Optimal Expectations

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Bayesian rationality

Non-Bayesian
Optimal Expectations

Framework
Discussion
Literature
Applications
Portfolio Choice
General Equilibrium
Consumption & Savings
Conclusion

rational expectations

Lucas rationality

Bayesian rationality

Non-Bayesian rational view
Optimal Expectations
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Biases: confirmation, optimism, overconfidence ....

Rational Expectations
- Lucas Rationality
- Bayesian Rationality

Non-Bayesian

Behavioral View
biases: confirmation, optimism, overconfidence ....

rational expectations

Lucas rationality

common priors

Harsanyi rationality

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non-common priors

Non-Bayesian
biases: confirmation, optimism, overconfidence ....

rational expectations

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common priors non-common priors

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Non-Bayesian

Critic: no disagreement no-trade theorem
Rational expectations

- Lucas rationality
  - Common priors
  - Harsanyi rationality
  - Bayesian rationality

Non-Bayesian

Behavioral view

- Critic: no disagreement
- no-trade theorem
- everything goes
- no structure

Biases: confirmation, optimism, overconfidence...
Overview: Three Main Elements

1. **Felicity at** \( t \): \( \hat{E}_t [U(c_1, \ldots, c_T)] \)
   - Agents care about utility flow today and
   - expected utility flows in the future
   \( \Rightarrow \) happier if more optimistic

2. **No split personality**
   - Distorted beliefs distort actions
   \( \Rightarrow \) better outcomes if more rational

3. **Optimal beliefs balance these forces**
   - Beliefs maximize well-being \( \frac{1}{T} E \left[ \sum_{t=1}^{T} \hat{E}_t [U(c_1, \ldots, c_T)] \right] \)
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Outline

1. Optimal Expectations Framework
2. Discussion
3. Related Literature
4. Applications
   - Portfolio Choice
   - General Equilibrium
   - Consumption & Savings
5. Conclusion
The General Framework

Actions: At each $t$ agent chooses $c_t$ to maximize felicity$_t$ given subjective beliefs $\hat{\pi} (s_t | s_{t-1})$, and resource constraints.

**Felicity at $t$:** $\hat{E}_t [U(c_1, ..., c_T)]$

with time-separable exponential discounting equals

$$\sum_{\tau=1}^{t-1} \beta^\tau u(c_\tau) + \beta^t u(c_t) + \hat{E}_t \left[ \sum_{\tau=t+1}^{T} \beta^\tau u(c_\tau) \right]$$

Note: $\beta$s for past consumption could be replaced with $\delta$. 
Utility Flow, Felicity and Well-being

\[ u(c_1) + \sum_{\tau=1}^{T-t} \beta^\tau u(c_{1+\tau}) \]
Utility Flow, Felicity and Well-being

\[ u(c_1) + \sum_{\tau=1}^{T-t} \beta^\tau u(c_{1+\tau}) \]

- Felicity at \( t = 1 \)
- Felicity at \( t = 2 \)
- Felicity at \( t = 3 \)

Well-being
Beliefs: At $t = 0$ optimal beliefs are $\hat{\pi}^{OE}(s_t|s_{t-1})$

that maximize

Well-being: $W = \frac{1}{T} E \left[ \sum_{t=1}^{T} \hat{E}_t [U(\cdot)] \right]$

subject to:

- agent behavior given these beliefs
- $\hat{\pi}^{OE}(s_t|s_{t-1})$ are probabilities
- $\hat{\pi}^{OE}(s_t|s_{t-1}) = 0$ if $\pi(s_t|s_{t-1}) = 0$
Two-period **Example** with Consumption at $t = 2$

<table>
<thead>
<tr>
<th></th>
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<th>$t = 2$</th>
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<tbody>
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<td>felicity in period 1</td>
<td></td>
<td>$\beta \hat{E}[u(c_2)]$</td>
</tr>
<tr>
<td>felicity in period 2</td>
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Actions maximize felicity: $\beta \hat{E}[u(c_2)]$

Beliefs maximize well-being: $W = \frac{1}{2} \beta \hat{E}[u(c_2)] + \frac{1}{2} \beta E[u(c_2)]$
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Actions maximize felicity: $\beta \hat{E}[u(c_2)]$

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Discussion

1. **Subjective probabilities are chosen once and forever**
   - Bayes’ Rule (LIE) holds,
   - Can be interpreted as choice of priors

2. If beliefs are objective, wellbeing = felicity
   - Only incentive to distort beliefs is anticipatory utility gain

3. Rational expectations are optimal *only if*
   - anticipatory utility does enter felicities or
   - anticipatory utility does not enter well-being \( W \).

