

#### Markus K. Brunnermeier

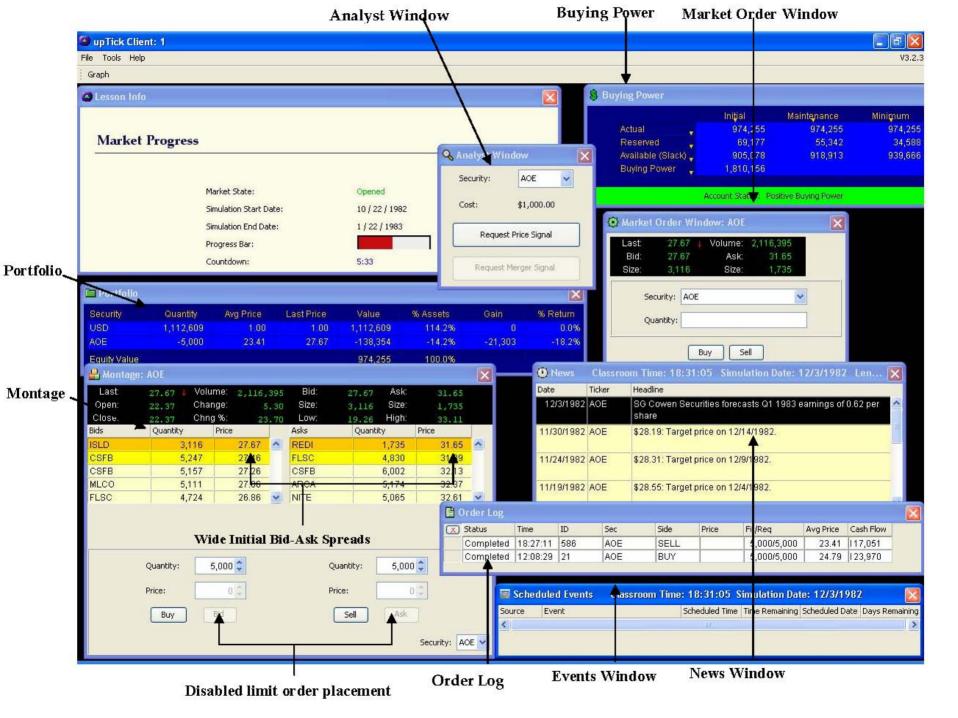
Preceptor: Filippos Papakonstantinou

**Princeton University** 

#### Introduction

#### Main Principles of Finance

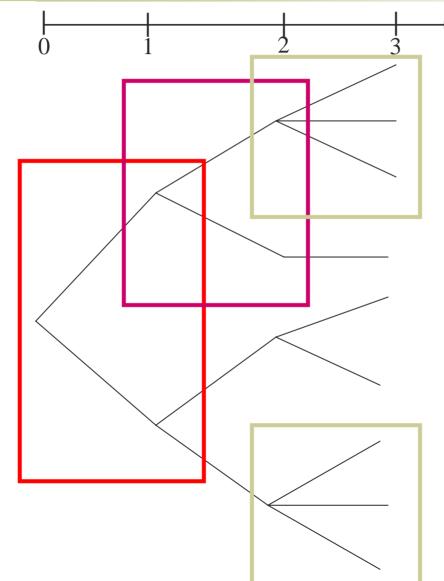
- One principle per lesson see syllabus
- Focus on institutional features (frictions matter)
- "UpTick" Trading software developed by
  - Joshua Coval (HBS)
  - Eric Stafford (HBS)
  - If software breaks down, we will switch to a standard lecture
- Student presentation



## Philosophy of UpTick

- Price is affected by
  - historical real price data
  - o trading of students
  - Price is loosely anchored around real historical price data
    - 1. Computer traders/market makers find it more and more profitable to trade towards historical price the further price deviates from historical time series
    - 2. Signals reveal historical price x periods ahead
    - 3. Final liquidity value equals historical price
- Realistic trading screen
  - Montage limit order book (shows bid-ask spread + market depth)
  - Event window
- Personal Calculator (Excel)

# Abstraction – Event tree



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#### Law of one Price, No risk-free Arbitrage

#### Law of one price (LOOP)

- Securities (strategies) with the same payoff in the future must have the same price today.
- Price of actual security = price of synthetic security
- No (risk-free) Arbitrage
  - There does not exists an arbitrage strategy that costs nothing today, but yields non-negative and a strictly positive future payoff in at least one future state/event AND
  - There does not exist an arbitrage strategy that yields some strictly positive amount today and has non-negative payoffs at later point in time.

