Problem Set 1

Fin 525: Financial Economics I
Part 1: Asset Pricing in Discrete Time

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Due Date: TBA

Problem 1

During the bagel hour on Thursday morning, Max (a fellow Ph.D. student) approaches you. He looks very tired and after your standard "How are you?", he answers only with a soft "OK". "What's the problem?", you reply. He admits that he had a hard time falling asleep last night since he could not find a difference between the three No-Arbitrage Conditions mentioned in the last Finance lecture. He also thinks that the Law of one Price (LOOP) and the No-Arbitrage Conditions (NAC) are essentially the same thing. Can you help him out? (Contrast the LOOP and NACs etc.)

Problem 2

Moritz thinks an illustrative example would really help to understand the concept of no arbitrage. Here it is:

Consider an economy with two assets. The payoffs are $x^1 = (10, -20, 60)$ and $x^2 = (20, 30, 10)$.

(a) Find the set P of all price pairs $p = (p_1, p_2)$ which are arbitrage free. What is the shape of P? Does the law of one-price hold?

(b) For each $p \in P$ find the set of all state price vectors $\pi = (\pi_1, \pi_2, \pi_3)$ consistent with those prices (i.e. $p = \pi X$).

(c) What is the space of all prices $\overline{P}$ that admit arbitrage? For each $\overline{p} \in \overline{P}$ exhibit a portfolio $\theta = (\theta_1, \theta_2)$ which gets something for nothing.

*Grader: E. Glen Weyl

Problem 3

There are three states of the world in period 1 and two securities. The first security, the " bond" , pays $(1, 1, 1)$, whereas the second security, the "stock" , pays $(0, 1, 1)$.

(a) Calculate the set of all payoffs that can be achieved by portfolios containing the bond and the stock.

From now on assume that there is no arbitrage.

(b) What restrictions does the vector (p^1, p^2) of the prices of the bond and the stock satisfy?

(c) Find all state prices that are compatible with $(p^1, p^2) = (x, y)$. What are the minimum and maximum prices that a security that pays $(1, 1, 0)$ can sell for? What about the security $(1, 0, 0)$?

(d) Identify all securities for which there is a unique price compatible with a given prices p^1 for the bond and p^2 for the stock. How does your answer compare with the answer to (a)? What is the economic interpretation?

(e) How many other securities would you have to know the price of so that you could then price any other security? Identify the set of all securities (or pairs/triplets...) such that if you knew their price, in addition to observing the price of the bond and the stock, you could then determine the price of any other security. How do these relate to the answer in parts (d) and (a)? What is the economic interpretation?

Problem 4

Consider a one-period model with $S = 2$ and two agents $I = 2$. Their utility function is

$$\begin{aligned} u^1(c) &= \log c_0 + \frac{1}{2}\left(\frac{1}{2}\log c_1 + \frac{1}{2}\log c_2\right) \\ u^2(c) &= \log c_0 + \frac{1}{3}\left(\frac{1}{2}\log c_1 + \frac{1}{2}\log c_2\right) \end{aligned}$$

and endowments $e^1 = (19/8, 1, 3)$, $e^2 = (21/8, 5, 3)$.

(a) Suppose there are two securities traded at date 0: the riskless bond and a security which pays 1 unit in state 1 and nothing in state 2. Find the financial markets equilibrium. (Hint: the computation can be kept short if you are clever.) What is the rate of interest? Give a careful economic interpretation of the portfolio strategy adopted by each agent.

(b) Suppose that only the riskless bond is traded at date 0. Find the financial markets equilibrium for this incomplete marke and compare the equilibrium allocation with that in 1. Comment on the differences. (Hint: it is not possible to avoid some tedious computations but the numbers are chosen so that you can find the equilibrium without help of a computer.) What economic role do complete markets play?

(c) Compute the difference in consumer welfare between the complete and incomplete market setting for each consumer. How large would the cost of completing the market have to be for it not to be economically efficient to do so? Some economists have argued that in settings of complete information, transactions and start-up costs are unlikely to lead to "much" market incompleteness. Interpret this in your own words; what bounds can we place on the "importance" of market incompleteness? If markets are too incomplete (relative to the cost of creating a market), what should we expect to happen?

(d) If costs of creating a market are unlikely to lead markets to be incomplete, what might? Be creative in listing some factors and try to justify them with some economic intuition or a simple model. What characteristics would each factor predict unmarketed claims would "look like"? Can you think of a case where market completeness might be harmful(in the absence of additional legal restraints on trading)?

Problem 5

If you have extra time, try writing your own problem based on or extending the problems of this week (perhaps jumping off from part d of Problem 4). Also, your thoughts, opinions or just rating of the problems above would be *extremely* useful.