Bubbles and Crashes

Dilip Abreu  
*Princeton University*

Markus K. Brunnermeier  
*Princeton University*

http://www.princeton.edu/~markus
Company X introduced a revolutionary wireless communication technology.

It not only provided support for such a technology but also provided the informational content itself.

It’s IPO price was $1.50 per share. Six years later it was traded at $ 85.50 and in the seventh year it hit $ 114.00.

The P/E ratio got as high as 73.

The company never paid dividends.
Story of RCA - 1920’s

- Company: Radio Corporation of America (RCA)
- Technology: Radio
- Year: 1920’s

- It peaked at $397 in Feb. 1929, down to $2.62 in May 1932,
Internet bubble? - 1990’s

**Internet bubble?**

**1990’s**

**NASDAQ Combined Composite Index**

<table>
<thead>
<tr>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>2000</td>
<td>3000</td>
<td>4000</td>
<td>5000</td>
<td>5000</td>
<td>4500</td>
<td>4000</td>
<td>3500</td>
<td>3000</td>
<td>2500</td>
<td>2000</td>
</tr>
</tbody>
</table>

Chart (Jan. 98 - Dec. 00)

- 38 day average

**NEMAX All Share Index (German Neuer Markt)**

<table>
<thead>
<tr>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1T</td>
<td>2T</td>
<td>3T</td>
<td>4T</td>
<td>5T</td>
<td>6T</td>
<td>7T</td>
<td>8T</td>
<td>2T</td>
<td>3T</td>
<td>4T</td>
<td>5T</td>
</tr>
</tbody>
</table>

Chart (Jan. 98 - Dec. 00) in Euro

- 38 day average

Loss of ca. **60 %**
from high of $ 5,132

Loss of ca. **85 %**
from high of Euro 8,583

- Are bubbles *recurrent*?
- What happened in March 2000?
- Further evidence of bubble
  - crash was not accompanied by fundamental news.
  - excess volatility
Do (rational) professional ride the bubble?

- South Sea Bubble (1710 - 1720)
  - *Isaac Newton*
    - 04/20/1720 sold shares at £7,000 profiting £3,500
    - re-entered the market later - ended up losing £20,000
    - “I can calculate the motions of the heavenly bodies, but not the madness of people”

- Internet Bubble (1992 - 2000)
  - *Druckenmiller* of Soros’ Quantum Fund didn’t think that the party would end so quickly.
    - “We thought it was the eighth inning, and it was the ninth”
  - *Julian Robertson* of Tiger Fund refused to invest in internet stocks
Pros’ dilemma

“The moral of this story is that irrational market can kill you …

Julian said ‘This is irrational and I won’t play’ and they carried him out feet first.

Druckenmiller said ‘This is irrational and I will play’ and they carried him out feet first.”

Quote of a financial analyst, *New York Times*

*April, 29 2000*
Suppose behavioral trading leads to mispricing.

Can mispricings or bubbles persist in the presence of rational arbitrageurs?

What type of information can lead to the bursting of bubbles?
Main Literature

- Keynes (1936) ⇒ bubble can emerge
  - “It might have been supposed that competition between expert professionals, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself.”

- Friedman (1953), Fama (1965)
  Efficient Market Hypothesis ⇒ no bubbles emerge
  - “If there are many sophisticated traders in the market, they may cause these “bubbles” to burst before they really get under way.”

- Limits to Arbitrage
  - arbitrageurs are myopic/short-lived and risk averse
    (DeLong et al. [DSSW], 1990a)
  - fund managers (arbitrageurs) face liquidation risk due to principal-agent problem (Shleifer & Vishny, 1997)
  - arbitrageurs exploit delayed reaction of feedback traders
    (DeLong et al. [DSSW], 1990b)
Market timing & Synchronization

1. Market timing problem
   - ride the bubble as long as possible (conflicting interest)
   - sell right before others

2. Bubble only bursts if more than $\kappa$ arbitrageurs attack (common action)

3. Arbitrageurs become sequentially aware of mispricing

Lack of synchronization → "delayed arbitrage" → Bubble persists
Model setup

- common action of \( \kappa \) arbitrageurs
- sequential awareness
  (random \( t_0 \) with \( F(t_0) = 1 - \exp\{-\lambda t_0\} \)).

\[
\nu = e^{gt_0 + r(t-t_0)}
\]

\[
p_t = e^{gt}
\]

paradigm shift
- internet 90’s
- railways
- etc.

maximum life-span of the bubble \( \tau \)

bubble bursts for exogenous reasons
Payoff structure

- **Cash Payoffs (difference)**
  - Sell ‘one share’ at $t-\Delta$ instead of at $t$.
    
