



# THE ITHEORY OF MONEY

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Updates: [http://www.princeton.edu/~markus/research/papers/i\\_theory\\_slides.pdf](http://www.princeton.edu/~markus/research/papers/i_theory_slides.pdf)

# || Motivation

- Unified framework to study financial and monetary stability
- Combines **intermediation (credit)** and **money** - **inside money**
- Value of money endogenous - store of value, liquidity
  - (Samuelson, Bewley, Kiyotaki-Moore...)
- Fisher (1933) **deflationary spiral** after *negative* productivity shock
  - Negative shock hits **asset** side of intermediaries' balance sheets and is amplified through leverage and volatility dynamics
  - Decline in inside money, leads to deflationary pressure that hits intermediaries' balance sheet on the **liability** side

*Difference to literature!*
- **Inside money and outside money**
  - "Endogenous" money multiplier =  $f(\text{health of intermediary sector})$
- Monetary policy (interest rates, open market operations)
  - Fills in demand for money when money multiplier contracts
  - Redistribution from/towards intermediary sector
  - Difference to New Keynesian framework

# Some Literature

- Medium of exchange (new) monetarists
- Store of value & liquidity
  - Samuelson's OLG Consumption smoothing
  - Bewley Precaution savings for
    - Scheinkman & Weiss uninsurable endowment shocks
    - Homstrom & Tirole to keep project running
    - Kiyotaki & Moore (2008) new investment opportunity + "resell constraint"
- Financial stability & monetary policy
  - Diamond & Rajan (2006)
  - Stein (2010)
  - Curdia & Woodford (2010) New Keynesian framework
- Economies with financial frictions
  - Bernanke, Gertler & Gilchrist, Kiyotaki & Moore, Geanakoplos, He & Krishnamurthy, Brunnermeier & Sannikov 2010

# Outline of Modeling Ideas

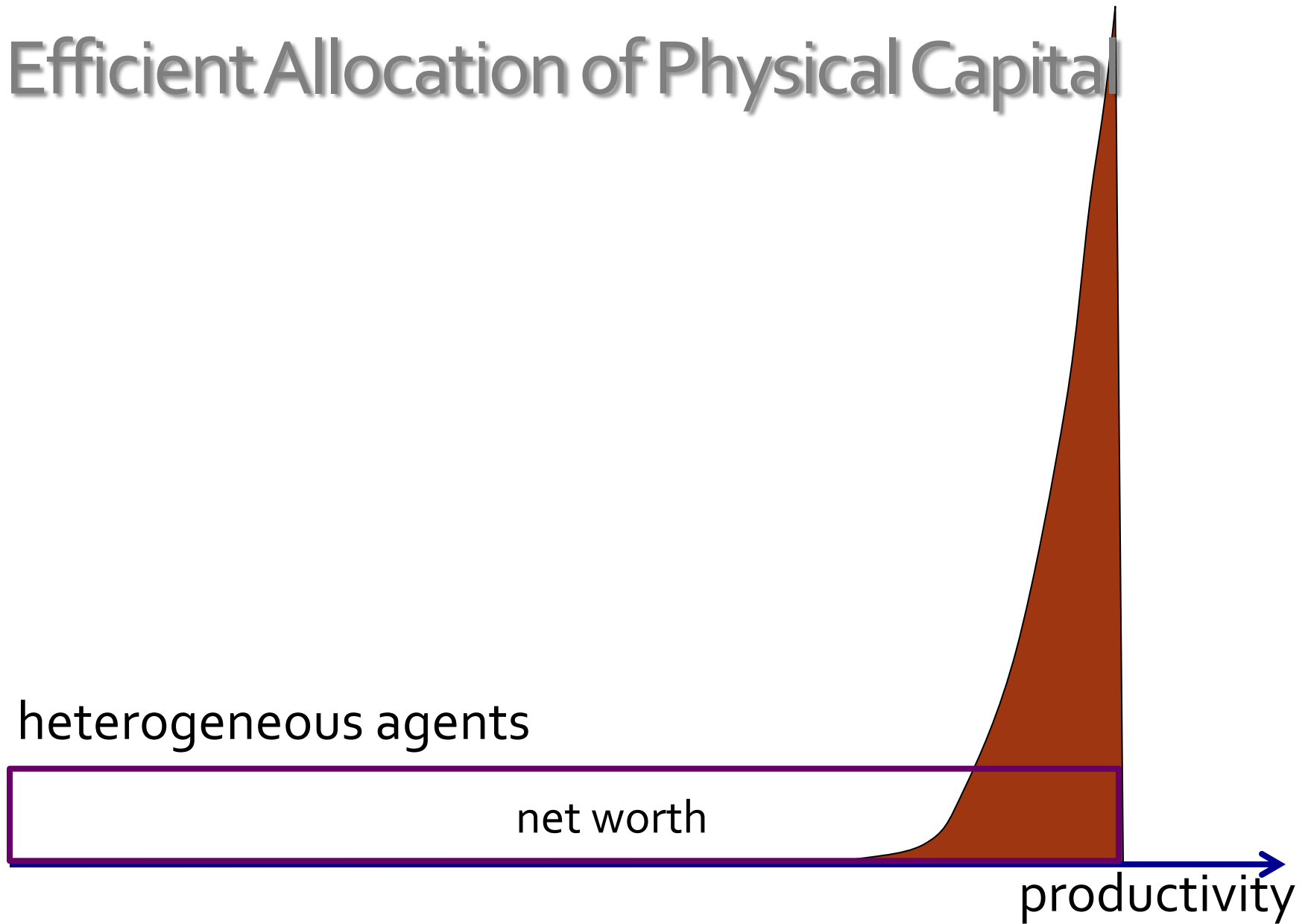
heterogeneous agents

net worth

productivity



# Efficient Allocation of Physical Capital



# Allocation with Extreme Financial Constraint

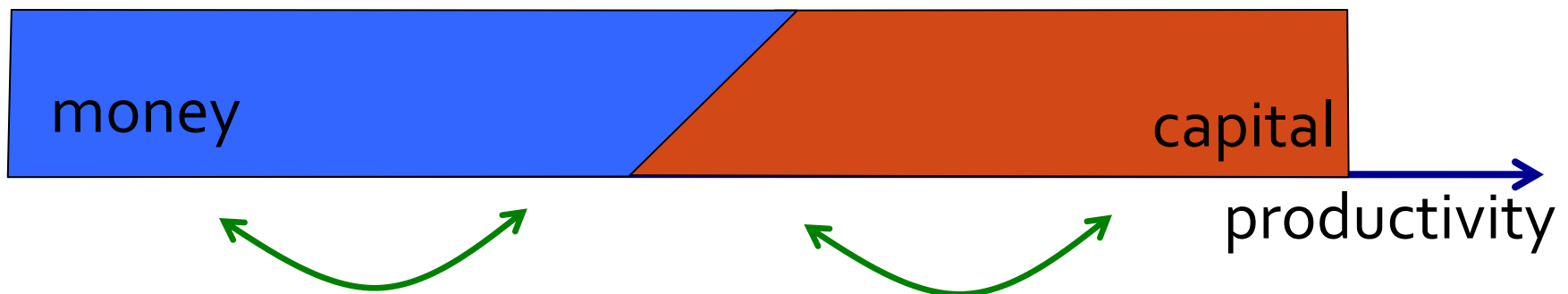
heterogeneous agents

capital

productivity

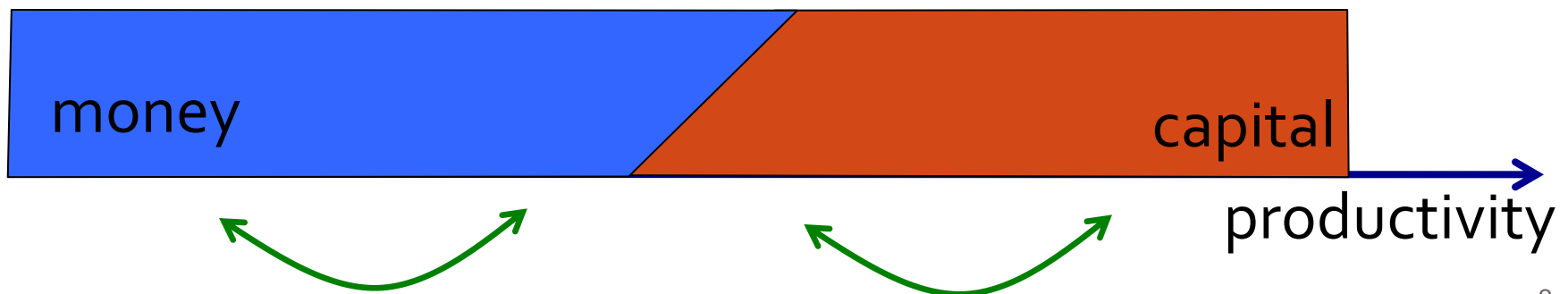
# Switching Types and Money

- Money (gold) intrinsically worthless
- Agents willing to hold money if someone (productive agents becoming unproductive) will want to hold money later



