The I Theory of Money*

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first version: Oct. 10, 2010

this version: June 5, 2011

Abstract

This paper provides a theory of money, whose value depends on the functioning of the intermediary sector, and a unified framework for analyzing the interaction between price and financial stability. Households that happen to be productive in this period finance their capital purchases with credit from intermediaries and from their own savings. Less productive household save by holding deposits with intermediaries (inside money) or outside money. Intermediation involves risk-taking, and intermediaries' ability to lend is compromised when they suffer losses. After an adverse productivity shock, credit and inside money shrink, and the value of (outside) money increases, causing deflation that hurts borrowers even further. An accommodating monetary policy in downturns can mitigate these destabilizing adverse feedback effects. Lowering short-term interest rates increases the value of long-term bonds, recapitalizes the intermediaries by redistributes wealth. While this policy helps the economy ex-post, ex-ante it can lead to excessive risk-taking by the intermediary sector.

*Preliminary and Incomplete. We are grateful to comments by seminar participants at Princeton, Bank of Japan, Philadelphia Fed, Rutgers, Toulouse School of Economics, Wim Duisenberg School, University of Lausanne, Banque de France-Banca d'Italia conference, University of Chicago, New York Fed, Chicago Fed, Central Bank of Chile, Penn State, Institute of Advanced Studies, Columbia University, University of Michigan, University of Maryland, Northwestern, Cowles General Equilibrium Conference, Renmin University, Johns Hopkins, Kansas City Fed, IMF, LSE, LBS, Bank of England and the Central Bank of Austria.

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1 Introduction

A theory of money needs a proper place for financial intermediaries. Financial institutions are able to create money, for example by lending to businesses and home buyers, and accepting deposits backed by those loans. The amount of money created by financial intermediaries depends crucially on the health of the banking system and on the presence of profitable investment opportunities in the economy. This paper proposes a theory of money and provides a framework for analyzing the interaction between price stability and financial stability. It therefore provides a unified way of thinking about monetary and macroprudential policy.

Since intermediation involves taking on some risk, a negative shock to productive agents also hits intermediary balance sheets. Intermediaries' individually optimal response is to lend less and accept fewer deposits. Hence, the amount of inside money in the economy shrinks. Because money serves as a store of value and the total demand for money changes little, the value of outside money increases inducing deflationary pressure. More specifically, in our model the economy moves between two polar cases: in one case the the financial sector is well capitalized: it can overcome financial frictions and is able to channel funds from less productive agents to more productive agents. Financial institutions through their monitoring role enable productive agents to issue debt and equity claims. The value of money is low. In contrast in the other polar case, in which the financial sector is undercapitalized, funds can only be transferred via outside money. Whenever an agent becomes productive he buys capital goods from less productive households using his outside money, and vice versa. That is, outside money allows de facto some implicit borrowing and lending. In this case outside money steps in for the missing financial intermediation. The value of money is high. A negative shock to productive agents lowers the financial sector's risk bearing capacity and hence brings us closer to the second case with high value of money. That is, a negative productivity shock leads deflation a la Fisher (1933). Since financial institutions accept demand deposits they are hit on both sides of their balance sheet. First, they are exposed to productivity shocks on the asset side of the balance sheet. Second, their liabilities grow after a negative shock as the value of money increases. This amplifies the initial shock even further, absent appropriate monetary policy. This leads to non-linear adverse feedback loop effects and liquidity spirals as studied in Brunnermeier and Sannikov (2010). In addition, the money multiplier is endogenous and declines with the health of the financial system.

We then study the effect of monetary policy on the intermediary sector. We focus on budget-neutral monetary policies that do not affect government spending, and consider both interest-rate policies (implemented by printing money) and open-market operations that change the maturity structure of government liabilities. We find that in the absence of long-term government bonds, interest rate policy on its own has no real effects on the economy. That is, if the monetary authority pays interest on reserves by printing money, the only effect is that the interest rate always equals the nominal inflation. This result holds because all debt is short-term in our model, and we do not allow for surprise changes in the interest-rate policy.

However, with long-term (government) bonds that pay a fixed rate of interest, interest-rate policy can have real effects. Then, if the monetary authority always sets a positive interest rate, bonds and cash are substitutes in the sense that unproductive households can use both of these assets to store wealth.[1] If the monetary authority lowers interest rates in downturns, then the value of long-term bonds rises. Intermediaries can hold these long-term nominal assets as a hedge against losses due to negative macro shocks. In other words, interest-rate cuts can help banks in downturns, as long as they hold nominal assets with longer maturities. We refer to this phenomenon as "stealth recapitalization" as it redistributes wealth. Importantly, this is however not a zero sum game. Of course, while this policy can help banks ex-post, by reducing further losses that they are exposed to, it can create extra risk-taking incentives ex-ante.

Related Literature. While in almost all papers, a negative productivity shock causes inflationary pressures, in our setting it induces deflationary pressure absent a monetary intervention. This is consistent with the empirical output-inflation patterns before 1960 under the (extended) Gold Standard, as e.g. documented by Cagan (1979). Like in monetarism (see e.g. Friedman and Schwartz (1963)), an endogenous reduction of money multiplier (given a fixed monetary base) leads to deflation in our setting. However, in our setting outside money is only an imperfect substitute for inside money. Intermediaries, either by channeling funds through or by underwriting and thereby enabling firms to approach capital markets directly, enable a better capital allocation and more economic growth. Hence, in our setting monetary intervention should aim to recapitalize undercapitalized borrowers rather than simply increase the money supply across the board. Another difference is that our approach focuses more on the role of money as a store of value instead of the transaction role of money. Overall, our approach is closer in spirit to banking channel literature, see e.g. Patinkin (1965), Tobin (1970), Gurley and Shaw (1955), Bernanke (1983) Bernanke and

[1] If the interest rate is 0, then of course perpetual bonds would have an infinite nominal price.

Blinder (1988) and Bernanke, Gertler and Gilchrist (1999).[2] Another distinct feature of our setting is that our effects arise despite the fact that prices are fully flexible. This is in sharp contrast to the New Keynesian framework, in which a nominal interest rate cut also lowers real rates and thereby induces households to consume more. Recently, Cordia and Woodford (2010) introduced financial frictions in the new Keynesian framework. In contrast, our framework focuses on the redistributional role of monetary policy, a feature we share with Scheinkman and Weiss (1986). In Kiyotaki and Moore (2008) money is desirable as it does not suffer from a resellability constraint, unlike capital in their model.