    \[
    p_{t-\Delta} e^{r\Delta} - p_t
    \]

    where  
    \[
    p_t = \begin{cases} 
    e^{gt} & \text{prior to the crash} \\
    e^{gt_0} + r(t-t_0) & \text{after the crash} 
    \end{cases}
    \]

  - Execution price at the time of bursting.

\[
\begin{aligned}
p_t^{\text{burst}} &= \begin{cases} 
    e^{gt} & \text{for first random orders up to } \kappa \\
    e^{gt_0} + r(t-t_0) & \text{all other orders}
\end{cases}
\end{aligned}
\]
Reputational penalty $cp_t$ for attacking if bubble does not burst
- relative performance evaluation
- draw downs

Risk-neutrality but max/min stock position
- max long position $\sigma = 0$
- max short position $\sigma = 1$
- due to capital constraints, margin requirements etc.

Definition 1: trading equilibrium
- Perfect Bayesian Nash Equilibrium
- Belief restriction: trader who attacks at time $t$ believes that all traders who became aware of the bubble prior to her also attack at $t$. 
introduction

model setup

arbitrageurs never burst bubble
graphical illustration
proof for symmetric strategies
lack of common knowledge

persistence of bubbles

endogenous crashes

public events

conclusion
Proposition 1: Suppose \( \theta_\kappa \geq -\ln(1 - \frac{\lambda}{g-r+c})(\frac{1}{\lambda}) \)

- existence of a unique trading equilibrium
- traders begin attacking after a delay of \( \tau^* < \bar{\tau} \) periods.
- bubble \textit{never} bursts due to endogenous selling pressure. It only bursts at \( t_0 + \bar{\tau} \).
Attack condition for $\Delta \to 0$ periods

If prob that bubble will burst in next instant $> 0$

If prob that bubble will burst in next instant $= 0$

$\Delta h(t|t_i) \leq (1 - \Delta h(t|t_i)) [g - r] + \Delta [g - r] + c [p(t)]$

$\Delta h(t|t_i) \geq (1 - \Delta h(t|t_i)) [g - r] + c [p(t)]$

RHS converges to $(g - r) + c$ as $t \to \infty$

\[ h(t|t_i) \geq \frac{1 - E[e^{-(g-r)(t-t_0)}]}{(g-r) + c} \]
Sequential awareness

Distribution of $t_0$

Distribution of $t_0 + \tau$
(bursting of bubble if nobody attacks)

trader $t_i$

since $t_i \leq t_0 + \theta$

since $t_i \geq t_0$

trader $t_{i'}$

trader $t_{i''}$

$t_0$

$t_{i'}$

$t_{i''}$

$t_0 + \tau$
Conjecture 1: Immediate attack

⇒ Bubble bursts at $t_0 + \theta \kappa$

when $\kappa$ traders are aware of the bubble

If $t_0 < t_i - \theta \kappa$, the bubble would have burst already.
Conj. 1 (ctd.): Immediate attack

⇒ Bubble bursts at \( t_0 + \theta \kappa \)

hazard rate of the bubble
\[ h = \frac{\lambda}{1 - \exp\{-\lambda (t_i + \theta \kappa - t)\}} \]

Distribution of \( t_0 \)
\[ \frac{\lambda}{1 - e^{-\lambda \theta}} \]

Distribution of \( t_0 + \theta \kappa \)

Bubble bursts for sure!
Recall the attack condition:

\[ h(t|t_i) \geq \frac{(g-r)+c}{1-E[e^{-(g-r)(t-t_0)}]} \]

⇒ Bubble bursts at \( t_0 + \theta \kappa \)

hazard rate of the bubble
\( h = \frac{\lambda}{1-e^{-\lambda(t_i + \theta \kappa - t)}} \)

Distribution of \( t_0 \)

lower bound: \((g-r) + c > \frac{\lambda}{1-e^{-\lambda \theta \kappa}}\)

Optimal time to attack \( t_i + \tau_i \)

⇒ “delayed attack is optimal”
no “immediate attack” equilibrium!
Conj. 2: Delayed attack by arbitrary $\tau'$

⇒ Bubble bursts at $t_0 + \theta \kappa + \tau' < t_0 + \tau$

- bubble appreciation
- bubble size

hazard rate of the bubble
$h = \lambda/(1-\exp\{-\lambda(t_i + \theta \kappa + \tau' - t)\})$

lower bound: $(g-r) + c > \lambda/(1-e^{\lambda \theta \kappa})$

⇒ attack is never successful
⇒ bubble bursts for exogenous reasons at $t_0 + \tau$
Formal analysis for symmetric strategies