No Arbitrage U LOOP Brunnermeier

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#### Arbitrage Strategy

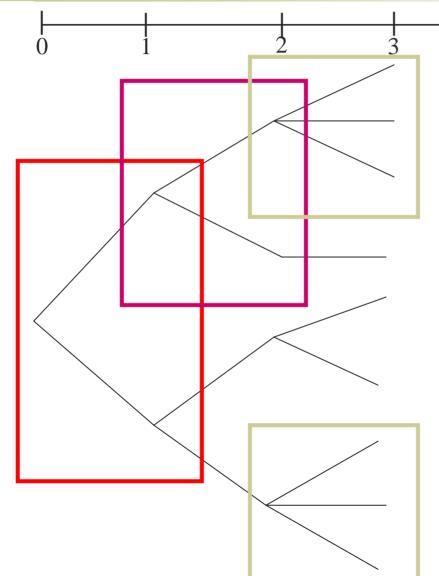
#### Static:

- acquire all positions at time t
- no retrades necessary

#### Dynamic:

- Future retrades are necessary for an arbitrage strategy
- Retrades depend on price movements

# Abstraction – Event tree, again



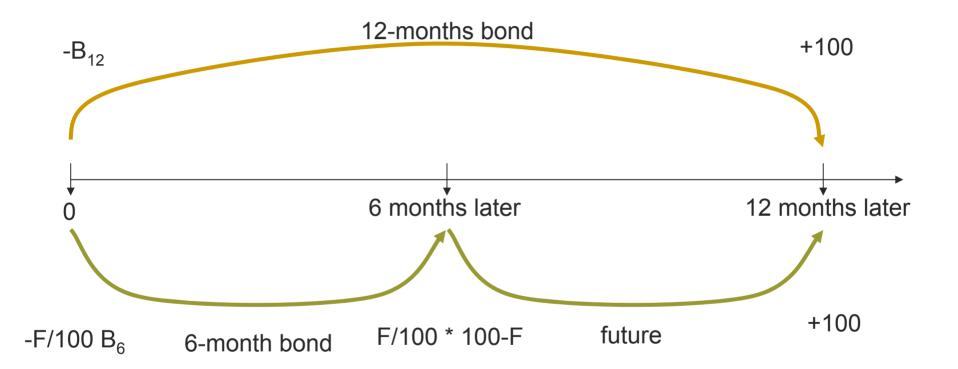
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# **Bond - Simplest Event Tree**

- A zero-coupon bond pays \$100 at maturity with no intermediate cashflows
- The future value (FV=\$100) and the present value (PV=bond price, B) are related by the following equation: PV x (1+R) = FV, where R is the periodic interest rate
- Equivalently, PV = FV / (1+R)
- The bond price is: B = \$100 / (1+R)

# **Bond Pricing Example**



$$1+r_{0,12} = (1+r_{0,6})(1+r_{6,12})$$

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### Law of One Price

#### Payoffs to purchasing the securities

	0	0.5	1
Long Bond	-B <sub>Long</sub>	0	100
Short Bond	-B <sub>Short</sub>	100	
Futures	0	-F	100

#### Suppose you want \$100 in one year

	0	0.5	1
Long Bond	-B <sub>Long</sub>	0	100
Buy 1 long-term bond			

# O 0.5 1 Short Bond -B<sub>Short</sub> x F/100 F Futures 0 -F 100 Net -B<sub>Short</sub> x F/100 0 100

Markus K. Brunnermeier 9/14/2006 2 ways of getting the same payoffs should have the same price:

# Synthetic Long-term Bond

- The pricing relation: B<sub>12</sub> = B<sub>6</sub> x F/100, can be rearranged to solve for any of the securities
  - The RHS represents a "**synthetic**" **long-term bond** (1 futures contract and F/100 short-term bonds)
- For example,  $F = B_{12} / B_6 \times 100$
- If this pricing relation does not hold, then there is a risk-free profit opportunity
  - a risk-free arbitrage

### **Bond Pricing Example**

What if you observe the following prices:

- Long Bond = \$94.50
- Short Bond = \$95.00
- Futures = \$98.00

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Synthetic LBond = BShort x F/100 = \$93.10

Aibiliage Tidde				
	0	0.5	1	
Sell 1 Long Bond	94.50	0.00	-100.00	
Buy 0.98 Short Bonds	-93.10	98.00	0.00	
Buy 1 Futures	0.00	-98.00	100.00	
Net	1.40	0.00	0.00	

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# **Example in International Setting**

- Any one of the following four securities:
  - Domestic bond
  - Foreign bond
  - Spot currency contract
  - Currency futures contract

can be replicated with the other three.

- Create a synthetic \$/£ futures contract using:
  - O US bond = \$95
  - UK bond =  $\pm 96$
  - Pounds spot = 1.50/E

# Bid-Ask Spread limits arbitrage

What is the market price for a security?