Within a three-period framework, Diamond and Rajan (2006) and Stein (2010) also address the role of monetary policy as a tool to achieve financial stability. More generally, there is a growing macro literature which also investigated how macro shocks that affect the balance sheets of intermediaries become amplified and affect the amount of lending and the real economy. These papers include Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), who study financial frictions using a log-linearized model near steady state. In these models shocks to intermediary net worths affect the efficiency of capital allocation and asset prices. However, log-linearized solutions preclude volatility effects and lead to stable system dynamics. Brunnermeier and Sannikov (2010) also study full equilibrium dynamics, focusing on the differences in system behavior near the steady state, and away from it. They find that the system is stable to small shocks near the steady state, but large shocks make the system unstable and generate systemic endogenous risk. Thus, system dynamics are highly nonlinear. Large shocks have much more serious effects on the real economy than small shocks. He and Krishnamurthy (2010) also study the full equilibrium dynamics and focus in particular on credit spreads. For a more detailed review of the literature we refer to Brunnermeier et al. (2010).

This paper is organized as follows. Section 2 informally describes the logical framework around which we construct our model. Section 3 frames the ideas from Section 2 into a basic model. Section 4 characterizes the equilibrium. Section 5 presents a computed example and discusses equilibrium properties, including capital and money value dynamics, the amount of lending through intermediaries, and the money multiplier. Section 6 introduces long-term bonds and studies the effect of interest-rate policies as well as open-market operations. Section 7 concludes.

[2]The literature on credit channels distinguishes between the bank lending channel and the balance sheet channel (financial accelerator), depending on whether banks or corporates/households are capital constrained. Strictly speaking our setting refers to the former, but we a agnostic about it and prefer the broader credit channel interpreation.

2 Informal Description of the Economy

In this section we describe informally how we think about money and intermediation. The goal is to explain the logic behind the main results and to lay out a general framework that we implement in the next section.

The economy is populated by intermediaries and heterogeneous households. The distribution of productivity among households does not match the distribution of wealth. As a result, productive households, whom we call entrepreneurs, need financing to be able to manage capital. In the absence of intermediaries there are extreme financial frictions: unproductive households with excess wealth cannot lend directly to financially constrained entrepreneurs. In the absence of money, these frictions lead to an extremely inefficient allocation of capital, in which each agent holds the amount of capital that is proportionate to his net worth. For example, if the wealth of entrepreneurs is only 1% of aggregate wealth, then they can hold only 1% of capital.

If household types are switching, then there can be a more efficient equilibrium with fiat money. Assume that there is a fixed supply of infinitely divisible money. Even though it is intrinsically worthless, in equilibrium money can have value by a mechanism which can be related to the models of Samuelson (1958) and Bewley (1980).[3] Crucially, in order for money to have value, enough agents should create demand for new savings through money to offset the supply of money by agents who want to spend it to consume. In our model, this demand stems from agents who suddenly become unproductive, and who want to exchange their capital to hold money. If so, then these agents supply capital and demand money, agents who stay unproductive supply money and demand output, and agents who stay productive supply output and demand more capital. In equilibrium, the relative prices of capital, money and output are determined so that all markets clear.

Money in equilibrium can lead to a more efficient allocation of capital. Even with extreme financial frictions that preclude borrowing and lending, the allocation of capital across agents does not have to be proportional to their net worths. In fact, it may be possible for entrepreneurs to hold *all* capital in the economy, while unproductive households hold money.

Nevertheless, when there is investment, the equilibrium with money is less efficient than

[3] In Samuelson (1958), young agents are willing to save their wealth in money because they expect that when they get old, they can trade money for consumption goods with the next generation of agents. In Bewley (1980), agents are willing to accumulate money in periods when they have high endowment because they expect to be able to trade money for consumption goods when their endowment is low, with agents who have high endowment in that period.

the efficient outcome that arises in the absence of financial frictions. Without borrowing and lending, the entrepreneurs' *demand* for capital is limited by their net worths. As a result, even with money in equilibrium, capital becomes undervalued, leading to underinvestment in capital. Furthermore, given low capital valuations, unproductive households may find it attractive to hold some capital, leading to an inefficient allocation.

We introduce intermediaries who can mitigate the financial frictions by facilitating lending from unproductive households to productive ones. Intermediaries can take deposits from unproductive households to extend loans to entrepreneurs.[4] Importantly, when they facilitate the flow of funds from unproductive agents to entrepreneurs, intermediaries must invariably be exposed to the risks of the projects they finance. They msut have some "skin in the game." The intermediaries' ability to perform their functions depends on their risk-bearing capacity. Because intermediaries are subject to the solvency constraint, their ability to absorb risks depends on their net worths, and so after losses they are less able to perform their functions.

Risk taking by intermediaries leads economic fluctuations between a two polar cases: One polar case looks like the benchmark without financial frictions and in the second regime there is no lending and hence money plays a much more important role. In the former regime, banks create a large quantity of inside money by lending freely. Unproductive agents have alternative ways to save other than holding outside money - they can hold deposits with intermediaries (or entrepreneur equity). As a result, outside money has low value. At the same time, easy financing leads to a high price of capital and high investment.

If an aggregate macro shock causes intermediaries to suffer losses, lending contracts, causing entrepreneurs to reduce their demand for capital. As a result, the price of capital and investment fall. At the same time, as the creation of inside money decreases, unproductive households bid up the value of outside money to satisfy their demand for savings. This leads to a collapse of the money multiplier and deflation.

When deposits with intermediaries are denominated in money rather than output, then deflation increases the value of liabilities of intermediaries. Thus, intermediaries are doubly hit: on the asset side because the value of capital that they finance decreases, and on the liabilities side because the real value of their obligations goes up in value.

We construct a model that captures these dynamic effects in the next section. In Section 4 we introduce in addition some long-term bonds and study how monetary policy and

[4]They can also help entrepreneurs issue outside equity directly to households. Although we do not allow for this possibility in our baseline model, we have explored it in an extension.

macroprudential policy can mitigate these adverse effects.

3 The Formal Baseline Model

We consider an infinite-horizon economy populated by heterogeneous households and intermediaries. Household types are denoted by $\omega \in \Omega$, where Ω could be an interval, or a finite set. Higher types are more productive.

In our baseline model, which we present in this section, we assume that there is a fixed amount of gold in the economy, which serves as money. Gold is intrinsically worthless, and the total quantity of gold is fixed.

Household Production Technologies. The technology of household type ω generates output at rate $a^\omega - \iota_t$ per unit of capital, where a^ω is the productivity parameter and ι_t^ω is the rate of investment. Capital is measured in efficiency units, and the quantity of capital evolves according to

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta^\omega)\, dt + d\epsilon_t^\omega. \tag{3.1}$$

Function Φ reflects a decreasing-returns-to-scale investment technology, with $\Phi(0) = 0$, $\Phi' > 0$ and $\Phi'' < 0$. That is, in the absence of investment, capital managed by household ω simply depreciates at rate δ^ω. The term $d\epsilon_t^\omega$ reflects Brownian fundamental shocks to technology ω. The technological shocks of types ω and $\omega' \in \Omega$ have covariance $\sigma(\omega, \omega')$, so that $\sigma(\omega, \omega)$ is the volatility of ϵ_t^ω. We assume that a^ω weakly increases in type ω, while δ^ω decreases in ω.