- Suppose endogenous selling pressure would burst bubble at $t = t_0 + \hat{\tau}$, where $\hat{\tau} < \tau$
  - for $t < t_i - \theta + \hat{\tau}$, $h(t | t_i) = 0$
  - for $t \geq t_i - \theta + \hat{\tau}$, $h(t | t_i) = \frac{\lambda}{1 - e^{-\lambda(t_i + \hat{\tau} - t)}}$
- (from attack condition) sell shares at
  $$t_i + \hat{\tau} + \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right]\left[\frac{1}{\lambda}\right]$$
- mass of arbitrageurs aware of the bubble $\max\{\frac{1}{\theta}(t - t_0), 1\}$
- mass of arbitrageurs attacking (= selling pressure)
  - $\max\{\frac{1}{\theta}(t - \hat{\tau} - \ln[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}][\frac{1}{\lambda}]) - t_0) , 1\} \}$
  - for $t = t_0 + \hat{\tau}$, $-\frac{1}{\theta \lambda} \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right] < \kappa$
  - contradiction!
April 13th - Pre-emption Lemma

- \( \hat{\tau} \) might depend on \( t_0 \) in asymmetric equilibria
  - Example: all start attacking on Friday, April 13th.

\[
\hat{\tau}^{13} = \begin{cases} 
\hat{\tau} & \text{if } t_0 > t^{13} - \theta \kappa \\
 t^{13} - t_0 & \text{if } t_0 \leq t^{13} - \theta \kappa 
\end{cases}
\]

- bubble would burst with strictly positive probability.
- selling pressure \( s_{t=13} > \kappa, p_{t=13} \) drops already.
- Individual incentive to attack a little bit earlier.

- Not an equilibrium!
Lack of common knowledge

⇒ standard backwards induction can’t be applied

\( t_0 \rightarrow t_0 + \theta \kappa \rightarrow t_0 + \theta \rightarrow t_0 + 2\theta \rightarrow t_0 + 3\theta \rightarrow \ldots \rightarrow t_0 + \kappa \theta \)

- everybody knows of the bubble
- everybody knows that everybody knows of the bubble
- everybody knows that everybody knows that everybody knows of the bubble

\( \kappa \) traders know of the bubble

(same reasoning applies for \( \kappa \) traders)
Related theoretical literature

- Asynchronized clocks
  - Halpern & Moses (1984) [computer science]
  - Morris (1995)
    - restricted strategy space: condition only on own clock
      no conditioning on calendar time, past payoffs, etc.

- Global Games
  (uniqueness of equilibrium in static games with strategic complementarities)
  - Carlson & van Damme (1994)
introduction

model setup

persistence of \textbf{bubbles}

\textbf{endogenous crashes}

endogenous life-span of the bubble

comparative statics

public events

conclusion
Endogenous crashes

**Proposition 3:** Suppose \( \theta_{\kappa} < \frac{1}{\lambda} \ln \left( 1 - \frac{\lambda e^{-(g-r)\tau}}{g-r+c} \right) \). 

- ‘most aggressive’ trading equilibrium.
- traders begin attacking after a delay of \( \tau^{**} \) periods.
- bubble **bursts** due to endogenous selling pressure at

\[
t_0 + \frac{-\ln \left[ 1 - \frac{g-r+c}{\lambda (1 - e^{-\lambda \theta_{\kappa}})} \right]}{g-r}
\]
Endogenous crashes

⇒ Bubble bursts at \( t_0 + \theta \kappa + \tau^{**} \)

lower bound: \((g-r) + c > \lambda/(1-e^{-\lambda \theta \kappa})\)

hazard rate of the bubble
\[ h = \lambda/(1-\exp\{-\lambda(t_i + \theta \kappa + \tau' - t)\}) \]
Endogenous crashes - deriving $\tau^{**}$

- In symmetric equilibrium trader $t_i = t_0 + \theta \kappa$ bursts the bubble.
- When he sells his shares his support of $t_0$ is $[t_i- \theta \kappa, t_i]$, hence his hazard rate is $h = \lambda / (1-\exp\{-\lambda \theta \kappa\})$ (1)
- The bubble bursts at $t_i = t_0 + \theta \kappa + \tau^{**}$, hence it bursts at a size of $e^{gt}(1-\exp\{-(g-r)(\theta \kappa + \tau^{**})\})$
  bubble appreciation/ size = $(g-r+c) / (1-\exp\{-(g-r)(\theta \kappa + \tau^{**})\})$ (2)
Comparative statics

