# Project Description: Financial Frictions and the Macroeconomy

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## 1 A Macroeconomic Model with a Financial Sector

In standard macroeconomic models, households directly invest without financial intermediaries. This approach can only yield realistic macroeconomic predictions if, in reality, there are no frictions in the financial sector. Yet, following the great depression, economists such as Fisher, Keynes and Minsky have attributed the economic downturn to the failure of financial markets. The current financial crises has underscored once again the importance of the financial sector for the business cycles. Consequently, incorporating a financial sector in a macroeconomic model is imperative.

A few papers, such as Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997) incorporate financial frictions in a macro model, but only study dynamics near the steady state. Yet, this approach does not fully capture the instabilities of the financial system and the fact that occasionally it enters volatile destructive episodes away from the steady state.

We build a model that captures these instabilities. Our model is rich enough to replicate system dynamics of earlier work by Bernanke, Gertler and Gilchrist, as well as Kiyotaki and Moore as special cases when we set aggregate shocks to be small. Then the system fluctuates near the steady state. However, for larger shocks, it may enter unstable liquidity spirals where dynamics are very different. In our setting, a negative shock leads to price declines through the erosion of net worth of the financial sector as well as through increased price volatility. It is this increase in price volatility that leads to deleveraging, hoarding, and fire sales. These self-reinforcing feedback loops lead to unstable system dynamics. The associated large drop in asset values impairs real investment and economic activity.

The novelties of our approach are threefold. (i) We characterize dynamics both close to and away from the long-run steady state, and argue that it is typical for the system to get into amplifying cycles away from the steady state, (ii) we explicitly take volatility effects into account, which allows us to model the deleveraging phenomena observed during crises periods, and (iii) we allow for securitization, i.e. part of the risk is passed on to outside investors. We use our model to identify fire sales and other externalities that quantify the degree of excessive leverage in good times. We conclude by arguing that these externalities should form the basis of future bank regulation and monetary policy. Thus, the focus on financial stability should complement the current emphasis of monetary policy on price stability (inflation).

## 1.1 Model setup

In our economy, output is produced at a rate

$$y_t^i = a_t k_t^i,$$

where $a_t$ is TFP (assume constant for the purpose of this proposal) and $k_t^i$ are 'efficiency units of capital'. If capital is managed by a (risk-neutral) expert $i$, it grows/depreciates at rate $g$ and gets affected by economy-wide Brownian shocks $dZ_t$. In addition, expert $i$'s capital is subject to idiosyncratic jump risk summarized by a compensated Poisson loss process $L_t^i$ with mean 0. That is, capital evolves according to

$$dk_t^i = g k_t^i \, dt + \sigma k_t^i \, dZ_t + k_t^i \, dL_t^i.$$

Losses after an idiosyncratic jump are characterized by the distribution function

$$F : [0,1] \mapsto [0,1]$$

which describes the percentage of capital that is recovered in the event that a loss occurs. The arrival rate of jumps is given by a non-decreasing function $\lambda\left(\sigma_t^p\right)$, where $\sigma_t^p$ is the current volatility of the price process.

Each financial expert only has limited wealth $n^i$ on his own and hence tries to raise funds from households in the form of debt and outside equity. Experts' agency problems limit the size of their balance sheets. As shocks occur and asset values vary, the experts' balance sheets expand and contract over the business cycle.

Households can also directly manage capital. Their capital dynamics differs only insofar as the drift term is only $g^* < g$ instead of $g$. Assuming a common discount rate $\rho$, in equilibrium the price $p_t$ of a unit of capital is between $a/\left(\rho - g^*\right)$ and $a/\left(\rho - g\right)$.

The agency problem is that experts may lower the growth rate of capital from $g$ to $g^*$, or sometimes cause losses that generate private benefits. We assume that the expert can convert one unit of physical capital into $b$ units of private financial benefit. Experts' agency problems arise since only the total value of capital $p_t k_t^i$, but not $g$ or the loss shocks, are contractible. As a result, the financial expert has to retain a fraction $\alpha_t = \frac{b}{p_t}$ of the asset value risk (in Bernanke-Gertler $\alpha_t = 1$). This conclusion can be justified using arguments from the literature on optimal long-term contracts, and so our equilibrium contracts are equivalent to optimal dynamic contracts of a certain class. Note that $\alpha_t$ increases and contracting become harder when prices are depressed.