Intermediation. Intermediaries can lend to productive households or invest in their equity. In our model equity investment in household ω works as if the intermediary were directly holding capital employed under the production technology of household ω, except for an additional monitoring cost. We express the monitoring cost of equity financing through an increased depreciation rate by ϖ.

Thus, intermediaries can improve efficiency in the economy in two ways: by channeling funds to the most productive technologies, and by diversifying household risks on their balance sheets. Intermediaries finance themselves borrowing from households.

Markets for Capital, Money and Consumption Goods. All markets are fully liquid. All agents are small, and can buy or sell unlimited quantities of capital, money and output in the market at any moment of time at current prices without making any price impact. The aggregate quantity of capital in the economy is denoted by K_t, and q_t is the price of one unit of capital in the units of *output.* The total quantity of gold is normalized to one, and the value of all gold in the units of output is denoted by P_t.

The aggregate amount of capital K_t depends on aggregate investment, the allocation of capital among the different technologies, and technology shocks. The price of capital q_t and the value of money P_t are determined endogenously from supply and demand.

Net Worths and Balance Sheets. The total net worth of all agents is

$$q_t K_t + P_t.$$

The main focus of our model is on the net worth of intermediaries is $N_t < q_t K_t + P_t$. The net worth of the intermediary sector N_t relative to the size of the economy K_t determines the risk-taking capacity of intermediaries, the amount of financing available to productive technologies and the creation of inside money.

To keep the model tractable, we ignore any effects that arise through changing wealth distribution among households. Specifically, we assume that wealth $q_t K_t + P_t - N_t$ is always distributed among households with an *exogenous* density $\theta(\omega)$, with

$$\int_\Omega \theta(\omega)\, d\omega = 1.$$

That is, even though shocks and heterogeneous productivity have different effects on the net worth of different households, we assume that household types are completely transient - they switch fast enough to eliminate any temporary wealth accumulation effects. At the beginning of each period, each household gets randomly reassigned to a new type according to the probability distribution $\theta(\omega)$.

Each household chooses how to allocate wealth between *its own* productive technology and money. Households may borrow and become levered, allocating a negative portfolio weight to money. Intermediaries invest in technologies of a range of household types ω. We denote the equilibrium allocation of capital across technologies, and between households and intermediaries through functions $\xi_t(\omega)$ and $\zeta_t(\omega)$ such that

$$\int_\Omega \xi_t(\omega)\, d\omega + \int_\Omega \zeta_t(\omega)\, d\omega = 1.$$

Function $\xi_t(\omega)$ describes the density of the allocation of capital across households, and $\zeta_t(\omega)$, the technology portfolio of intermediaries. Given the allocation of capital, the aggregate amount of capital in the economy follows

$$\frac{dK_t}{K_t} = \underbrace{\left(\int_\Omega (\xi_t(\omega) + \zeta_t(\omega))g^\omega(q_t)\, d\omega - \varpi \int_\Omega \xi_t(\omega)\, d\omega\right)}_{\mu_t^K} dt + \underbrace{\int_\Omega d\epsilon_t^\omega (\xi_t(\omega) + \zeta_t(\omega))\, d\omega}_{d\epsilon_t^K}. \quad (3.2)$$

Optimal Investment. Given the current price of capital q_t, the optimal investment rate ι_t is identical across all agents. It solves

$$\max_\iota\ \Phi(\iota)q_t - \iota,$$

where $\Phi(\iota)$ is rate at which new capital is created and q_t is the price of capital. Since $\Phi'' < 0$, the solution is uniquely pinned down by the first-order condition $\Phi'(\iota)q_t = 1$. We denote the solution by $\iota(q_t)$, and the resulting net output rates and capital growth rates by

$$c^\omega(q_t) = a^\omega - \iota(q_t) \quad \text{and} \quad g^\omega(q_t) = \Phi(\iota(q_t)) - \delta^\omega.$$

Returns on Capital and Money. We denote the law of motion of the price of capital q_t by

$$\frac{dq_t}{q_t} = \mu_t^q\, dt + d\epsilon_t^q. \quad (3.3)$$

As a result, household of type ω earns return on capital of

$$dr_t^\omega = \underbrace{\frac{c^\omega(q_t)}{q_t}}_{\text{dividend yield}}\ dt + \underbrace{(g^\omega(q_t) + \mu_t^q + \mathrm{Cov}(d\epsilon_t^\omega, d\epsilon_t^q))}_{\text{expected capital gains rate}}\ dt + \underbrace{d\epsilon_t^\omega + d\epsilon_t^q}_{\text{risk}}.$$

The capital gains rate and risk, which equal $d(k_t q_t)/(k_t q_t)$, and are derived from (3.1) and (3.3) using Ito's lemma.

Intermediaries get a return of

$$dr_t^\omega - \varpi\, dt \quad (3.4)$$

from investing in technology ω. The return on money is given by the law of motion of P_t,

$$dr_t^M = \frac{dP_t}{P_t} = \mu_t^M\, dt + d\epsilon_t^M. \quad (3.5)$$

Preferences and Utility Maximization. All agents have *logarithmic* utility with a common discount rate of ρ. Logarithmic utility has two convenient properties that significantly simplify the analysis of our model.

1. Any agent with logarithmic utility consumes ρ times his net worth.

2. The solution of the optimal portfolio choice problem of an agent with log utility has a very simple characterization. On the margin, any small change in asset allocation around the optimum has

$$\textit{marginal expected return} = \mathrm{Cov}(\textit{marginal risk}, \textit{net worth risk at optimum}). \tag{3.6}$$

   In this equation, net worth risk is measured per dollar of net worth. We demonstrate further the mechanics of applying this condition.

Importantly, the consumption and investment decisions of an agent with logarithmic utility are completely myopic, irrespective of future investment opportunities. The solution procedure for more general preferences specifications is more complicated, and requires the introduction of the agents' value functions and the Bellman equation.

In addition, to control the size of the intermediary sector in our model, we assume that intermediaries may retire. Intermediaries have a difficult job, which requires effort. Upon retirement, they relax and receive an immediate utility boost of x. We denote the retirement rate of experts by $N_t d\Xi_t$.

The Evolution of N_t. Since we know the portfolio of technologies $\zeta_t(\omega)$, in which intermediaries invest, the return of each technology (3.4) and the promised return to depositors (3.5), the law of aggregate intermediary net worth is

$$dN_t = N_t\, dr_t^M - C_t\, dt + q_t K_t \int_\Omega \zeta_t(\omega)(dr_t^\omega - \varpi\, dt - dr_t^M)\, d\omega - N_t d\Xi_t. \tag{3.7}$$

In this equation, C_t is the aggregate rate of intermediary consumption.

