Lecture 04: State-price BETA Model

Prof. Markus K. Brunnermeier

Overview

- Risk-adjustment in payoffs
- Risk-adjustment in returns
- State price beta model
- Different specific asset pricing models

Risk-adjustment in payoffs

$$p = E[mx^j] = E[m]E[x] + Cov[m,x]$$

Since $1=E[mR]$, the risk free rate is $R^f = 1/E[m]$

$$\mathbf{p = E[x]/R^f + Cov[m,x]}$$

Remarks:

(i) If risk-free rate does not exist, R^f is the shadow risk free rate

(ii) In general $Cov[m,x] < 0$, which lowers price and increases return

Risk-adjustment in Returns

$E[mR^j]=1$ $\quad\quad$ $R^f\, E[m]=1$

$\Rightarrow E[m(R^j-R^f)]=0$

$E[m]\{E[R^j]-R^f\} + Cov[m,R^j]=0$

$$\mathbf{E[R^j] - R^f = - Cov[m,R^j]/E[m]} \qquad \mathbf{(2)}$$

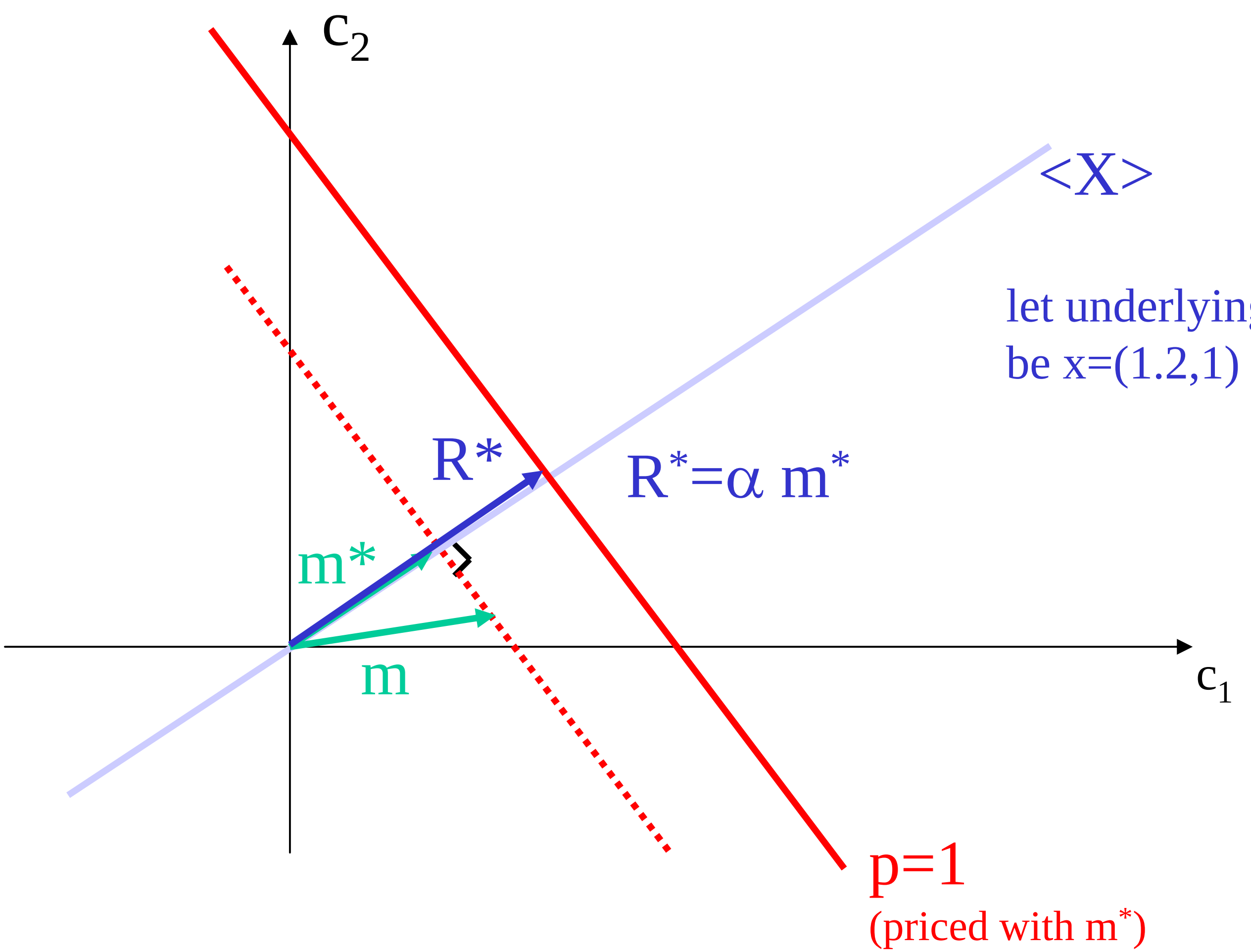
also holds for portfolios *h*

Note:

- risk correction depends only on Cov of payoff/return with discount factor.
- Only compensated for taking on systematic risk not idiosyncratic risk.

State-price BETA Model

shrink axes by factor $\sqrt{\pi_s}$



State-price BETA Model

$$\mathbf{E[R^j] – R^f = - Cov[m,R^j]/E[m] \qquad (2)}$$

also holds for all portfolios *h* and

we can replace m with m^*

Suppose (i) $Var[m^*] > 0$ and (ii) $R^* = \alpha\, m^*$ with $\alpha > 0$

$$E[R^h] – R^f = - Cov[R^*,R^h]/E[R^*] \qquad (2')$$

Define $\boldsymbol{\beta^h := Cov[R^*,R^h]/ Var[R^*]}$ for any portfolio *h*