In addition, verifying idiosyncratic loss shocks is costly. To ensure that the financial expert does not divert funds by pretending to have suffered a large loss shock (which he has the opportunity to at an arrival rate of $\Lambda$), the creditor verifies the loss shock at a certain cost, whenever the expert defaults on his debt. This specification follows BGG, who adopted Townsend (1979)'s costly state verification framework. In equilibrium, default occurs solely due to idiosyncratic loss shocks. The expected verification costs drive an interest rate spread between the risky and the risk-free interest rate. Specifically, the expected loss in the event that a loss shock drives the value of assets below the value of debt is $V\int_0^{D/V}\left(\frac{D}{V}-x\right)dF(x)$, where $V$ is the asset value and $D$ is the debt level. As in Bernanke-Gertler, we take the verification costs to be proportional to the recovered value, i.e. $V\int_0^{D/V}(cx)\,dF(x)$.

To capture additional volatility effects, we allow the intensity of losses $\lambda\left(\sigma_t^p\right)$ to depend on the volatility of asset prices $p$. This assumption is consistent with the general idea that interest rate spreads and margins are set by debt holders who worry about potential losses (which depend on volatility). It can be justified through an informal story that idiosyncratic shocks have to do with liquidity (such as the difficulty to find an acceptable buyer and having to sell assets at fire-sale prices). Note that this assumption would not be meaningful under steady-state analysis, since price volatility is constant near the steady state.

Note that by varying the specification of the verification costs, several other models are also captured by our framework. Kiyotaki-Moore assume that financial experts can borrow only up to fraction $\theta$ of the market value of assets. Thus, someone with net worth $n_t$ can hold at most $1/(1-\theta)\,n_t$ worth of assets, by financing $\theta/(1-\theta)\,n_t$ of the assets with debt and the rest, $n_t$, with personal wealth. This is captured in our framework by setting the verification costs to zero up to a certain level and infinity afterwards. Alternatively, one can assume that margins are set equal to the value-at-risk (VaR) as in Shin (2010). In Brunnermeier and Pedersen (2009), margins increase with endogenous price volatility. These effects are captured in our model through the dependence of potential losses on price volatility.

Financial activity affects investment and the level of economic growth. We measure the size of the economy by the number of physical assets $K_t$, which can be financed through intermediaries, or directly owned by households. For tractability, we focus on a scale-invariant setting, so that the ratio $\eta_t = N_t/K_t$ is the key determinant of the availability of financing in the economy. We refer to $\eta_t$ as financial capital.

Our model exhibits interesting dynamics and instabilities even without securitization, i.e. when $\alpha_t = 1$. Securitization only amplifies some of the effects, since contracting becomes more difficult when prices are depressed.

### 1.2 Studying the wealth dynamics

Let us discuss our model in more detail, and move on to system dynamics. We assume that everybody is risk-neutral and has discount rate $\rho$. The net worth of an individual expert evolves according to

$$dn_t = -\left(\rho + \xi_t\right)\left(p_t k_t - n_t\right)\ dt + a_t k_t\ dt + d\left(p_t k_t\right),$$

where $p_t k_t - n_t$ is debt, $\xi_t$ is the interest rate spread on debt, $a_t k_t$ is output and $d(p_t k_t)$ is capital gain. Idiosyncratic shocks cancel out in the aggregate, yielding the law of motion of total expert net worth $N_t$. We combine it with the law of motion of aggregate capital $K_t$ to derive the drift and volatility of $\eta_t = N_t/K_t$ in the following equation, using Ito's lemma:

$$d\eta_t = \mu_t^\eta\ dt + \sigma_t^\eta\ dZ_t.$$

As we already mentioned, $\eta_t$ is the key equilibrium state variable. Three functions characterize the competitive equilibrium: prices $p\left(\eta_t\right)$, the fraction of assets held by experts $\psi\left(\eta_t\right)$ and expert value function $f\left(\eta_t\right) n_t$. Expert value is proportional to net worth, since experts are small and competitive. In choosing the sizes of their positions $p_t k_t$, experts will take the laws of motion of $\eta_t$ and $f\left(\eta_t\right)$ as given. We anticipate that $p\left(\eta_t\right)$ is an increasing function, $\psi\left(\eta_t\right) = 1$ except when $\eta_t$ gets small and $f\left(\eta_t\right)$ is a decreasing function.

Intuitively, experts make greater profit per dollar of capital when $\eta_t$ is small and prices are depressed.