Equilibrium Definition. An *equilibrium* is characterized by price processes q_t and P_t, the allocation $\xi_t(\omega)$ and $\zeta_t(\omega)$ of aggregate capital K_t, consumption rates of all agents and the rate of intermediary retirement, which map any history of Brownian shocks $\{Z_s,\ s \in [0,t]\}$ into the state of the economy at time t. The following conditions have to hold

(i) All markets, for capital, money and consumption goods, clear

(ii) Intermediaries choose consumption, investment $\zeta(\omega)$ and retirement to maximize utility and

(iii) Households of each type ω choose consumption and the allocation of wealth between money and capital to maximize utility

Scale Invariance and Markov Equilibria. Intuitively, because our setting is scale-invariant, the severity of financial frictions in our economy is quantified by the ratio of expert net worth to the size of the economy,

$$\eta_t = \frac{N_t}{K_t}.$$

We expect two economies in which the ratio η_t is the same look like scaled versions of one another, and look for a Markov equilibrium with the state variable η_t. In a Markov equilibrium, the price of capital $q_t = q(\eta_t)$ and the allocation of capital $\{\zeta_t(\omega), \xi_t(\omega)\} = \{\zeta(\eta_t, \omega), \xi(\eta_t, \omega)\}$ are functions of η_t, while the value of money is proportional to K_t, i.e. $P_t = p(\eta_t)K_t$.

The laws of motion of P_t and $p_t = p(\eta_t)$ are related by

$$dr_t^M = \frac{dP_t}{P_t} = \underbrace{\left(\mu_t^p + \mu_t^K + \mathrm{Cov}(d\epsilon_t^p, d\epsilon_t^K)\right)}_{\mu_t^M} dt + \underbrace{d\epsilon_t^p + d\epsilon_t^K}_{d\epsilon_t^M}, \quad \text{where} \quad \frac{dp_t}{p_t} = \mu_t^p \, dt + d\epsilon_t^p. \tag{3.8}$$

4 Equilibrium Conditions

In this section we derive equilibrium conditions, and develop a method to compute equilibria numerically. We focus on the following conditions from our definition of equilibria:

(i) The market-clearing conditions - two equations guarantee that all three markets clear, by Walras' law

(ii) Intermediaries borrow money to gain exposure to *each* technology ω optimally

(iii) Each household type ω chooses the allocation of wealth between money and its own technology optimally

Besides these conditions, we embed the optimal consumption rates into the market-clearing conditions directly, and embed the optimal retirement rate of intermediaries into the law of motion of the state variable η_t.

Market-Clearing Conditions. The condition that the market for consumption goods has to clear is

$$\rho(q(\eta_t) + p(\eta_t)) = \int_\Omega (\zeta(\eta_t, \omega) + \xi(\eta_t, \omega))\, a^\omega \, d\omega - \iota(q(\eta_t)), \tag{4.1}$$

since $(q_t + p_t)K_t$ is aggregate net worth of all agents, and the right hand side of (4.1) is net equilibrium output per unit of capital.

The market-clearing condition for capital is just

$$\int_\Omega \xi(\eta_t, \omega)\, d\omega + \int_\Omega \zeta(\eta_t, \omega)\, d\omega = 1. \tag{4.2}$$

Intermediaries' Optimal Portfolio Choice. The optimal portfolio choice conditions are based on equation (3.6). The marginal expected return rate from an investment in technology ω, financed by borrowing money, is $E[dr_t^\omega - \varpi\, dt - dr_t^M]$. The marginal risk of this investment is $d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M$, and, from (3.7), the risk of intermediary net worths is

$$d\epsilon_t^N = d\epsilon_t^M + \frac{q_t}{\eta_t} \int_\Omega \zeta_t(\omega')(d\epsilon_t^{\omega'} + d\epsilon_t^q - d\epsilon_t^M)\, d\omega'. \tag{4.3}$$

Therefore, intermediaries choose their portfolios optimally if and only if

$$E[dr_t^\omega - \varpi\, dt - dr_t^M] \leq \mathrm{Cov}(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M, d\epsilon_t^N), \tag{4.4}$$

with equality if intermediaries invest a nonzero amount in technology ω, i.e. $\zeta(\eta_t, \omega) > 0$.

Households' Optimal Portfolio Choice. Household of type ω chooses how to allocate its wealth between its own technology and money. The marginal expected return from an additional allocation to capital is $E[dr_t^\omega - dr_t^M]$, while the marginal risk of this investment is $d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M$. To determine the risk of this household's net worth, note that if holds capital with value $\xi(\eta_t, \omega) q_t K_t$ on the net worth of $\theta(\omega)(q_t + p_t - \eta_t)K_t$. Therefore, the household faces risk per unit of net worth of

$$d\epsilon_t^M + \frac{\xi(\eta_t, \omega) q_t}{\theta(\omega)(q_t + p_t - \eta_t)}(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M).$$

The optimal portfolio choice condition of household ω is

$$E[dr_t^\omega - dr_t^M] \leq \operatorname{Cov}\left(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M, d\epsilon_t^M + \frac{\xi(\eta_t,\omega)q_t}{\theta(\omega)(q_t+p_t-\eta_t)}(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M)\right), \quad (4.5)$$

with equality if the household invests a positive fraction of wealth in its technology, i.e. $\xi(\eta_t,\omega) > 0$.

The Law of Motion of η_t. The drift and volatility of q_t and p_t that enter equilibrium conditions (4.4) and (4.5) depend on the law of motion of the state variable η_t. Proposition 1 allows us to reduce the equilibrium conditions to a system of differential equations for $q(\eta)$ and $p(\eta)$, together with the portfolio choice processes $\{\zeta(\eta_t,\omega), \xi(\eta_t,\omega)\}$.

Proposition 1. *In equilibrium η_t follows*

$$\frac{d\eta_t}{\eta_t} = (\mu_t^p - \rho + \operatorname{Cov}(d\epsilon_t^\eta - d\epsilon_t^p, d\epsilon_t^\eta))\ dt + d\epsilon_t^\eta - d\Xi_t, \quad \text{where } d\epsilon_t^\eta = d\epsilon_t^N - d\epsilon_t^K. \quad (4.6)$$

Proof. From (4.4) and (4.3),

$$\frac{q_t}{\eta_t}\int_\Omega \zeta_t(\omega)(dr_t^\omega - \varpi dt - dr_t^M)d\omega = \frac{q_t}{\eta_t}\int_\Omega \zeta_t(\omega)\operatorname{Cov}(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M, d\epsilon_t^N)d\omega = \operatorname{Cov}(d\epsilon_t^N - d\epsilon_t^M, d\epsilon_t^N).$$

Therefore, from (3.7) and (3.8), the law of motion of N_t is given by

$$\frac{dN_t}{N_t} = \left(\mu_t^p + \mu_t^K + \operatorname{Cov}(d\epsilon_t^p, d\epsilon_t^K) - \rho + \operatorname{Cov}(d\epsilon_t^N - d\epsilon_t^M, d\epsilon_t^N)\right)\ dt + d\epsilon_t^N - d\Xi_t. \quad (4.7)$$