4. Different memory discounting in felicity
   - Paper’s results hold qualitatively for any memory discounting
   - But can introduce additional incentives to bias beliefs
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Frictionless Extreme

Why optimal expectations?

- It is optimal: “as if” interpretation
- Parents/Upbringing affects (prior) beliefs
- Neuroscientific “story”:
  
 prefrontal cortex exerts effort to reduce overoptimism

(subconscious process)

Payoff: biases are endogenous

- biases are small when distort behavior a lot
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Related Literature

1. Adam Smith (1776)
   “That the chance of gain is naturally overvalued, …”
   “That the chance of loss is frequently undervalued, …”

2. Anticipatory utility (‘Pleasure of Expectation’):
   • Bentham, Hume, Böhm-Barwerk, Marshall, Loewenstein,
   • Geanakopolis-Pearce-Stacchetti, Caplin-Leahy

3. Models of belief distortions:
   • cognitive dissonance (Akerlof-Dickens),
   • agents choose beliefs (Yariv and Landier),
   • intrapersonal (confidence) games (Bénabou-Tirole),
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   • follow up: link to prospect theory (Gollier), (Glaeser)
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Applications

- **Portfolio choice**
  - Preference for skewed returns

- **General equilibrium**
  - Endogenous heterogeneous prior beliefs
  - Equity premium puzzle versus long shot phenomena

- **Consumption-savings problem with stochastic income**
  - Optimism and overconfidence in future income
  - Consumption profiles concave due to “news”
  - Choose incomplete consumption insurance

- **Optimal timing of a single task**
  - Procrastination, planning fallacy, context effect
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Portfolio Choice

• Setup

1. Two period problem:
   invest in period 1, consume in period 2
2. Two assets:
   a risk-free asset, return $R$; a risky asset, return $R + Z$
3. Uncertainty:
   $S > 2$ states, $\pi_s > 0$ for $s = 1$ to $S$,
   $Z_s < Z_{s+1}$, $Z_1 < 0 < Z_S$
4. $c \geq 0$ in all states
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Portfolio Choice

Stage 2: Agent  \( \max_{\alpha} \beta \sum_{s=1}^{S} \hat{\pi}_s u \left( R + \alpha Z_s \right) \)

\[ \text{FOC: } 0 = \sum_{s=1}^{S} \hat{\pi}_s u' \left( R + \alpha Z_s \right) Z_s \implies \alpha^* \left( \hat{\pi} \right) \]

Stage 1: Choose \( \hat{\pi}_s \) to maximize well-being

\[ \frac{1}{2} \beta \sum_{s=1}^{S} \hat{\pi}_s u \left( R + \alpha^* Z_s \right) + \frac{1}{2} \beta \sum_{s=1}^{S} \pi_s u \left( R + \alpha^* Z_s \right) \]

\( \text{felicity at } t = 1 \)

\( \text{‘average’ utility at } t = 2 \)

\[ \text{FOC: } \frac{\beta}{2} (u_s - u_{s'}) = \frac{\beta}{2} \sum_{s=1}^{S} \pi_s u' \left( R + \alpha^* Z_s \right) Z_s \frac{d\alpha^*}{d\hat{\pi}_{s'}} \]

\( \text{benefits of anticipation} \)

\( \text{costs of changed behavior} \)
Portfolio Choice

Stage 2: Agent $\max_\alpha \beta \sum_{s=1}^{S} \hat{\pi}_s u (R + \alpha Z_s)$

FOC: $0 = \sum_{s=1}^{S} \hat{\pi}_s u' (R + \alpha Z_s) Z_s \Rightarrow \alpha^*(\hat{\pi})$

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\underline{felicity at } t = 1 \quad \underline{\text{‘average’ utility at } t = 2}$

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\underline{benefits of anticipation} \quad \underline{costs of changed behavior}
Portfolio Choice

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\]

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\]

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\]

- **Benefits of anticipation**
- **Costs of changed behavior**
Proposition Excess risk taking due to optimism

(i) Agents are optimistic about states with high portfolio payout if $\alpha^* > 0$, 
if $\alpha^* > 0$, \[ \sum_{s=1}^{S} (\hat{\pi}_s - \pi_s) u'(R + \alpha^* Z_s) Z_s > 0; \]
if $\alpha^* < 0$, \[ \sum_{s=1}^{S} (\hat{\pi}_s - \pi_s) u'(R + \alpha^* Z_s) Z_s < 0. \]

