became aware of the mispricing more than $\tau_e$ periods ago observe (look out for) public events.
Proposition 5: In ‘responsive equilibrium’
Sell out a) always at the time of a public event $t_e$,
   b) after $t_i + \tau^{e*}$ (where $\tau^{e*} < \tau^*$),
   except after a failed attack at $t_p$, re-enter the market
   for $t \in (t_e, t_e - \tau_e + \tau^{e*})$.

Intuition for re-entering the market:
- for $t_e < t_0 + \eta \kappa + \tau_e$ attack fails, agents learn $t_0 > t_e - \tau_e - \eta \kappa$
- without public event, they would have learnt this only at $t_e + \tau_e - \tau^{e*}$.
  - the existence of bubble at $t$ reveals that $t_0 > t - \tau^{e*} - \eta \kappa$
  - that is, no additional information is revealed till $t_e - \tau_e + \tau^{e*}$
  - density that bubble bursts for endogenous reasons is zero.
Role of information

- Only unanticipated public news can burst a bubble.
- News which is considered as important can be more important than real fundamental news.
- Fads and fashions in information.
introduction
model setup
preliminary analysis
persistence of bubbles
public events
price cascades and rebounds
conclusion
Price cascades and rebounds

- **Price drop as a synchronization device** (public event).
  - through psychological resistance line
  - by more than, say 5 %

- **Exogenous price drop**
  - after a price drop
    - if bubble is ripe
      - bubble bursts and price drops further.
    - if bubble is not ripe yet
      - price bounces back and the bubble is strengthened for some time.
Price cascades and rebounds (ctd.)

Proposition 6: 

Sell out a) after a price drop if \( \tau_i \geq \tau_p(H_p) \)

b) after \( t_i + \tau^{**} \) (where \( \tau^{**} < \tau^* \) ),

re-enter the market after a rebound at \( t_p \)

for \( t \in (t_p, t_p - \tau_p + \tau^{**}) \).

- attack is costly, since price might jump back
  \( \Rightarrow \) only arbitrageurs who became aware of the
  bubble more than \( \tau_p \) periods ago attack the bubble.

- after a rebound, an endogenous crash can be
temporarily ruled out and
  hence, arbitrageurs re-enter the market.

- Even sell out after another price drop is less likely.
Conclusion

- **Bubbles**
  - Dispersion of opinion among arbitrageurs causes a synchronization problem which makes coordinated price corrections difficult.
  - Arbitrageurs time the market and ride the bubble.
  - $\Rightarrow$ Bubbles persist

- **Crashes**
  - can be triggered by unanticipated news without any fundamental content, since
  - it might serve as a synchronization device.

- **Rebound**
  - can occur after a failed attack, which temporarily strengthens the bubble.
Formal analysis for symmetric strategies

- Suppose endogenous selling pressure would burst bubble at $t = t_0 + \hat{\tau}$, where $\hat{\tau} < \tau$
  - for $t < t_i - \theta + \hat{\tau}$, $h(t|t_i) = 0$
  - for $t \geq t_i - \theta + \hat{\tau}$, $h(t|t_i) = \frac{\lambda}{1 - e^{-\lambda(t_i + \hat{\tau} - t)}}$
- (from sell out condition) sell shares at
  $$t_i + \hat{\tau} + \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right]\left[\frac{1}{\lambda}\right]$$
- mass of arbitrageurs aware of the bubble $\max\left\{\frac{1}{\theta}(t - t_0), 1\right\}$
- mass of arbitrageurs attacking (= selling pressure)
  - $\max\left\{\frac{1}{\theta}(t - \hat{\tau} - \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right]\left[\frac{1}{\lambda}\right] - t_0), 1\right\}$
  - for $t = t_0 + \hat{\tau}$
    - $\frac{-1}{\theta \lambda} \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right] < \kappa$
  - contradiction!
Endogenous crashes

⇒ Bubble bursts at $t_0 + \theta \kappa + \tau^*$

- bubble appreciation
- bubble size

lower bound: $(g-r) + c > \lambda/(1-e^{\lambda \theta \kappa})$

hazard rate of the bubble

$h = \lambda/(1-\exp{-\lambda(t_i + \theta \kappa + \tau^* - t)})$

$t_i - \eta$
$t_i - \eta \kappa$
$t_i$
$t_i - \eta + \eta \kappa + \tau^{**}$
$t_i + \tau^{**}$
$t_i + \eta \kappa + \tau^{**}$