Using (3.2) and Ito's lemma,

$$\frac{d(1/K_t)}{1/K_t} = (\operatorname{Var}(d\epsilon_t^K) - \mu_t^K)\, dt - d\epsilon_t^K.$$

Since $\eta_t = N_t(1/K_t)$, we have

$$\frac{d\eta_t}{\eta_t} = \frac{dN_t}{N_t} + \frac{d(1/K_t)}{1/K_t} - \operatorname{Cov}(d\epsilon_t^N, d\epsilon_t^K)\, dt =$$

$$\left(\mu_t^p + \operatorname{Cov}(d\epsilon_t^p + d\epsilon_t^K - d\epsilon_t^N, d\epsilon_t^K) - \rho + \operatorname{Cov}(d\epsilon_t^N - d\epsilon_t^M, d\epsilon_t^N)\right)\ dt + d\epsilon_t^N - d\epsilon_t^K - d\Xi_t,$$

which simplifies to (4.6). □

Because the decision to retire is a real-option problem for experts, they exercise the option when η_t reaches a sufficiently high critical level η^*, at which competition among experts makes them indifferent between retiring or not. As a result, the state variable η_t follows (4.6) with $d\Xi_t = 0$ when $\eta_t < \eta^*$. The process ξ_t makes η_t reflect at η^*.

Differential Equations and Boundary Conditions. Equilibrium conditions (4.1), (4.2), (4.4) and (4.5) can be converted into a system of differential equations for $q(\eta)$, $p(\eta)$, as well as $\{\zeta(\eta_t,\omega), \xi(\eta_t,\omega)\}$, using Ito's lemma. Terms $d\epsilon_t^q$, $d\epsilon_t^p$, as well as μ_t^q and μ_t^p, can be expressed in terms of the drift and volatility of η_t as follows

$$d\epsilon_t^q = \frac{q'(\eta_t)\eta_t}{q(\eta_t)} d\epsilon_t^\eta, \quad d\epsilon_t^p = \frac{p'(\eta_t)\eta_t}{p(\eta_t)} d\epsilon_t^\eta,$$

$$\left(1 - \frac{p'(\eta_t)\eta_t}{p(\eta_t)}\right) \mu_t^p = \frac{p'(\eta_t)\eta_t}{p(\eta_t)} \left(\left(1 - \frac{p'(\eta_t)\eta_t}{p(\eta_t)}\right)(\sigma^\eta)^2 - \rho\right) + \frac{1}{2}\frac{p''(\eta_t)\eta_t}{p(\eta_t)}(\sigma^\eta)^2,$$

$$\mu_t^q = \frac{q'(\eta_t)\eta_t}{q(\eta_t)} \left(\mu_t^p - \rho + \left(1 - \frac{p'(\eta_t)\eta_t}{p(\eta_t)}\right)(\sigma^\eta)^2\right) + \frac{1}{2}\frac{q''(\eta_t)\eta_t}{q(\eta_t)}(\sigma^\eta)^2,$$

where $(\sigma^\eta)^2 = \mathrm{Var}(d\epsilon_t^\eta)$.

The boundary conditions are as follows. The value of η^* is uniquely pinned down by the parameter x, the utility boost that intermediaries receive upon retirement. In order to prevent abnormal profits or losses at the reflection point η^*, we need that

$$d\epsilon_t^q = d\epsilon_t^p \quad \Leftrightarrow \quad \frac{p'(\eta^*)}{p(\eta^*)} = \frac{q'(\eta^*)}{q(\eta^*)}.$$

The level of $q(\eta^*)$ has to be chosen to match the autarky boundary condition at $\eta = 0$, which corresponds to the equilibrium without intermediaries.

5 An Example

In this version of the paper, we present an example from a simplified version of the model with three types $\Omega = \{\omega_L, \omega_M, \omega_H\}$. The distribution of wealth among households is given by $\theta(\omega_L) = 65\%$, $\theta(\omega_M) = 35\%$ and $\theta(\omega_H) = 0$. All three production technologies lead the same gross output rate $a^\omega = 1$, but different depreciation rates that satisfy $\delta^{\omega_L} = \infty$, $\delta^{\omega_M} = 3\%$ and $\delta^{\omega_H} + \varpi = 0\%$. Note that, since the most productive households have zero net worth, we only need to know their productivity net of the monitoring cost. The monitoring cost

ϖ is large enough so that intermediaries do not want to invest in type ω_M technology. The investment function is $\Phi(i) = \sqrt{0.02i}$, and shocks to the agent's technologies are given by

$$d\epsilon_t^M = 0, \quad d\epsilon_t^H = \sigma dZ_t + \sigma' dZ_t',$$

where the Brownian motion Z_t represents the aggregate shocks, and Z_t' represent idiosyncratic shocks, which are independent and fully diversifiable across individual agents.

The three household types play very specific distinct roles in this example. The low types create a demand for money. The middle types, who do not get financing from the intermediaries, exist to ensure that the average of the shocks that hit the economy is distinct from the average of the shocks that hit intermediary assets. Intermediaries invest in projects with higher and more concentrated risk compared the risk of their liabilities, i.e. money, which in the long run has the same risk as the economy as a whole, that is, $d\epsilon_t^K$.[5]

Regarding preferences, we set the discount rate to $\rho = 5\%$, and select the intermediaries' effort cost x to induce them to retire at $\eta^* = 2.1$.

Figure 1 presents market prices of money and capital, as well as capital allocation, intermediary leverage and the volatility of η_t.

As we expected, the price of capital increases with η while the price of money decreases with η. Intermediaries are able to hold more capital for higher η, i.e. $\zeta(\eta, \omega_H)$ is increasing in η. At the same time, when η is low, then lower price of capital encourages intermediaries to take on greater leverage. As $\eta \to 0$, the prices of capital and money converge to $q(0) = \theta(\omega_M)a^{\omega_M}/\rho = 7$ and $p(0) = \theta(\omega_L)a^{\omega_M}/\rho = 13$, since η_t stays at zero if it ever reaches 0. Both $q'(\eta)$ and $p'(\eta)$ are large near $\eta = 0$, which creates significant amplification in the depressed region. A negative macro shock causes the price of capital to fall and the price of money to rise, hitting intermediaries on both asset and liability sides of their balance sheets. As a result, the volatility of η_t, which reflects endogenous risk, is high near $\eta = 0$.

Figure 2 displays the expected rate of economic growth, investment, the volatility of capital and money, and $\mu_t^q - \mu_t^p$, which enters the expected return of a levered position in capital. As expected, economic growth and investment are higher when η_t is larger. The bottom left panel shows the reaction of the values of capital and money to aggregate shocks. Capital becomes cheaper after a negative macro shock as expert demand for capital decreases following a contraction of their balance sheets. On the other hand, a negative macro shock

[5] If there were only two household types, unproductive and productive, then there would be an equilibrium in which q_t and p_t are constant, and both intermediary assets and liabilities are perfectly hedged with risk $d\epsilon_t^K$.