Regression $R^h_s = \alpha^h + \beta^h (R^*)_s + \varepsilon_s$ with $Cov[R^*,\varepsilon]=E[\varepsilon]=0$

State-price BETA Model

(2) for R^*: $E[R^*]-R^f=-Cov[R^*,R^*]/E[R^*]$

$=-Var[R^*]/E[R^*]$

(2) for R^h: $E[R^h]-R^f=-Cov[R^*,R^h]/E[R^*]$

$= - \beta^h \; Var[R^*]/E[R^*]$

$$\mathbf{E[R^h] - R^f = \beta^h \, E[R^* - R^f]}$$

where $\beta^h := Cov[R^*,R^h]/Var[R^*]$

very general – but what is R^* in reality?

Different Asset Pricing Models

$p_t = E[m_{t+1} \, x_{t+1}]$ $\Rightarrow$ $E[R^h] - R^f = \beta^h \, E[R^* - R^f]$

where $\beta^h := Cov[R^*, R^h]/Var[R^*]$

where $m_{t+1} = f(\cdot, \ldots, \cdot)$

f(·) = asset pricing model

General Equilibrium

$f(\cdot) = MRS / \pi$

Factor Pricing Model

$a + b_1 \, f_{1,t+1} + b_2 \, f_{2,t+1}$

CAPM

$a + b_1 \, f_{1,t+1} = a + b_1 \, R^M$

CAPM

$R^* = R^f \, (a + b_1 R^M)/(a + b_1 R^f)$

where R^M = return of market portfolio

Is $b_1 < 0$?

Different Asset Pricing Models

- Theory
  - ➢All economics and modeling is determined by
    $$m_{t+1} = a + \mathbf{b}' \mathbf{f}$$
  - ➢Entire content of model lies in restriction of SDF
- Empirics
  - ➢m^* (which is a portfolio payoff) prices as well as m (which is e.g. a function of income, investment etc.)
  - ➢ measurement error of m^* is smaller than for any m
  - ➢Run regression on *returns* (portfolio payoffs)! (e.g. Fama-French three factor model)