These three equilibrium descriptors are found from three equations: (1) the household break-even condition when $\psi(\eta_t) < 1$,

$$\rho(p_t k_t)\, dt = a_t k_t \; dt + E[d(p_t k_t)], \text{ given the growth/depreciation rate of capital } g^*,$$

(with inequality $\geq$, when $\psi(\eta_t) = 1$, i.e. when households do not earn a required rate or return on this investment), (2) the Bellman equation

$$\rho(f(\eta_t)\, n_t) \;\; dt = \phi\left[n_t - f(\eta_t)\, n_t\right] \;\; dt + \max_k E\left[d(f(\eta_t)\, n_t)\right],$$

and (3) the first-order condition with respect to $k$ from the Bellman equation. In the Bellman equation, parameter $\phi$ refers to expert exit rate, which we introduce to make direct comparisons with the steady-state analysis of Bernanke-Gertler. With Poisson intensity $\phi$ each expert is hit by a shock that forces him to exit and consume his wealth.

In the first-order condition for the Bellman equation, the term $E[d(f(\eta_t)\, n_t)]$ illustrates the precautionary motive to hoard cash. Since profit opportunities $f(\eta_t)$ are negatively correlated with the amount of expert capital $n_t$, experts have stronger incentives to hoard cash when market volatility is high.

## 1.3 Dynamics beyond the steady state analysis and non-linear volatility effects

Now, to illustrate equilibrium dynamics, we solve the model first for the case without aggregate uncertainty. This case, which arises in the limit as $\sigma$ gets close to 0, allows us to make a direct analogy to BGG and KM. Unlike these models, we do not restrict our attention to the system around the steady state, but solve for the whole dynamics. Figure 1 illustrates the full dynamics for $\sigma = 0$ and a steady state value of $\eta$ around 80.

Near the steady state, the drift of $\eta$ is directed towards the steady state, and is proportional to the distance away from the steady state. Figure 2 that we borrowed from Kiyotaki (1998) illustrates reversion to the steady state by this type of dynamical system.

However, the solution starts looking nonlinear away from the steady state, and we can anticipate the kind of nonlinear dynamics that we get once we add volatility. Price volatility depends on the slope of the price function with respect to $\eta$. Note that the slope

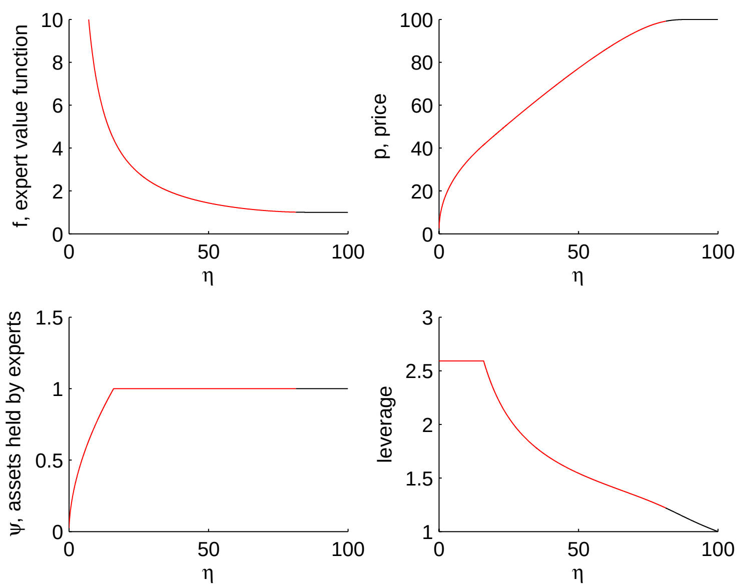


Figure 1: $f(\eta)$, $p(\eta)$, $\psi(\eta)$ and leverage as functions of $\eta$ for $\sigma \to 0$ with a steady state around $\eta = 80$. For $\sigma \to 0$ leverage is countercyclical, but in our setting with positive $\sigma$ leverage becomes procyclical for $\eta$ below 18 due to precautionary hoarding.

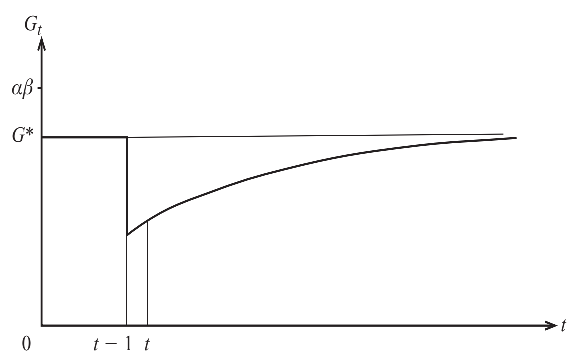


Figure 2: Convergence to steady state after temporary shock for $\sigma \to 0$.

$p'(\eta_t)$ is quite low near the steady state, due to low borrowing costs. As $\eta$ falls below the steady state, prices become depressed due to higher borrowing costs, and $p'(\eta_t)$ gets larger. When $\eta$ is so low that non-expert households have to buy some of the assets (for $\eta$ below 18), prices are depressed further and $p'(\eta_t)$ gets even higher.