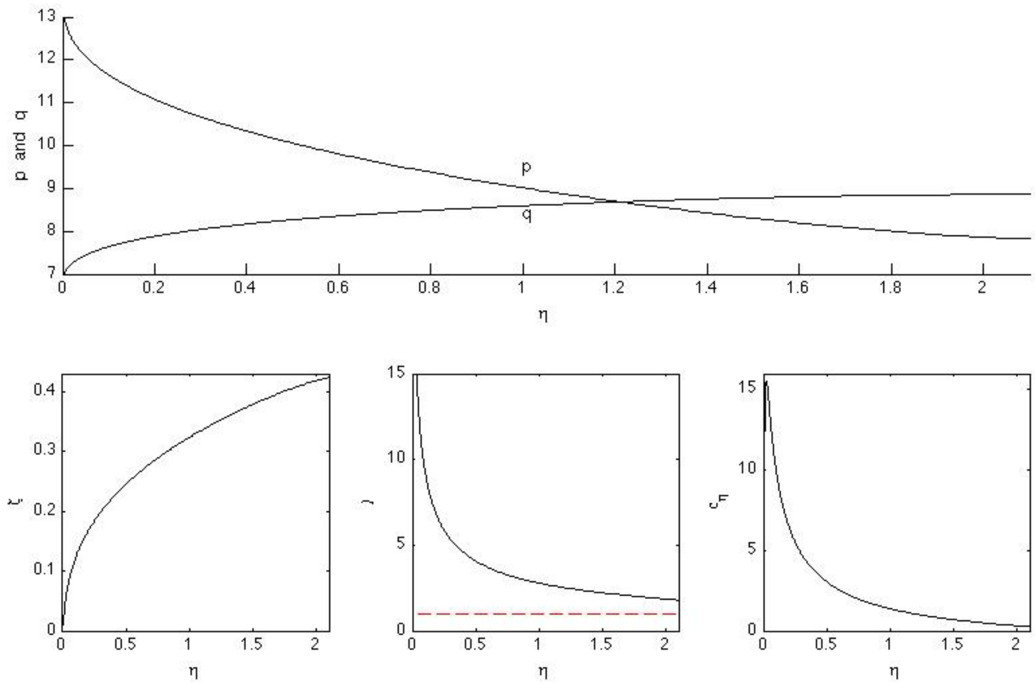

Figure 1: Asset prices, capital allocation, leverage and σ_t^η in dynamic equilibrium.

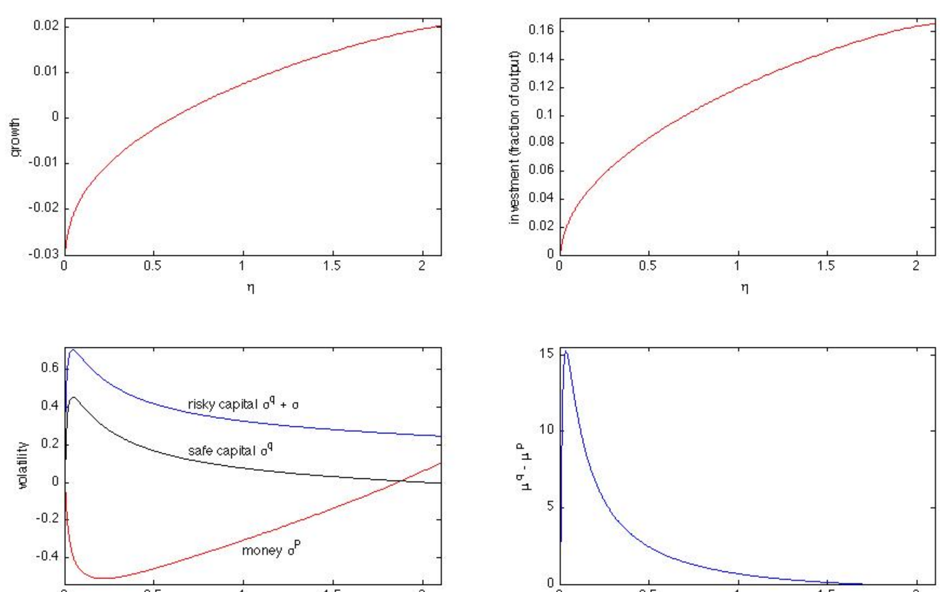

Figure 2: Economic growth, investment, asset price volatilities and returns in equilibrium.

generally increases the value of money, except for η_t near η^*. As intermediary balance sheets contract, the amount of inside money they create decreases and, to offset this effect, the value of outside money has to rise. Note also that the volatility of a levered position in capital is highest when η is low. This volatility is due to endogenous risk, which is created as the macro shocks get amplified through the intermediaries' balance sheets. High volatility leads to a high required risk premium. Hence, $\mu_t^q - \mu_t^p$ is highest when volatility due to endogenous risk is highest.

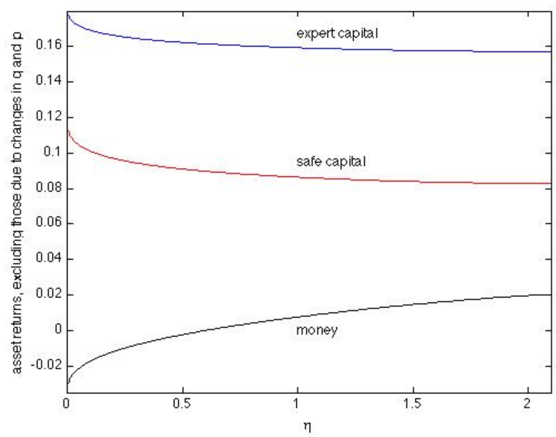

Figure 3: Fundamental components of the returns on capital and money.

The impact of μ_t^q and μ_t^p on the returns of capital and money can be significant. For η_t between 0 and 0.5, these valuation effects easily overwhelm the effects of the changes in fundamentals on the return on money and capital. Recall that the fundamental return on capital stems from output and investment while the fundamental return on money stems from economic growth, i.e. money buys a fixed fraction of the economy in the long run. Figure 3 displays the fundamental components of the return on capital held by intermediaries and type ω_M households, as well as money. This component of the return on capital decreases in η as capital becomes more expensive. At the same time, the return on money increases with the expected rate of economic growth.

Importantly, Figure 4 displays various money quantities in our equilibrium. The top panel of Figure 4 exhibits the value of all outside money $p(\eta)$, as well as the total value of money and the value of inside money. As η increases, intermediaries invest more and create more inside money. As the value of outside money decreases at the same time, the money multiplier expands, as shown on the bottom panel of Figure 4.