# Switching Types and Money

- Money (gold) intrinsically worthless
- Agents willing to hold money if someone (productive agents becoming unproductive) will want to hold money later
- Inefficiencies
  - Allocation (money does not generate any income)
  - Underinvestment (price of capital and hence investment is low)





# Two polar cases

Economy	Assets	Value of fiat money
Frictions (severe)	No claims	high
Frictionless	Issue claims <ul style="list-style-type: none"><li>• Debt</li><li>• Equity</li></ul>	low

# Two polar cases introducing intermediaries

Economy	Assets	Value of fiat money		Intermediaries' capitalization
Frictions (severe)	No claims	high		defunct
Frictionless	Issue claims <ul style="list-style-type: none"><li>• Debt</li><li>• Equity</li></ul>	low		perfect

## ■ Role of intermediaries

- Relax financing constraint by monitoring productive agents
- Have to take on productive agent's equity risk (so that they have incentive to monitor)
- Intermediation depends on their ability to absorb risk net worth of intermediaries

# Intermediaries and lending

- Monitoring technology  
Diamond (1984)  
Homstrom-Tirole (1997)

## intermediaries

Assets	Liabilities
loans to entrepreneurs	deposits
entrepreneur equity	net worth

## heterogeneous agents

deposits  
money

The diagram illustrates the flow of funds between intermediaries and heterogeneous agents. The top part shows a balance sheet for intermediaries, where assets (loans to entrepreneurs and entrepreneur equity) equal liabilities (deposits and net worth). The bottom part is a graph with a horizontal axis representing time. A blue area at the bottom represents 'money', which increases over time. Above it, a purple area represents 'deposits', which also increases. The total height of the purple and blue areas represents the total funds available to intermediaries, which then flow into the 'loans to entrepreneurs' part of the balance sheet.

# Intermediaries and lending

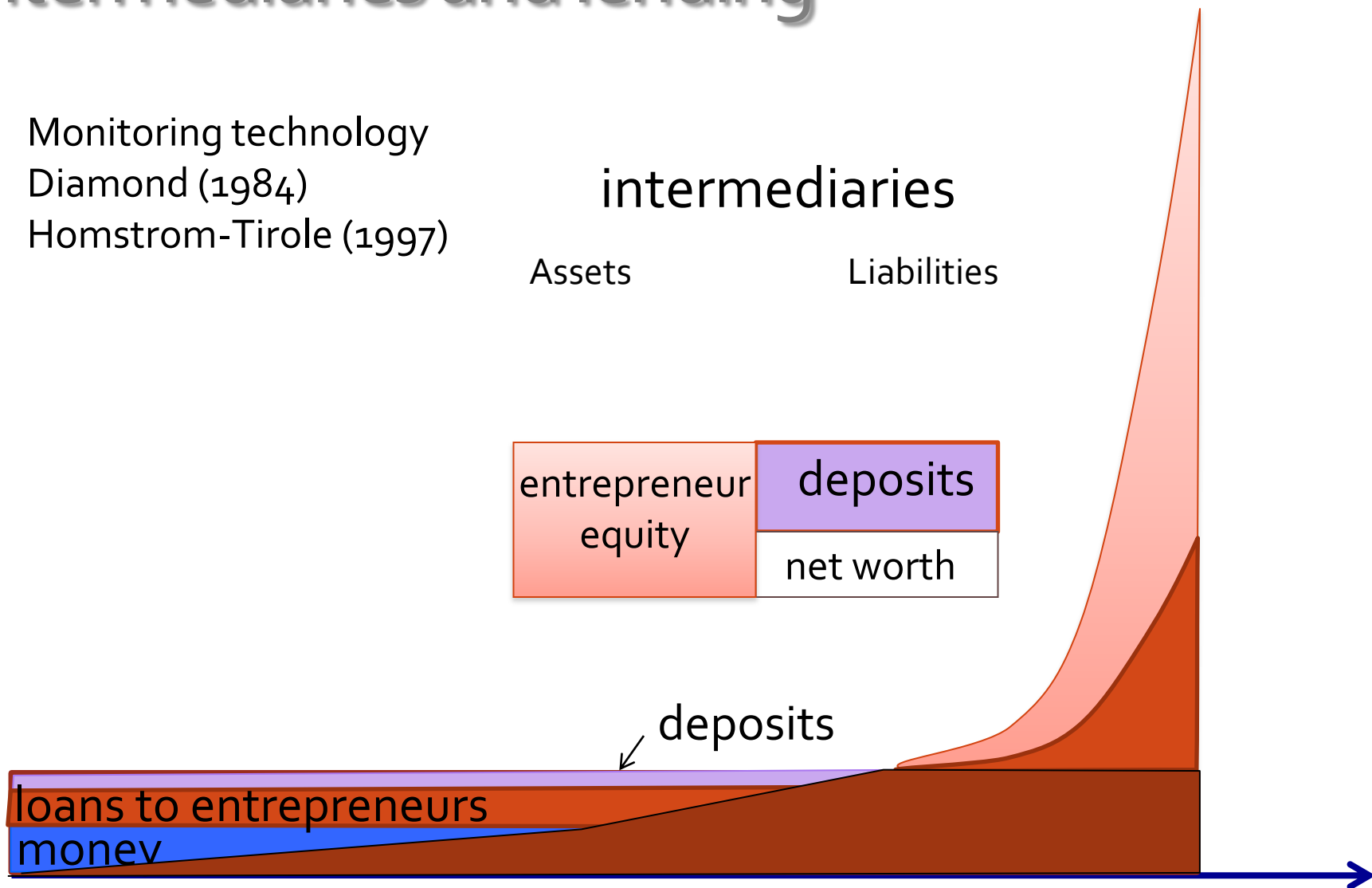
- Monitoring technology  
Diamond (1984)  
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## intermediaries