Once we add aggregate shocks, when $\sigma$ is small the shapes of these functions will change little, but when $\sigma$ gets larger some of the effects get amplified. By Ito's lemma, price volatility equals $p'(\eta_t)\,\sigma_t^\eta$. The volatility of $\eta$ is in turn increasing in price volatility, because experts are exposed to price risk. Thus, the effect of $p'(\eta_t)$ on price volatility is nonlinear and a liquidity spiral is at work. This logic is captured by the following formula, which arises from our analysis,

$$\sigma_t^p = \frac{p'(\eta_t)\,\sigma\,(\psi_t p_t - \eta_t)}{1 - \psi_t p'(\eta_t)}.$$

Note that $p'(\eta_t)$ enters both the numerator and the denominator. In fact, when $p'(\eta_t)$ reaches a sufficiently high finite level, $\sigma_t^p$ becomes infinite.

We have not yet computed solutions of this model with $\sigma > 0$, but we can already predict that equilibrium dynamics for $\eta$ sufficiently far below the steady state will be highly nonlinear. To see the differences in dynamics, recall that steady-state dynamics describes a mean-reverting dynamical system with constant volatility, and in which the rate of mean reversion is proportional to the distance away from the steady state. This type of dynamics is highly stable, leads to countercyclical leverage in crisis times, and generates a long-run distribution of $\eta$ that is normally distributed around the steady state. In other words, large deviations from the steady state are highly unlikely.

In contrast, our model predicts an equilibrium in which the volatility becomes very large once the system shifts far enough below the steady state. Moreover, higher borrowing costs, reduced positions, and precautionary motives lead to deleveraging, bound the experts' profits and lower the growth rate of expert capital/the rate of reversion towards the steady state. As a result, this dynamical system involves episodes in which the value of $\eta$ becomes very low and the price distribution exhibits fat tails. We conclude that the financial system is prone to crises, and that predictions of steady-state analyses may be misleading.

Figure 3 illustrates the drift, volatility and the long-run distribution of $\eta$ that we expect in the true equilibrium versus its steady-state log-linearization.

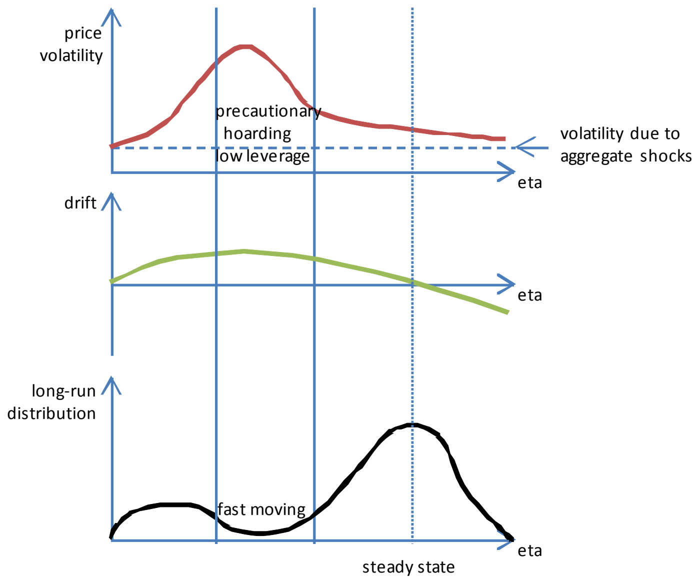


Figure 3: price volatility, drift of $\eta$, and long-run distribution of $\eta$ as a function of $\eta$.

## 1.4 Externalities

Another benefit of our model is that it can address the question whether there is "too much" leverage in the financial system. To answer this question, we identify several externalities that an individual expert does not take into account when levering up. For example, a fire-sale externality emerges since an individual expert takes the covariance between the next period's $\eta_t$ and $p_t$ as given, while all experts together affect this covariance term. We argue that these externalities should be the focus of any future financial regulation and should guide optimal monetary policy.

**Remark.** There are a number of important papers related to our proposal, which we could not discuss in detail for the lack of space. Importantly, He and Krishnamurthy (2008), Lorenzoni (2007) and Garleanue and Pedersen (2009) also study financial frictions in a dynamic model, but are primarily concerned with asset price implications. Christiano, Motto and Rostagno (2007) incorporate the steady-state analysis of the financial sector into large macro models, and study both theoretically and empirically the propagation of various shocks in a log-linearized framework. Cordia and Woodford (2009) study optimal monetary policy rules in such a framework.

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