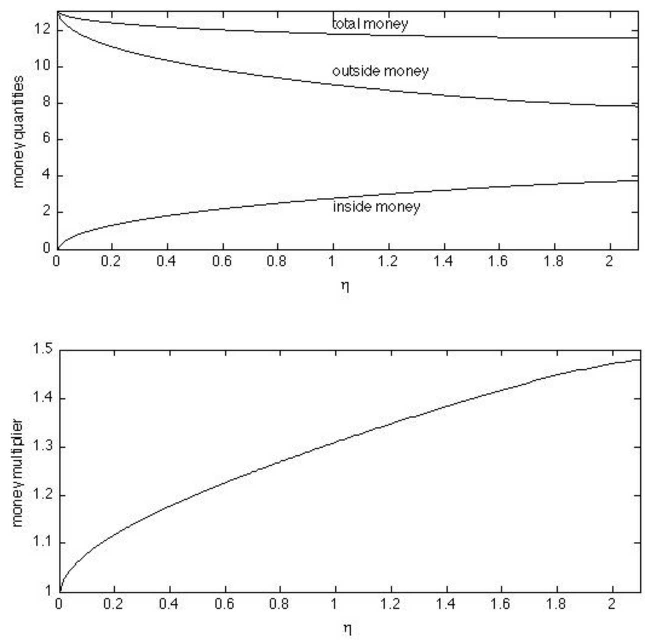

Figure 4: Inside and outside money, and the money multiplier.

6 Long-Term Bonds and Monetary Policy

In this section we analyze how a central bank can affect the equilibrium through monetary policy that involves setting short-term interest rates and using open-market operations to change the composition of monetary instruments. As a theoretical exercise, we would like to isolate monetary policies from more general fiscal policies. We do not consider policies that involve government spending, taxation, or explicit redistribution. Rather, we consider a central bank that only has the authority to print money and can issue perpetual long-term bonds, securities that promise a perpetual stream of monetary payments in the future. Interest on long-term bonds can be financed, again, by printing money or selling more long-term bonds.

Unlike in our benchmark model, with active monetary policy the amount of money in the economy is no longer fixed. The quantity of money changes over time, because anybody (households or intermediaries) can deposit money with the central bank to earn a nominal interest rate of r_t. In addition, the central bank issues perpetual long-term bonds and sells them in the open market in exchange for money. The central bank can also print money to repurchase some of the bonds. We summarize the policy that affects the composition of outstanding monetary instrument through a process b_t, where $b_t K_t$ is the *real* market value of all long-term bonds outstanding. Thus, two policy instruments - short-term interest rates and open-market operations - are summarized by a pair of processes $\{r_t, b_t\}$.

For a given policy, we denote the equilibrium *nominal* price of long-term bonds per unit coupon rate by B_t. For example, if $r_t = r$ for all t, then the nominal value of a bond that pays 1 in perpetuity is $B_t = 1/r$. If instead in downturns the central bank lowers r_t, then we expect B_t to increase. We denote the equilibrium law of motion of B_t by

$$\frac{dB_t}{B_t} = \mu_t^B \, dt + \epsilon_t^B. \tag{6.1}$$

With long-term bonds, the total *real* wealth in the economy is given by

$$(q_t + p_t + b_t)K_t,$$

where $p_t K_t$ still denotes the real market value of money and $b_t K_t$ is the real market value of long-term bonds.

The stochastic laws of motion of both p_t and b_t,

$$\frac{db_t}{b_t} = \mu_t^b \, dt + \epsilon_t^b, \tag{6.2}$$

reflect both the changes in the real value of money and long-term bonds, as well as the open-market operations conducted by the central bank. One has to take those central bank transactions into account to figure out exactly how the total values of money and bonds outstanding are related to the returns that money and bonds earn. The following proposition disentangles the algebra of open-market operations.

Proposition 2. *Given our notation, the real return on money can be expressed as*

$$dr_t^M = (\mu_t^p + \mu_t^b + \mu_t^K + \mathrm{Cov}(d\epsilon_t^p + d\epsilon_t^b, d\epsilon_t^K))\, dt + d\epsilon_t^p + d\epsilon_t^b + d\epsilon_t^K - \frac{b_t}{p_t + b_t}(dr_t^B - dr_t^M), \tag{6.3}$$

where the difference between the real returns on bonds and money is

$$dr_t^B - dr_t^M = \left(1/B_t - r_t + \mu_t^B + \mathrm{Cov}(d\epsilon_t^B, d\epsilon_t^M)\right)\, dt + d\epsilon_t^B. \tag{6.4}$$

Proof. To see why equation (6.4) holds, consider an agent who borrows money to buy bonds. This agent receives interest on bonds of $1/B_t$ per dollar investment, and has to pay interest

r_t on borrowed money. The agent is also exposed to the fluctuations of the price of bonds relative to money, $\mu_t^B\, dt + d\epsilon_t^B$, but is perfectly hedged to the fluctuations of the real value of money. The fluctuations in the value of money add to the agent's return only insofar as they are correlated to the fluctuations of the nominal price of bonds, leading to the term $\mathrm{Cov}(d\epsilon_t^B, d\epsilon_t^M)$.

Regarding (6.3), note that an investment into all monetary instruments in the economy (money and long-term bonds) earns a return of

$$\frac{d((p_t + b_t)K_t)}{(p_t + b_t)K_t} = (\mu_t^p + \mu_t^b + \mu_t^K + \mathrm{Cov}(d\epsilon_t^p + d\epsilon_t^b, d\epsilon_t^K))\, dt + d\epsilon_t^p + d\epsilon_t^b + d\epsilon_t^K. \tag{6.5}$$

Of this portfolio, fraction $b_t/(p_t + b_t)$ is invested in bonds, so

$$\frac{d((p_t + b_t)K_t)}{(p_t + b_t)K_t} = dr_t^M + \frac{b_t}{p_t + b_t}(dr_t^B - dr_t^M). \tag{6.6}$$

Combining (6.5) and (6.6), we get (6.3). □

Equilibrium Equations. We are now ready to write down equilibrium equations, which are analogous to those in Section 4. The only difference is that now, agents are free to hold not only money, but also bonds. To keep things simple, we consider monetary policies that are Markov in the state variable η_t. We are particularly interested in policies that lower the short-term interest rate in downturns and raise it in booms.