Assets

Liabilities

entrepreneur equity	deposits
	net worth

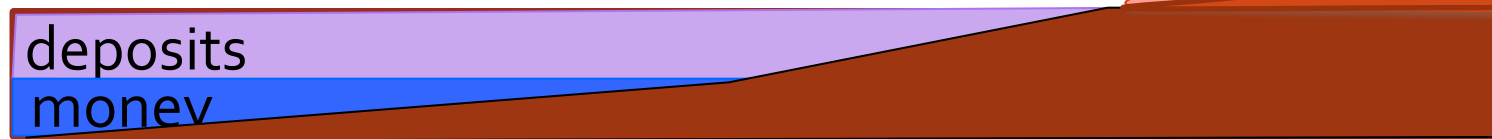


# Intermediaries and lending

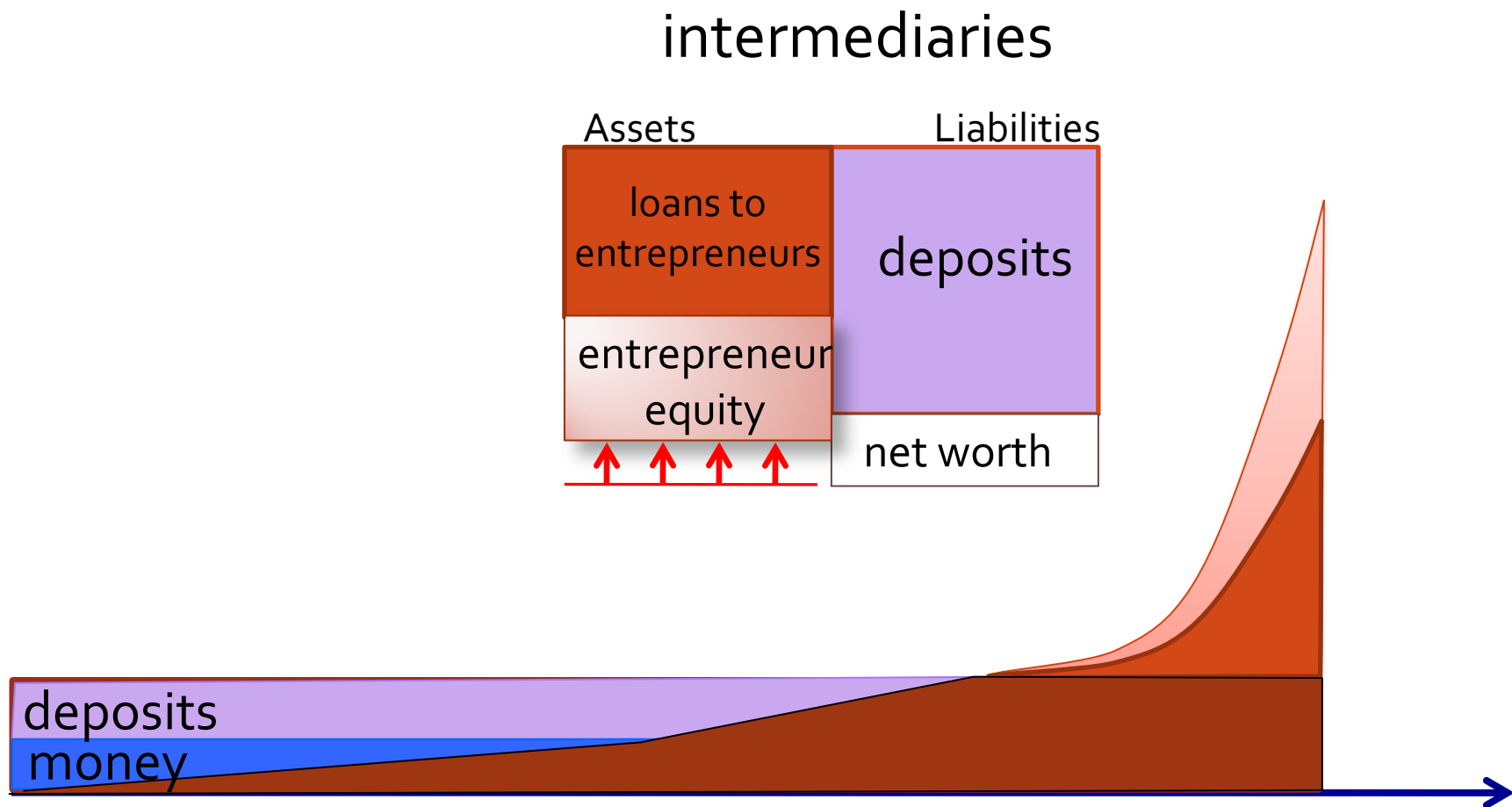
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## intermediaries

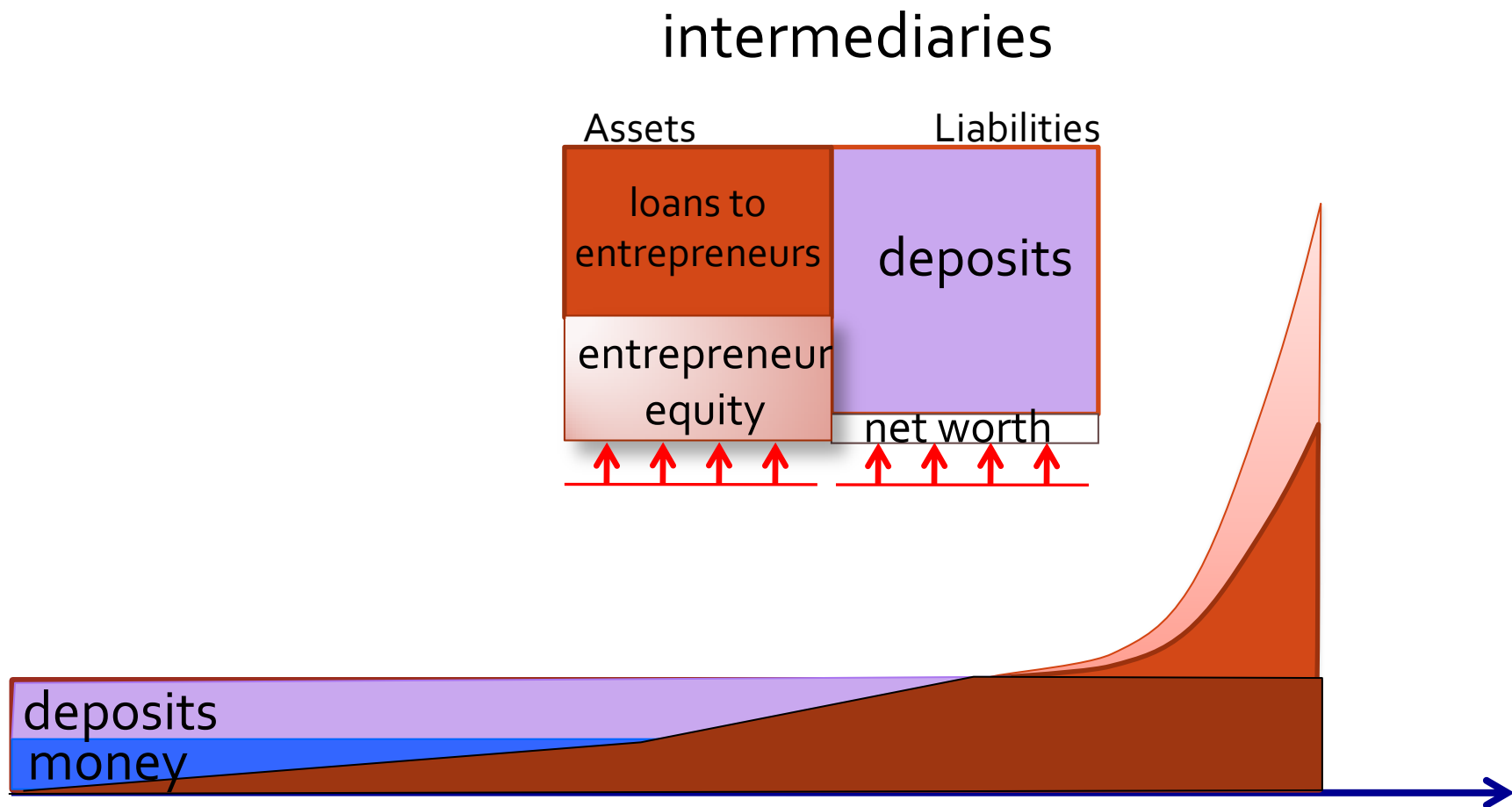
Assets	Liabilities
loans to entrepreneurs	deposits
entrepreneur equity	net worth