- Ask the market price to buy
- Bid or offer the market price to sell
- These are the prices at which a **market order** will be executed
- If we view the midpoint as the "fair value", then ½ x (Ask-Bid) = transaction cost per unit traded
  - A round-trip market order transaction will pay the full spread
- If the transaction size exceeds quantity being offered at the best bid or ask?
  - T-cost is an increasing function of order size
- UpTick records the difference between a trade's average transaction price and mid-price prevailing immediately

Markus K. prior to the trade as the trade's transaction cost. Brunnermeier 9/14/2006 1-15

# Arbitrage with Bid-Ask Spread

- The law of one price holds exactly only for transactable prices (i.e. within the bounds)
- Pricing relation: BLong = BShort x F/100

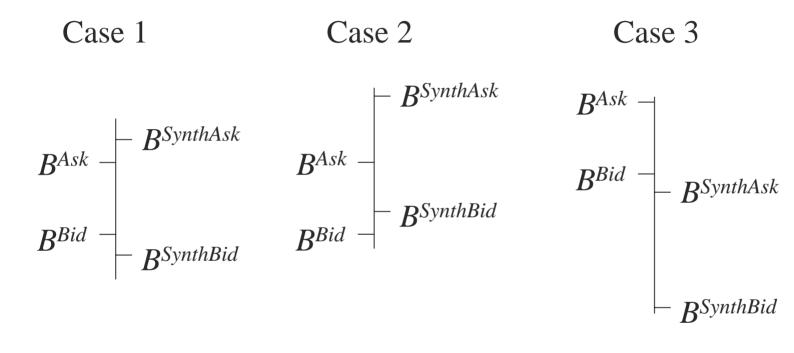
$$B_{1-yr}^{Synthetic} = \frac{F}{100} \cdot B_{_{6-mo}}$$

Total cost of buying the Long Bond synthetically:

$$B_{1-yr}^{SyntheticASK} = \frac{F^{ASK}}{100} \cdot B_{_{6-mo}}^{ASK}$$

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# Arbitrage with Bid-Ask Spread



- Buy and sell direct
- No arbitrage

- Buy direct; Sell synthetic
- No arbitrage

- Buy synthetic; sell direct
- Arbitrage

### Margins limit arbitrage

#### Positive size is limited

- Long an asset
  - m% \* p \* x ≥ marked-to-market wealth
- Short an asset
  - Sell asset, receive p = \$100
  - Put p + m\*p in margin account
  - Use up m\*p of your own financial wealth
- Cross-Margining
  - Netting: Only perfectly negatively correlated assets
  - Portfolio margin constrained
    - If better hedge one can take larger positions

### More on Margins

- How much leverage should your broker allow you?
  - Depends on interest they charge  $\leftarrow \rightarrow$  risk they are willing to bear
- Most brokers charge an interest rate that is close to the Federal Funds rate (riskfree rate)
- Hence, from broker's perspective the loan must be close to riskfree (very small probability of you defaulting)
  - Broker requires equity cushion sufficient to keep the loan close to riskfree, subject to constraints imposed by the Federal Reserve and exchanges
  - **Cross-margining/Netting:** Most brokers give preferred margin terms to clients with low total portfolio risk
  - upTick requires 50% margin to initiate most equity and bond positions
  - upTick evaluates the overall risk of portfolios rebates some of the reserved equity for perfectly offsetting positions

# More on Margins

\$

No constraints

Initial Margin (50%) \_\_\_\_\_ Reg. T 50 %

- Can't add to your position;
- Not received a margin call.

#### Maintenance Margin (35%) NYSE/NASD 25% long 30% short

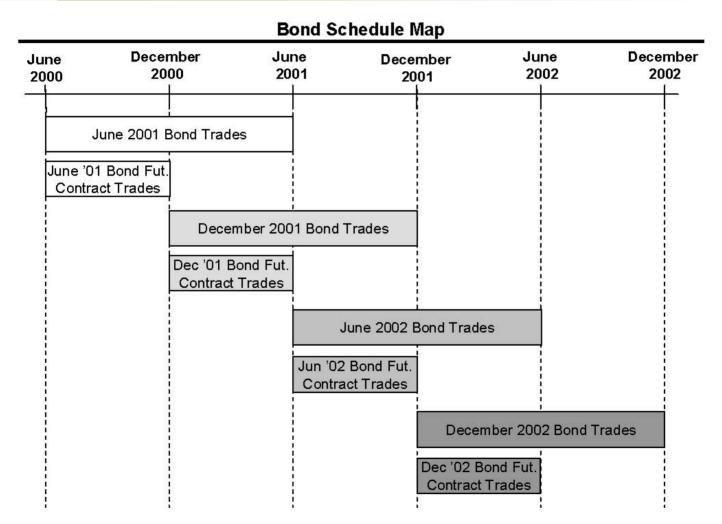
- Fixed amount of time to get to a specified point above the maintenance level before your position is liquidated.
- Failure to return to the initial margin requirements within the specified period of time results in forced liquidation.

#### Minimum Margin (25%)

• Position is always immediately liquidated

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### Simulation – Law of One Price



#### **Three simulations**

#### 1. Equal liquidity for all three assets

- o 12-month bond
- 6-month bond
- o Future
- 2. 12-month bond is less liquid
- 3. 6-month bond is less liquid
  + negative endowment in 6-month bond