We denote the fraction of bonds allocated to intermediaries by ζ_t^B, and the density of bond holdings across household types by $\xi_t^B(\omega)$. Then the market-clearing condition for bonds is

$$\zeta_t^B + \int_\Omega \xi_t^B(\omega)\, d\omega = 1. \tag{6.7}$$

While the market-clearing condition for capital is still (4.2), the market-clearing condition for output changes slightly to

$$\rho(q_t + p_t + b_t) = \int_\Omega (\zeta_t(\omega) + \xi_t(\omega))\, a^\omega\, d\omega - \iota(q_t). \tag{6.8}$$

To write down the optimal portfolio choice conditions, note that intermediaries are exposed to the risk of

$$d\epsilon_t^N = d\epsilon_t^M + \frac{q_t}{\eta_t}\int_\Omega \zeta_t(\omega')(d\epsilon_t^{\omega'} + d\epsilon_t^q - d\epsilon_t^M)\, d\omega' + \frac{b_t}{\eta_t}\zeta_t^B\, d\epsilon_t^B \tag{6.9}$$

per unit of net worth, while household of type ω face the risk

$$d\epsilon_t^{N,\omega} = d\epsilon_t^M + \frac{\xi_t(\omega)q_t}{\theta(\omega)(q_t + p_t - \eta_t)}(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M) + \frac{\xi_t^B(\omega)b_t}{\theta(\omega)(q_t + p_t - \eta_t)}\, d\epsilon_t^B. \tag{6.10}$$

The optimal portfolio choice equations are

$$E[dr_t^\omega - \varpi\, dt - dr_t^M] \le \mathrm{Cov}(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M, d\epsilon_t^N), \quad (= \text{ if } \zeta_t(\omega) > 0), \tag{6.11}$$

$$E[dr_t^B - dr_t^M] \le \mathrm{Cov}(d\epsilon_t^B, d\epsilon_t^N), \quad (= \text{ if } \zeta_t^B > 0), \tag{6.12}$$

$$E[dr_t^\omega - dr_t^M] \le \mathrm{Cov}\left(d\epsilon_t^\omega + d\epsilon_t^q - d\epsilon_t^M, d\epsilon_t^{N,\omega}\right), \quad (= \text{ if } \xi_t(\omega) > 0), \tag{6.13}$$

$$\text{and} \quad E[dr_t^B - dr_t^M] \le \mathrm{Cov}\left(d\epsilon_t^B, d\epsilon_t^{N,\omega}\right), \quad (= \text{ if } \xi_t^B(\omega) > 0). \tag{6.14}$$

The Effects of Monetary Policy. In our model, monetary policy can mitigate downturns in two ways. First, by increasing the money supply in downturns, reducing the deflationary spiral. Second, by redistributing wealth towards intermediaries, whose holdings of long-term bonds increase in value due to cuts in interest rates. Both of these effects appear when the central bank lowers r_t when η_t goes down.

To see why the deflationary spiral is mitigated by interest rate cuts in downturns, note that the demand for money stems from the unproductive households' desire to save. In the baseline model, when the intermediary sector creates less inside money in downturns, household demand for outside money goes up. That raises the value of outside money, leading to deflation. With a central bank, unproductive households can use both money and long-term bonds to save. The central bank can counteract the contraction of the money multiplier in downturns by cutting interest rates, and thereby increasing the value of long-term bonds. That satisfies the households' demand for savings, and eases the deflationary pressure.

The redistributing effects arise because intermediaries can hold long-term bonds to hedge losses in downturns. The increase in the value of long-term bonds offsets some of the losses that intermediaries suffer on their assets. Thus, interest rate cuts in downturns recapitalize intermediaries. Also, due to off-setting risks of capital and long-term bonds, intermediaries can absorb more capital risk in downturns, so they can lend more and create more inside money. Of course, policies that help intermediaries in downturns may create greater incentives for risk-taking ex-ante.

7 Conclusion

// to be completed //

8 Bibliography

Bernanke, B. (1983) "Non-Monetary Effects of the Financial Crisis in the Propagation of the Great Depression", *American Economic Review*, Vol. 73, 257-276.

Bernanke, B. and A. Blinder (1989) "Credit, Money, and Aggregate Demand", *American Economic Review, Papers and Proceedings*, Vol. 78, No. 2, 435-439.

Bernanke, B. and M. Gertler (1989) "Agency Costs, Net Worth, and Business Fluctuations", *American Economic Review*, Vol. 79, No. 1, 14-31.

Bernanke, B., M. Gertler and S. Gilchrist (1999) "The Financial Accelerator in a Quantitative Business Cycle Framework", *Handbook of Macroeconomics* (Chapter 21), 1341-1393.

Bewley, T. (1980) "The Optimum Quantity of Money", in *Models of Monetary Economics*, eds. J. Kareken and N. Wallace. Minneapolis, Minnesota: Federal Reserve Bank.

Brunnermeier, M. K., T. Eisenbach, and Y. Sannikov (2010) "Macroeconomics with Financial Frictions: A Survey", in *Proceedings of the Econometric Society World Congress 2010 in Shanghai*, eds. E. Dekkel, Cambridge University Press.

Brunnermeier, M. K. and Y. Sannikov (2010) "A Macroeconomic Model with a Financial Sector", working paper, Princeton University.

Cagan, P. (1979) *Persistent Inflation: Historical and Policy Essays*, Columbia university Press.

Cordia, V. and M. Woodford (2010) "Credit Spreads and Monetary Policy", working paper, Columbia University.

Diamond, D. W. (1984) "Financial Intermediation and Delegated Monitoring", *The Review of Economic Studies*, 51(3), 393414.

Diamond, D. and R. Rajan (2006) "Money in a Theory of Banking",' *American Economic Review*, March 2006.

Friedman, M. and A. J. Schwartz. (1963) *A Monetary History of the United States, 1867-1960*, Princeton University Press.

Gurley, J and E. Shaw (1955) "Financial Aspects of Economic Development", *American Economic Review*, September 2006, 515-538.

He, Z., and A. Krishnamurthy (2010) "Intermediary Asset Prices", working paper, Northwestern University.

He, Z., and A. Krishnamurthy (2010) "A Model of Capital and Crises", forthcoming in *Review of Economic Studies.*

Holmstrom, B., and J. Tirole (1997): "Financial Intermediation, Loanable Funds, and the Real Sector", *The Quarterly Journal of Economics*, 112(3), pp.663691.

Kiyotaki, N. and J. Moore (1997) "Credit Cycles", *Journal of Political Economy*, Vol. 105, No. 2, 211-48.

Kiyotaki, N. and J. Moore (2008) "Liquidity, Business Cycles, and Monetary Policy", working paper, Princeton University.

Patinkin D. (1965) *Money, Interest and Prices: An Integration of Monetary and Value Theory*, Harper & Row NY.

Samuelson, P. A. (1958) "An exact consumption loan model of interest, with or without the social contrivance of money" *Journal of Political Economy* 66, 467-82.

Stein, J. (2010) "Monetary Policy as Financial-Stability Regulation", working paper, Harvard University.

Scheinkman, J. A. and L. Weiss (1986) "Borrowing Constraints and Aggregate Economic Activity", Econometrica, Vol. 54, 23-45

Tobin, J. (1969) "A General Equilibrium Approach to Monetary Theory", Journal of Money, Credit and Banking, Vol. 1, No. 1, 15-29.