# || Negative Macro Shocks



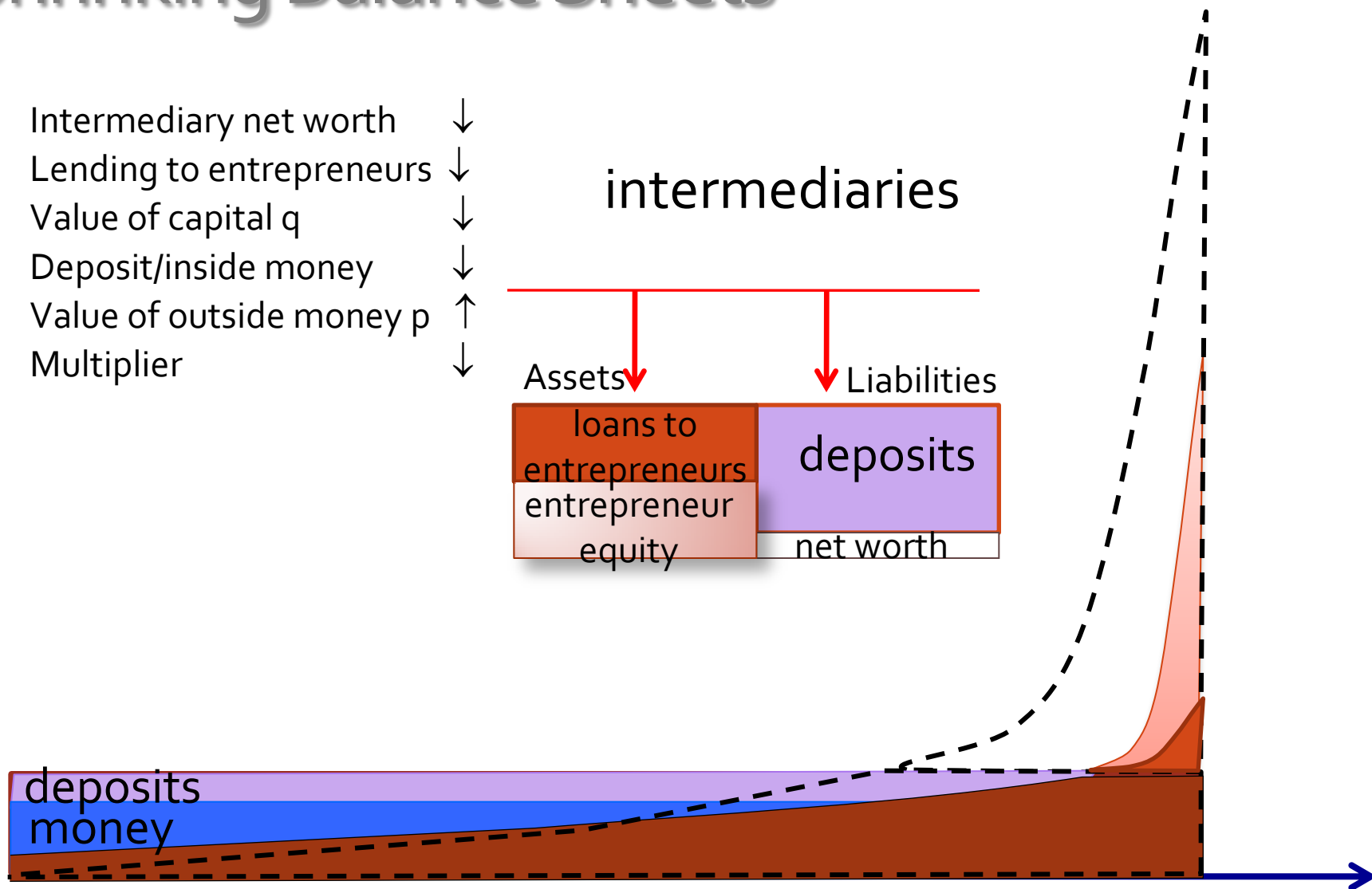
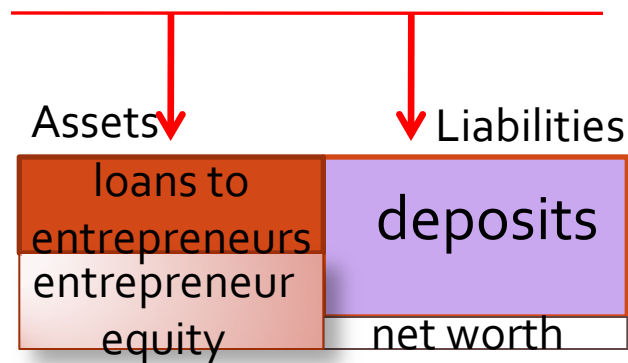
# || Negative Macro Shocks



# Shrinking Balance Sheets

- Intermediary net worth ↓
- Lending to entrepreneurs ↓
- Value of capital  $q$  ↓
- Deposit/inside money ↓
- Value of outside money  $p$  ↑
- Multiplier ↓

intermediaries





# Overview

- **Passive** monetary policy: “Gold standard”
  - Quantity of outside money fixed
  - Interest rate zero
  - When a negative macro shock hits intermediaries
    - quantity of inside money shrinks
    - value of outside money increases - deflationary spiral
    - intermediaries are hit on the liability side
- **Active Monetary Policy**
  - Introduce long-term bond
    - Short-term interest rate policy
    - Value of long-term bonds rises in downturns – substitute for reduction of inside money
    - Asset purchase and OMO
  - Redistributational effects
- Comparison to New Keynesian and Monetarism

# The Model: Technology

consumption rate

Output:  $y_t^\omega = a^\omega k_t^\omega = (c_t^\omega + i_t^\omega) k_t^\omega$

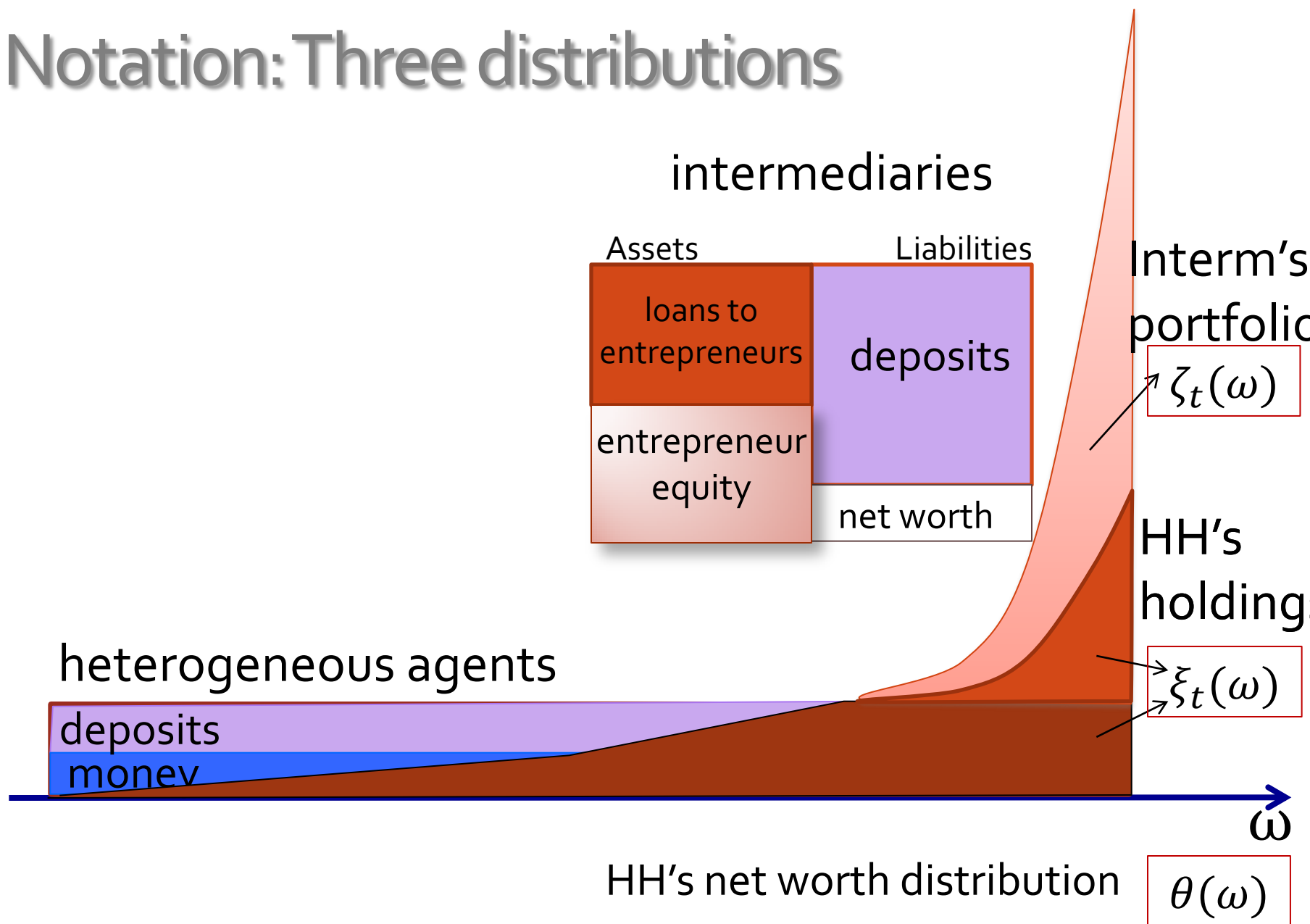
Capital:  $dk_t^\omega = (\Phi(i_t^\omega) - \delta^\omega)k_t dt + d\varepsilon_t^\omega$   
 $\Phi(0) = 0, \Phi' > 0, \Phi'' < 0$   
 $Cov[\varepsilon_t^\omega, \varepsilon_t^{\omega'}]$