- Role of information dispersion $\lambda, \theta$
  - Prior distribution of $t_0$ \( F(t_0) = 1 - e^{-\lambda t_0} \)
    - the smaller $\lambda$, the larger the life span of bubble
    - $\lambda \to \infty \Rightarrow t_0 = 0$, no info dispersion $\Rightarrow$ no bubble
    - $\lambda \to 0 \Rightarrow$ distributions $\to$ uniform \([\text{lifespan} - \ln(1-\theta \kappa(g-r+c))/(g-r)]\)
  - Dispersion of opinion $\theta$
    - as $\theta \uparrow \Rightarrow$ bubble’s life-span $\uparrow$
    - for $\theta \geq -\frac{1}{\kappa} \ln\left(1 - \frac{\lambda - \lambda e^{-(g-r)\kappa}}{g-r+c}\right)\left(\frac{1}{\lambda}\right) \Rightarrow$ exogenous crash
- Role of momentum traders $\kappa \Rightarrow$ same as for $\theta$
- Excess growth rate $(g - r) \uparrow$ [2 effects]
  - instantaneous appreciation effect $\uparrow \Rightarrow$ life span of bubble $\uparrow$
  - size of bubble (past appreciation) $\uparrow \Rightarrow$ life span of bubble $\downarrow$
Pre-scheduled vs. unanticipated public news

Pre-scheduled public events

- news is unknown, but timing is fixed in advance. (FOMC Meetings etc.)
- “martingale news”: correctly reflected in the price
- \( \Rightarrow \text{pre-scheduled news will only move price by its fundamental content, but not beyond.} \)
  - Why? It cannot serve as a synchronization device.
  - If it would, then the bubble would burst with strictly positive probability on this date. In this case arbitrageurs have incentive to attack slightly earlier (same as Friday 13th of July)
Pre-scheduled vs. unanticipated public news

Unanticipated public events

- pre-emption argument does not apply!
- can serve as synchronization device.
- there are millions of public events (weather, etc.)
- viewing something as a public event is also a coordination problem in itself.

Extended setting

- focus on news with \textit{no} informational content (sunspots).
- public event occurs with Poisson density $\lambda_p$.
- Arbitrageurs who are aware of the bubble become increasingly worried about it over time.
  - only traders who became aware of the mispricing more than $\tau_p$ periods ago observe (look out for) public events.
Public events & Market rebounds

Proposition 5:

**Attack**
- a) always at the time of a public event $t_p$,
- b) after $t_i + \tau^{***}$ (where $\tau^{***} < \tau^{**}$),

except after a failed attack at $t_p$, **re-enter** the market for $t \in (t_p, t_p - \tau_p + \tau^{***}) \cap (t_i + \tau^p, t^i)$.

Intuition for re-entering the market:

- for $t_p < t_0 + \theta \kappa + \tau_p$ attack fails, agents learn $t_0 > t_p - \tau_p - \theta \kappa$
- without public event, they would have learnt this only at $t_p + \tau_p - \tau^{***}$.
  - the existence of bubble at $t$ reveals that $t_0 > t - \tau^{***} - \theta \kappa$
  - that is, no additional information is revealed till $t_p - \tau_p + \tau^{***}$
  - density that bubble bursts for endogenous reasons is zero.
Role of information

- Only unanticipated public news can burst a bubble.
- News which is considered as important can be more important than real fundamental news.
- Fads and fashions in information.
Price cascades and rebounds

- **Price drop as a synchronization device** (public event).
  - through psychological resistance line
  - by more than, say 5 %

- **Exogenous price drop**
  - after a price drop
    - if bubble is ripe
      - bubble bursts and price drops further.
    - if bubble is not ripe yet
      - price bounces back and the bubble is strengthened for some time.
Price cascades and rebounds (ctd.)

Proposition 6:

**Attack**
a) after a price drop if $\tau_i \geq \tau'_p$
b) after $t_i + \tau^{****}$ (where $\tau^{****} < \tau^{**}$),

re-enter the market after a rebound at $t'_p$
for $t \in (t'_p, t'_p - \tau'_p + \tau^{****}) \cap (t_i + \tau'_p, 1)$.

- attack is costly, since price might jump back
  ⇒ only arbitrageurs who became aware of the bubble
  more than $\tau'_p$ periods ago attack the bubble.
- after a rebound, an endogenous crash can be temporarily ruled out and
  hence, arbitrageurs re-enter the market.
Conclusion

- **Bubbles**
  - Dispersion of opinion among arbitrageurs causes a synchronization problem which makes coordinated price corrections difficult.
  - Arbitrageurs time the market and ride the bubble.
  - $\Rightarrow$ Bubbles persist

- **Crashes**
  - can be triggered by unanticipated news without any fundamental content, since
  - it might serve as a synchronization device.

- **Rebound**
  - can occur after a failed attack, which temporarily strengthens the bubble.