(ii) Agents go even more long (short) than agent with RE or in the opposite direction 
if $E[Z] > 0$, then $\alpha^* > \alpha^{RE} > 0$ or $\alpha^* < 0$; 
if $E[Z] < 0$, then $\alpha^* < \alpha^{RE} < 0$ or $\alpha^* > 0$;
Preference for Skewed Returns

- **Empirical Phenomena:**
  - Horse race long shots: Golec and Tamarkin (1998)
  - Lottery demand: Garrett and Sobel (1999)
  - Security design? Swedish lottery bonds, PS-Lotteriesparen

- **Setup**
  - 2 states with payoffs: $Z_1 < 0 < Z_2$,
  - hold variance and mean fixed and $E[Z] < 0$
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Proposition Skewness

An agent with an unbounded utility function holds some of the asset even though its mean payoff is negative if the payoff is sufficiently skewed.

• Remark:
  • Agent goes long for large $\pi_1$ even though $E[Z] < 0$, since
    • there is not much room to short and distort beliefs
    • shorting becomes very risky
General Equilibrium

- Empirical Phenomena:
  - betting & gambling
  - high trading volume (stock and FX market)
  - home bias
  ⇐ endogenous heterogeneous prior beliefs?
  - negatively skewed: equity premium puzzle
  - positively skewed: IPO underperformance, long-shots

- Setup:
The portfolio choice problem with
  - A continuum of agents with identical endowments
  - A fixed supply of ‘bonds’ with normalization $R = 1$
  - The risky asset in zero net supply: $1 + Z_s = \frac{1 + c_s}{P_e}$
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Proposition Heterogeneous Priors

For $S > 2$ agents split into two groups with different beliefs

(i) Optimists with $\hat{E}^i [Z^{OE}] > 0$ and $\alpha^{OE,i} > 0 = \alpha^{RE}$
(ii) Pessimists with $\hat{E}^j [Z^{OE}] < 0$ and $\alpha^{OE,j} < 0$

both groups trade against each other and $\{\hat{\pi}^i\} \neq \{\pi\} \neq \{\hat{\pi}^j\}$.

• Example
  • $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ with $\gamma = 3$,
  • $\pi_1 = 0.25$, $\pi_2 = 0.75$,
  • $\varepsilon_1 = -0.6$, $\varepsilon_2 = 0.2$ so $P^{RE} = 1$. 
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Figure: Wellbeing as a function of subjective beliefs, $\hat{\pi}_2$
In this example, as we vary the economic environment, beliefs change . . .

\[ P^{OE} > P^{RE} = 1 \quad \text{if payoff is positively skewed (long-shots, IPO)} \]

\[ P^{OE} < P^{RE} = 1 \quad \text{if payoff is negatively skewed (stock market)} \]

**Conjecture**

For multi-asset case with positive net supply:
- Heterogeneity in beliefs is less pronounced.
- Agents invest in different skewed assets (forgo diversification benefits to hold skewed assets.)

*Complicates Aggregation:*
Representative agent has different preference structure from individual (possibly identical) investors.
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Empirical Phenomena:
- households **expect** upward sloping consumption profile (Barsky et al. (1997))
- **actual** average consumption growth is non-positive and profiles are concave (Gourinchatas & Parker (2002))

Setup:
- Finite-lived agent, quadratic utility $u(c_t) = ac_t - \frac{1}{2} bc_t^2$,
- one risk-free asset, $R\beta = 1$,
- i.i.d. income:

  *Objective prob.*: $y_t$ independent over time $\prod \left( y_t | y_{t-1} \right)$
  $d\prod(y_t) > 0$ for all $y \in [\underline{y}, \bar{y}]$.

  *Subjective prob.*: $\hat{\Pi} \left( y_t | y_{t-1} \right) \geq 0$ for all $y \in [\underline{y}, \bar{y}]$.
Optimal Consumption

Euler equation:

\[
c_t \left( A_t, y_t \right) = \hat{E} \left[ c_{t+1} \left( A_{t+1}, y_{t+1} \right) | y_t \right]
\]

Consumption rule:

\[
c_t^* \left( y_t \right) = \frac{1 - R^{-1}}{1 - R^{-(T-t)}} \left( A_t + y_t + \sum_{\tau=1}^{T-t} R^{-\tau} \hat{E} \left[ y_{t+\tau} | y_t \right] \right)
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Note: \( c_t^* \) depends only on \( \hat{E} \left[ y_{t+\tau} | y_t \right] \) (not higher moments)
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Optimal Beliefs

So ⇒ Variance only lowers anticipatory utility, but does not affect $c$
⇒ OE exhibit no uncertainty for quadratic utility.