## heterogeneous agents

Outside money (gold)  
is in fixed supply

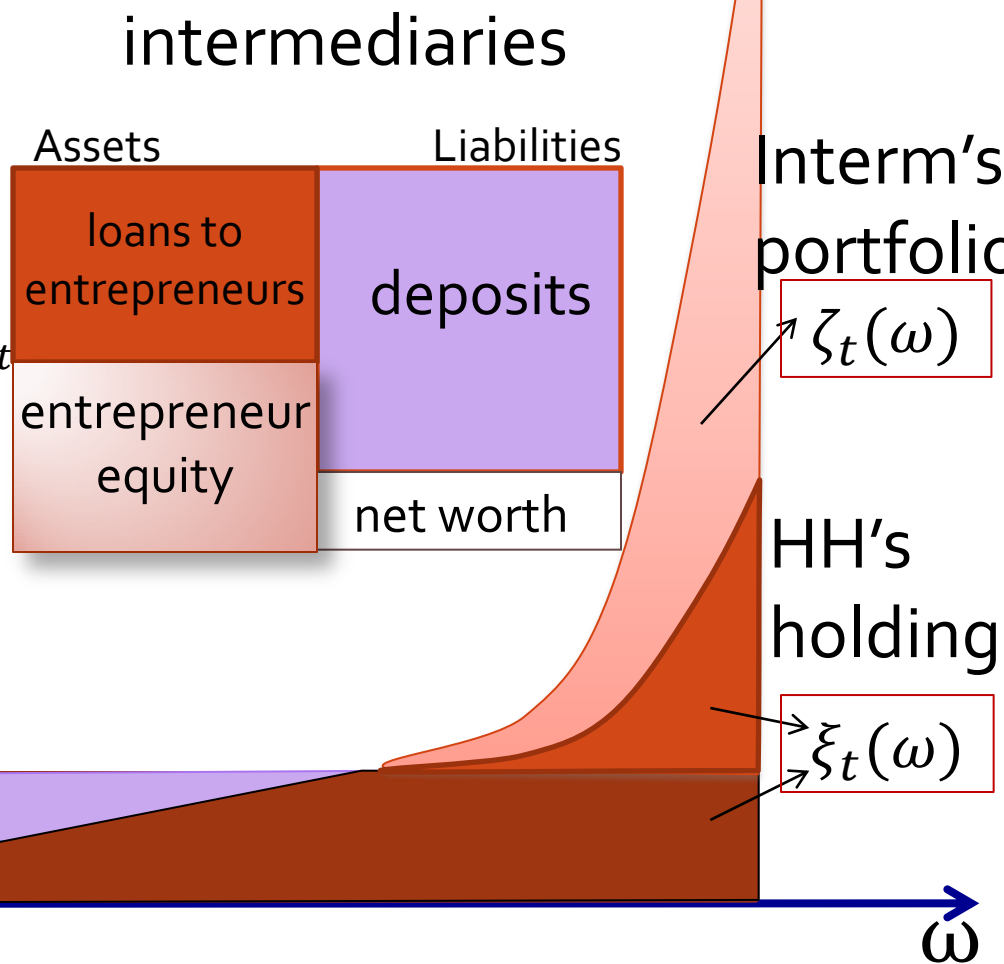
$$\omega$$

# Notation: Three distributions



# Scale Invariance

- Allocation of capital
  - $\int \zeta_t(\omega) d\omega + \int \xi_t(\omega) d\omega = 1$
  - All capital in the economy  $= K_t$
  - Capital value (in output)  $= q_t K_t$
- Outside money supply  $= 1$ 
  - Value of money (in output)  $= P_t = p_t K_t$



- $\theta(\omega)(q_t K_t + P_t - N_t)$  HH's net worth distr.  $\int \theta(\omega) d\omega = 1$

$$\theta(\omega)$$

# || The Model: Preferences

- All agents have logarithmic utility with discount rate

$$E \left[ \int_0^{\infty} e^{-\rho t} \log c_t dt \right]$$

- Retirement: intermediary gets utility boost, when it decides to become a household forever
- Implications of log utility:
  - Consumption =  $\rho \times \text{net worth}$
  - Required return =  $\text{Cov}[\text{asset risk}, \text{net worth risk}]$
  - Consumption is independent of investment opportunity
  - Asset demands are myopic  
(no Mertonian hedging demand, no precautionary motive)

# Equilibrium Definition

- For each history of shocks  $\{\{d\varepsilon_s^\omega\}_\omega, s \in [0, t]\}$ 
  - $q_t$  the price of physical capital
  - $P_t = p_t K_t$  the value of money
  - $\zeta_t(\omega), \xi_t(\omega)$  the allocation of capital
  - $i_t^\omega$  the rate of entrepreneurs investment
  - rates of consumption of all agents
  - Retirement rate of intermediaries

such that

- Given prices all agents choose portfolios & consumption to maximize utility, intermediaries choose optimally when to retire
- Markets for capital, money and consumption goods clear

# Derivation - Roadmap

- Individual choices
  - $c_t = \rho * \text{net worth}$
  - $i_t^\omega$
  - Required excess return = Cov [asset risk, net worth risk]
    - Postulate:  $dq_t = \mu_t^q dt + d\varepsilon^q$  and  $dp_t = \mu_t^p dt + d\varepsilon^p$
- Market clearing
  - Endogenously determines  $\mu_t^q, d\varepsilon_t^q, \mu_t^p, d\varepsilon_t^p$
- Derive  $\mu_t^q, d\varepsilon_t^q, \mu_t^p, d\varepsilon_t^p$  as functions of  $\eta$
- Need law of motion of  $\eta$ 
  - Depends on postulated price processes  $q_t$  and  $p_t$  (fixed point)

# Internal investment decision

$$dk_t^\omega = (\Phi(i_t^\omega) - \delta^\omega)dt + d\varepsilon_t^\omega$$

- Given the price of capital  $q_t$ , the optimal investment solves

$$\max_i \Phi(i)q_t - i \Rightarrow i^*(q_t)$$

- Determines for each HH  $\omega$ 
  - $c^\omega(q_t) = a^\omega - i^*(q_t)$
  - $g^\omega(q_t) = \Phi(i^*(q_t)) - \delta^\omega$



# Return on physical capital

- If  $dq_t = \mu_t^q q_t dt + q_t d\varepsilon_t^q \leftarrow$  endogenous

$$dr_t^\omega = \underbrace{\left( \frac{c^\omega(q_t)}{q_t} \right)}_{\text{dividend yield}} + \underbrace{g^\omega(q_t) + \mu_t^q + \text{Cov}[d\varepsilon_t^\omega, d\varepsilon_t^q]}_{\text{capital gains rate}} dt + \underbrace{(d\varepsilon_t^\omega + d\varepsilon_t^q)}_{\text{risk (endogenous + exogenous)}}$$

# Return on Money

- In the “long-run”

$$\frac{dK_t}{K_t} = \underbrace{\int (\zeta(\omega) + \xi(\omega)) g^\omega(q_t) d\omega}_{\mu_t^K} + \underbrace{\int \zeta(\omega) + \xi(\omega) d\varepsilon_t^\omega}_{d\varepsilon_t^K}$$

If  $dp_t = \mu_t^p p_t dt + p_t d\varepsilon_t^p \leftarrow$  endogenous

then a dollar invested in money earns return

$$dr_t^M = (\mu_t^K + \mu_t^p + Cov[d\varepsilon_t^K, d\varepsilon_t^p])dt + \underbrace{d\varepsilon_t^K + d\varepsilon_t^p}_{d\varepsilon_t^M}$$

# Intermediaries' "Risk Balance Sheet"

Assets

Liabilities

$$q_t K_t \int \zeta_t(\omega) (d\varepsilon_t^q + d\varepsilon_t^\omega) d\omega$$

$$\left( q_t K_t \int \zeta_t(\omega) d\omega - N_t \right) d\varepsilon_t^M$$

$$N_t d\varepsilon_t^N$$

$$dN_t = -\rho N_t dt + N_t dr_t^M$$

$$+ q_t K_t \int \zeta_t(\omega) \text{Cov}[d\varepsilon_t^q + d\varepsilon_t^\omega - d\varepsilon_t^M, d\varepsilon_t^N] d\omega dt$$

$$+ q_t K_t \int \zeta_t(\omega) (d\varepsilon_t^q + d\varepsilon_t^\omega - d\varepsilon_t^M) d\omega$$

$$d\eta_t = d(N_t/K_t) = \dots$$

# Equilibrium Conditions

1. Market clearing for **capital goods** and **bonds**

$$\int \zeta_t(\omega) d\omega + \int \xi_t(\omega) d\omega = 1$$

2. Market clearing for **output**:

$$\int (\zeta_t(\omega) + \xi_t(\omega)) c^\omega(q_t) d\omega = \rho(q_t + p_t)$$

3. Valuation of capital  $\omega$  -- **return = Cov(risk, net worth risk)**

- Intermediaries

$$E[dr_t^\omega - dr_t^M] \leq \text{Cov}[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^N] \quad (= \text{if } \zeta_t(\omega) > 0)$$

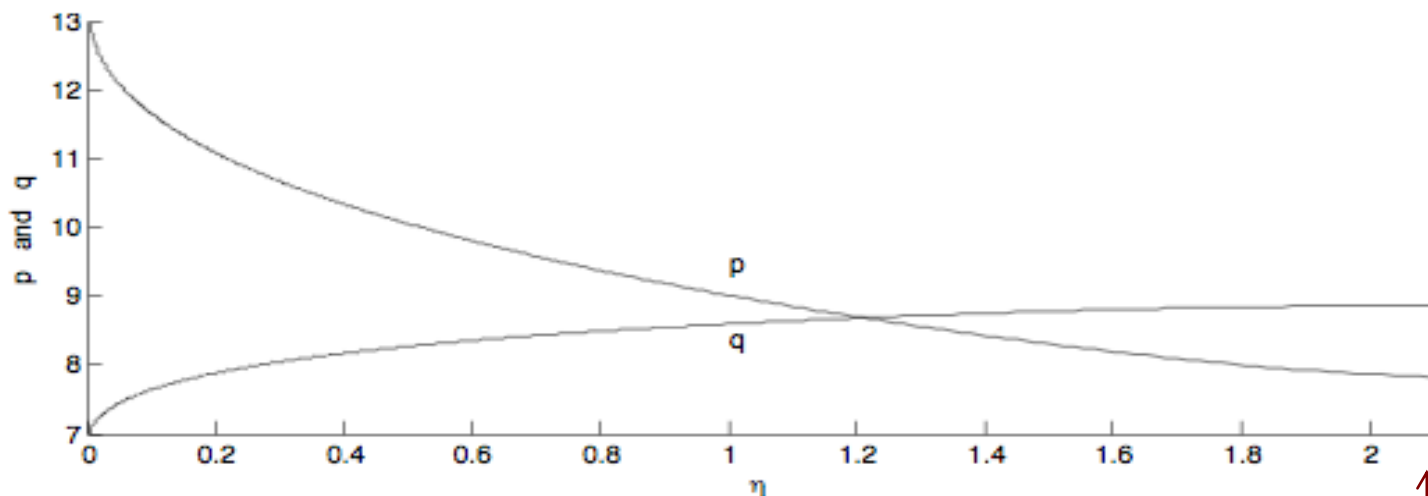
- HH  $\omega$

$$E[dr_t^\omega - dr_t^M] \leq \text{Cov}[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^{HH-N}] \quad (= \text{if } \xi_t(\omega) > 0)$$

# || Simplified Example

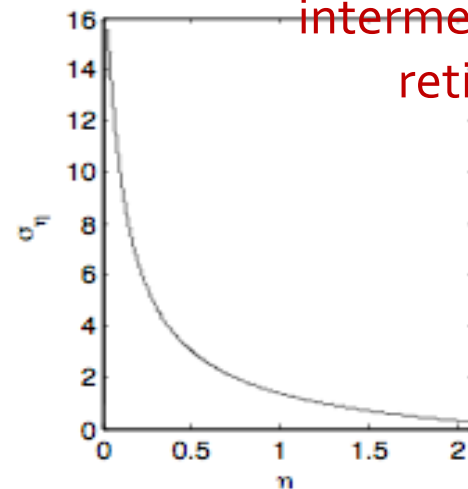
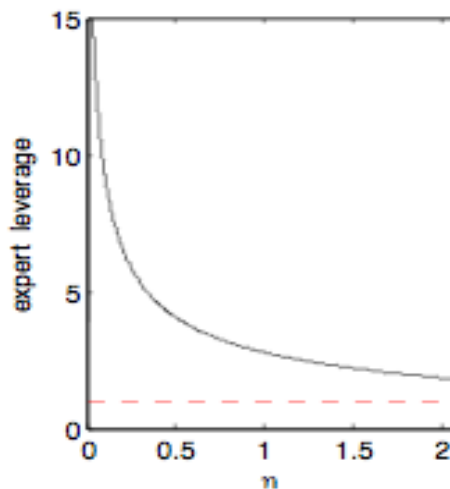
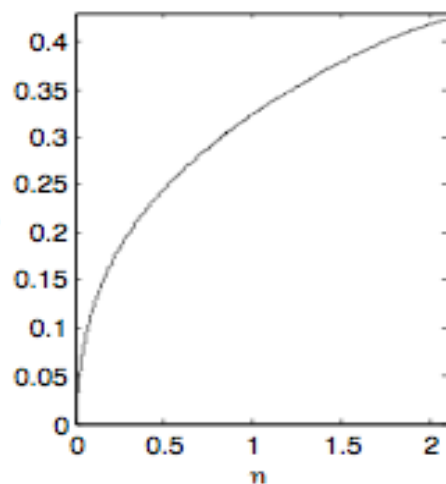
- Three household types  $\omega$  only
  - Low: very bad technology, hold money
  - Medium: risk-free technology, prefer to hold capital over money
  - High: risky production – low net worth
  
- Intermediaries choose to invest only in the most productive technology (due to high monitoring cost)

# Example



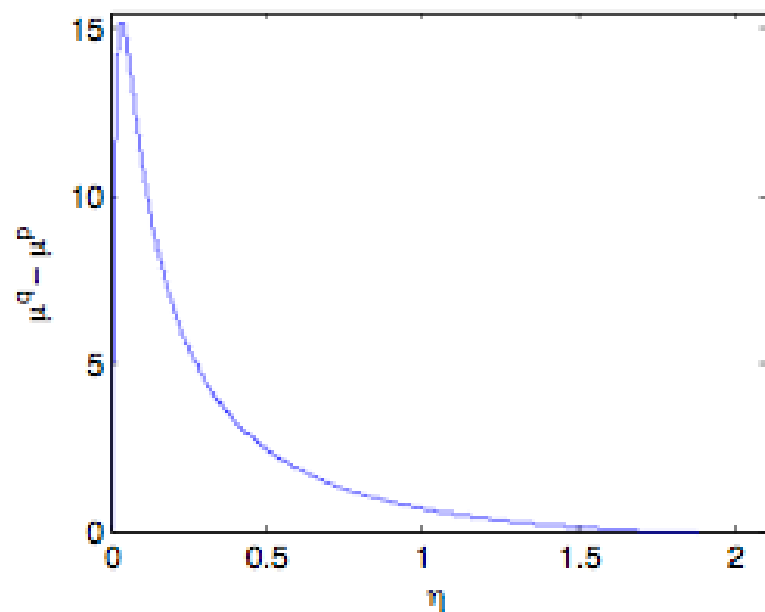
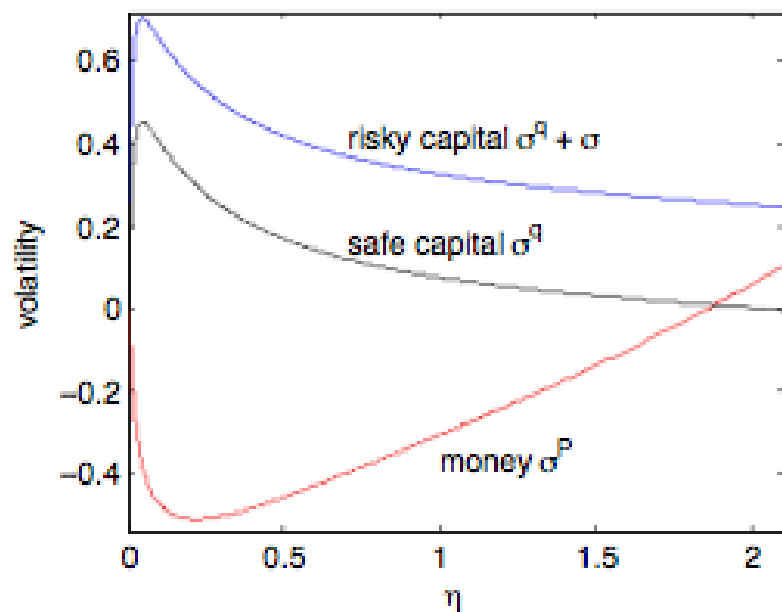
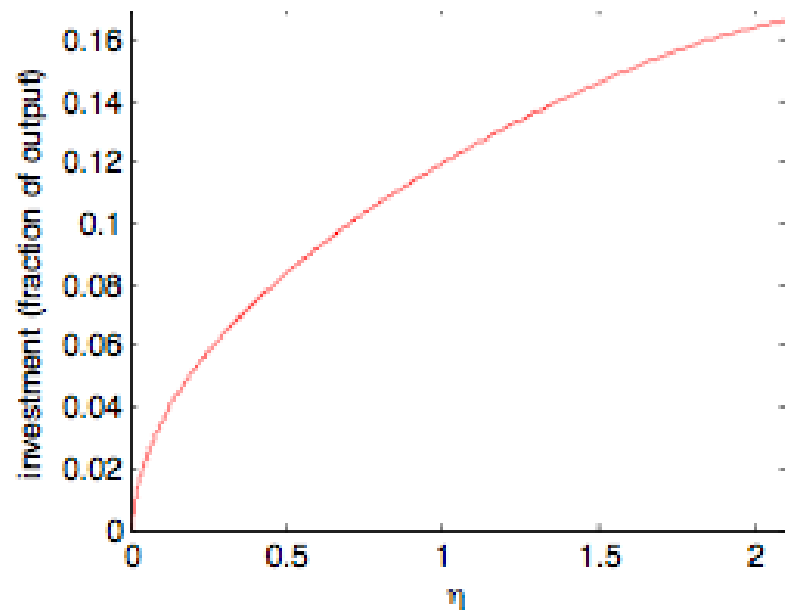
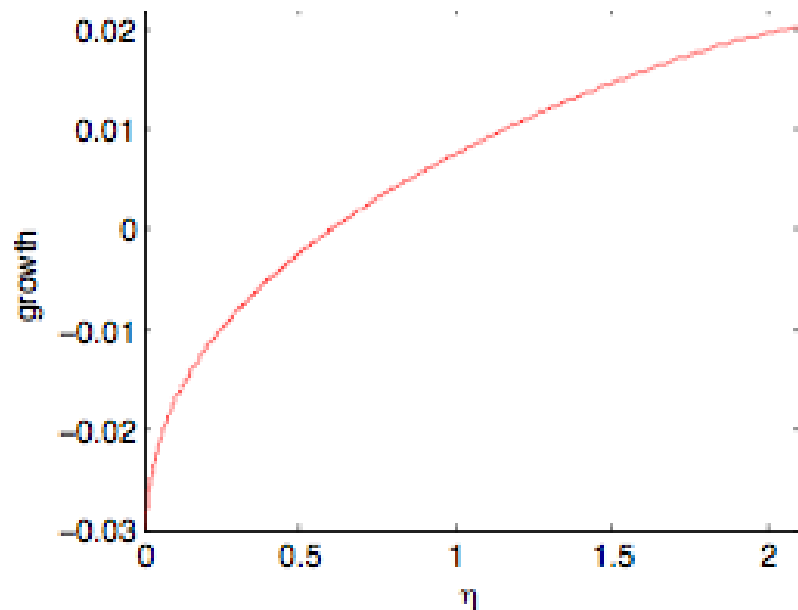
↑  
some