Therefore

$$\hat{E}\left[u(c^*_t) | y_t \right] = u\left(\hat{E}\left[c^*_t | y_t \right] \right)$$

Note: agents who expect risk have the same behavior and lower felicity
Optimal Beliefs

So $\Rightarrow$ Variance only lowers anticipatory utility, but does not affect $c$

$\Rightarrow$ OE exhibit no uncertainty for quadratic utility.

Therefore

$$\hat{E} \left[ u \left( c_{t+\tau}^* \right) | y_t \right] = u \left( \hat{E} \left[ c_{t+\tau}^* | y_t \right] \right)$$

Note: agents who expect risk have the same behavior and lower felicity
Certainty + Euler equation ⇒ wellbeing simplifies to

\[
\frac{1}{T} \sum_{t=1}^{T} \psi_t E \left[ u \left( c_t^* \left( y_t \right) \right) \right]
\]

and FOC implies an actual consumption path of

\[
c_t^* \left( y_t \right) = \frac{a}{b} - \frac{\psi_{t+\tau}}{\psi_t} R^\tau \left( \frac{a}{b} - E \left[ c_{t+\tau} \left( y_{t+\tau} \right) | y_t \right] \right)
\]

where \( \psi_t = \beta^{t-1} \left( 1 + \sum_{\tau=1}^{T-t} (\beta^\tau + (\beta \delta)^\tau) \right) \)
Figure: Consumption Path
Figure: Consumption Path
optimal expectations
Brunnermeier
& Parker
framework
discussion
literature
applications
portfolio choice
general equilibrium
consumption & savings
conclusion

expected consumption path for agent with optimal expectations at $t = 1$

average consumption path

overconsumption (overoptimism)

Figure: Consumption Path
Reduce consumption since income in $t=2$ was lower than expected.

Figure: Consumption Path
average consumption path

Initial over-consumption (overoptimism)

consumption at $t = 3$
for agent with optimal expectations

expected consumption path at $t = 3$

Figure: Consumption Path
Figure: Consumption Path

average consumption path

Initial over-consumption (overoptimism)

Figure: Consumption Path
Figure: Consumption Path

- $c_{OE}(t)$: Initial over-consumption (overoptimism)
- $c_{RE}(t)$: Average consumption path
Proposition Undersaving

For all \( t < T \)

(i) \[ \hat{E} \left[ \sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} \mid y_t \right] > E \left[ \hat{E} \left[ \sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} \mid y_t \right] \right] \]

(ii) \[ c^*_t \left( y_{t} \right) > E \left[ c^*_{t+1} \left( y_{t+1} \right) \mid y_t \right] \]

(iii) \[ \hat{E} \left[ c^*_{t+1} \left( y_{t+1} \right) \mid y_t \right] > E \left[ c^*_{t+1} \left( y_{t+1} \right) \mid y_t \right] \]

(iv) as \( T \to \infty \), \( c^*_t \left( y_{t} \right) \to c^{RE}_t \left( y_{t} \right) \)

- Model predictions
  - optimism and overconfidence
  - consumption profile hump-shaped
  - agent surprised by declining consumption on average
  - “overconsumption” declines with costs (length of life)
Proposition Undersaving

For all $t < T$

(i) $\hat{E} \left[ \sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} | y_t \right] > E \left[ \hat{E} \left[ \sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} | y_t \hat{E} \left[ \sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} | y_t \right] \right] \right]$

(ii) $c^*_t (y_{t-1}) > E \left[ c^*_t+1 (y_{t+1}) | y_t \right]$

(iii) $\hat{E} \left[ c^*_t+1 (y_{t+1}) | y_t \right] > E \left[ c^*_t+1 (y_{t+1}) | y_t \right]$ \(\Rightarrow\)

(iv) as $T \rightarrow \infty$, $c^*_t (y_{t-1}) \rightarrow c^*_{RE} (y_{t-1})$

- Model predictions
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Conclusion

- Rational expectations are sub-optimal:
  - Agents with rational beliefs makes the ex post best decisions
  - but agents that care about the future can be happier with some optimism
  - Utility gain determines biases

- Optimal expectations is a structural model of non-rational beliefs
  - beliefs are most distorted when decision errors are small
  - beliefs are most distorted when “dream” benefits are largest
  - excess risk taking due to optimism, preference for skewness
  - endogenous heterogenous beliefs; agreeing to disagree
  - overconfidence, optimism, and lack of consumption insurance
  - subjective procrastination, planning fallacy, context effect
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