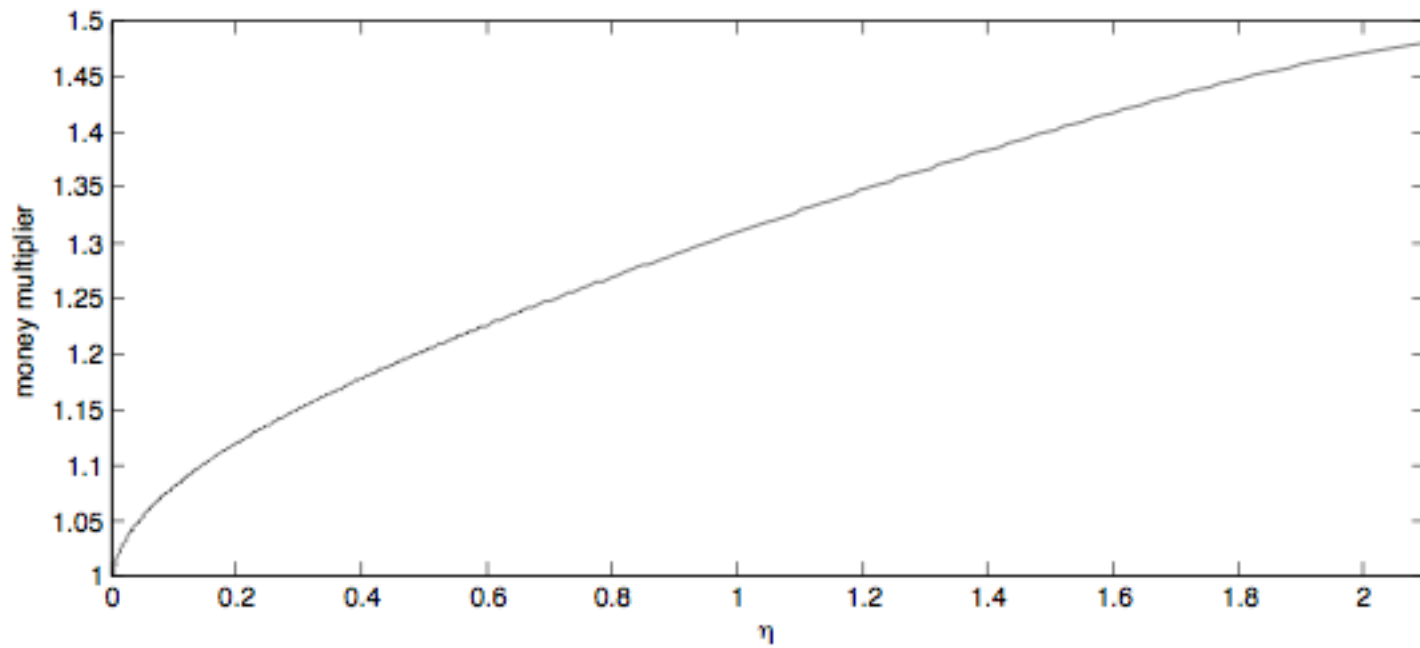
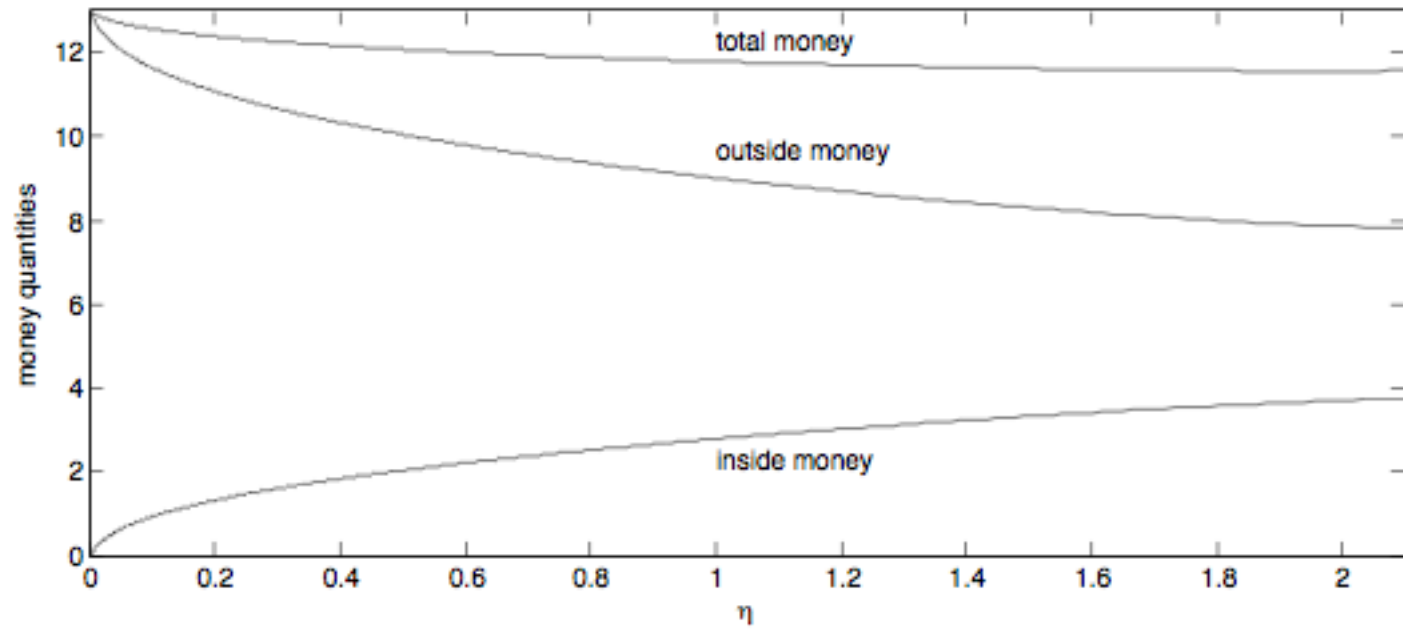
intermediaries  
retire



allocation to  
the most  
productive  
technology

$$\alpha = 1, r = 5\%, \delta^H = 0, \delta^M = 3\%, \sigma = 25\%, \theta^L = .65, \theta^M = 35\%, \theta^H = 0\%, \Phi(i) = (0.02i)^{1/2}$$







# Observations

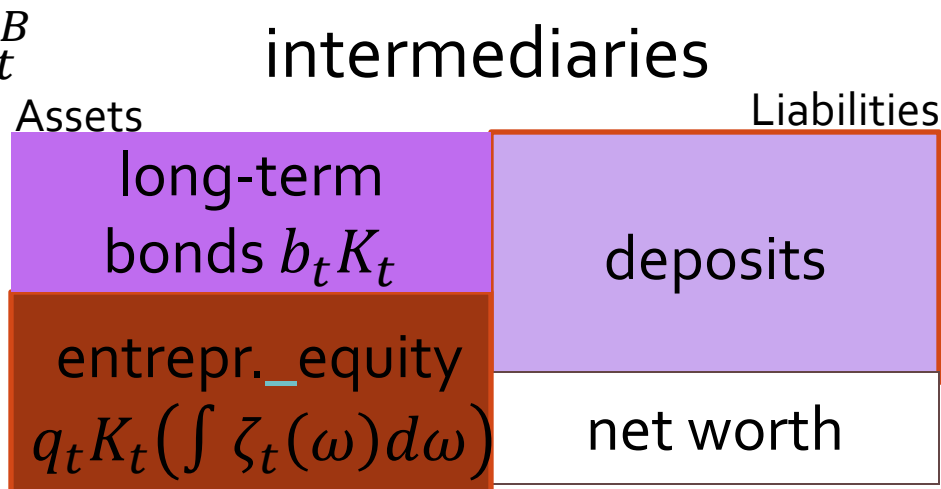
- As  $\eta$  goes down:
- Intermediaries take on less risk, competition decreases
- Price of capital  $q$  and investment,  $i(q)$ , decrease
- Capital is allocated less efficiently
- Unproductive households hold less inside money (loans to intermediaries/entrepreneurs) and more outside fiat money
- Price of outside money goes up (deflation)
- Additional source of amplification in economy with money:
  - value of assets fall
  - value of liabilities increase (due to deflation)

# Monetary Policy

- So far, Gold Standard
  - outside money fixed,
  - pays no interest
  - no central bank
- Introduce consol (perpetual) bond
  - pays interest rate in ST (outside) money
- Monetary Policies
  - Short-term interest rate policy
    - Central bank accepts deposits & pays interest rate (by printing money)
      - E.g. short-term interest rate is lowered when  $\eta$  becomes small
    - Budget neutral policies (at any point in time)
  - Asset purchase program
    - Bond – open market operations (OMO)

# Money and Long-term Bond

- Policy instruments (functions of  $\eta_t$ )
  - Central bank pays interest  $r_t \geq 0$  on money (by printing)
  - Sets total outstanding value  $b_t K_t$  of perpetual bond (by transacting)
- Endogenous market reaction
  - Price of long-term bond (in money, per unit coupon rate)
  - $dB_t = \mu_t^B B_t dt + B_t d\varepsilon_t^B$
  - $q_t$  = price of capital
  - $p_t K_t$  = value of money



# Disentangling Money and Bonds

- Return on money:  $dr_t^M = \mu_t^M dt + d\varepsilon_t^M$
- Price of bond:  $\frac{dB_t}{B_t} = \mu_t^B dt + d\varepsilon_t^B$  ( $\frac{1}{B_t}$  is current yield)
- Return on bonds:  

$$dr_t^B = dr_t^M + \left(\frac{1}{B_t} - r_t + \mu_t^B + Cov[\varepsilon_t^B, \varepsilon_t^M]\right)dt + d\varepsilon_t^B$$
- All monetary instruments:  $\frac{d(p_t+b_t)K_t}{(p_t+b_t)K_t} = dr_t^M + \frac{b_t}{p_t+b_t}(dr_t^B - dr_t^M)$   

$$= (\mu_t^p + \mu_t^b + \mu_t^K + Cov[\varepsilon_t^p + \varepsilon_t^b, \varepsilon_t^K])dt + d\varepsilon_t^p + d\varepsilon_t^b + \varepsilon_t^K$$
- Collecting shocks:  $d\varepsilon_t^M + \frac{b_t}{p_t+b_t}d\varepsilon_t^B = d\varepsilon_t^p + d\varepsilon_t^b + \varepsilon_t^K$

# Equilibrium Conditions

1. Market clearing for **capital goods** and **bonds**

$$\int \zeta_t(\omega) d\omega + \int \xi_t(\omega) d\omega = 1, \quad \zeta_t^B + \int \xi_t^B(\omega) d\omega = 1$$

2. Market clearing for **output**:

$$\int (\zeta_t(\omega) + \xi(\omega)) c^\omega(q_t) d\omega = \rho(q_t + p_t + b_t)$$

3. Valuation of capital  $\omega$  -- **return = Cov(risk, net worth risk)**

$$E[dr_t^\omega - dr_t^M] \leq \text{Cov}[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^N] \quad (= \text{if } \zeta_t(\omega) > 0)$$

$$E[dr_t^\omega - dr_t^M] \leq \text{Cov}[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^{HH-N}] \quad (= \text{if } \xi_t(\omega) > 0)$$

4. Valuation of bonds

$$E[dr_t^B - dr_t^M] = \text{Cov}[d\varepsilon_t^B, d\varepsilon_t^N] \quad (\text{assuming } \zeta_t^B > 0)$$

$$E[dr_t^B - dr_t^M] \leq \text{Cov}[d\varepsilon_t^B, d\varepsilon_t^{HH-N}] \quad (= \text{if } \xi_t^B(\omega) > 0)$$

# Short-term interest rate

- Without long-maturity assets changes in short-term interest rate have no effect
  - Interest rate change equals instantaneous inflation change
- With bonds: of all monetary instruments, fraction  $p_t/(p_t+b_t)$  is cash and  $b_t/(p_t+b_t)$  are bonds
  - deflationary spiral is less pronounced because as  $\eta$  goes down, growing demand for money is absorbed by increase in value of long-term bonds
  - also, intermediaries hedge risks better by holding long-term bonds
  - however, intermediaries also have greater incentives to increase leverage/risk-taking ex-ante
- Effectiveness of monetary policy depend on maturity structure (duration) of government debt





	New Keynesian	I-Theory
Risk build-up phase		Endogenous due to accommodating monetary policy
Net worth dynamics	zero profit      no dynamics	dynamic
State variables	Many exogenous shocks Intermediation/friction shock	Endogenous intermediation shock
Monetary policy rule	Taylor rule (is approximately optimal only if difference in $u'$ is well proxied by output gap) <ul style="list-style-type: none"><li>• spreads</li><li>• credit aggregates (?)</li></ul>	Depends on signal quality and timeliness of various observables
Policy instrument	Short-term interest rate + expectations	Short-term interest rate <b>+ long-term bond</b> + expectations
Role of money	In utility function (no deflation spiral)	Storage Precautionary savings





	Monetarism	I-Theory
Focus	Price stability	Price and Financial stability
Theory	Quantity theory of money $P*Y = v*M$  Transaction role of money	Distribution of wealth (liquidity, balance sheet)  endogenous money multiplier
Monetary aggregates	Mo (Brunner, Meltzer)  M1-2(Friedman,Schwartz) Inside and outside money are <i>perfect substitutes</i>	Outside money is only <i>imperfect substitute</i> for inside money (intermediation)  Bank underwriting ( <i>credit lines</i> ) is substitute to bank deposits (difficult to measure M1-3 in a meaningful way)
Monetary policy	Constant growth of M2 (Friedman)	Recapitalize banks through monetary policy Switch off deflationary pressure

# Conclusion

- Unified macromodel to analyze both
  - Financial stability
  - Monetary stability
    - Liquidity spirals
    - Fisher deflation spiral
  - GDP drops are associated with deflation (not inflation) absent monetary policy
- Capitalization of banking sector is key state variable
  - Price stickiness plays no role (unlike in New Keynesian models)
- Monetary policy rule
  - Redistributational feature
  - Time inconsistency problem – “Greenspan put”
- Further research
  - “Minsky cycle”

# Intermediaries and lending

- Monitoring technology  
Diamond (1984)  
Homstrom-Tirole (1997)

