Princeton Initiative 2011
Macro, Money and Finance
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Motivation

- **Aim:** Bridge the gap between
  - Macro/money research
  - Finance research

- **Financial sector helps to**
  - overcome financing frictions and
  - channels resources
  - creates money

... but

- Credit crunch due to adverse feedback loops & liquidity spirals
  - Non-linear dynamics

- **New insights to monetary and international economics**
Friday
- Macro models with financial frictions in continuous time
- Financial frictions: Empirical evidence

Saturday
- Demand for liquid assets, Money and Bubbles
- Funding liquidity risk (rollover risk)
- Bubbles

Sunday
- Fiscal Theory of the Price Level
- International: Global Liquidity and Capital control
Systemic risk – a broad definition

- Systemic risk build-up during (credit) bubble ... and materializes in a crisis
  - “Volatility Paradox” \(\rightarrow\) contemp. measures inappropriate
- Spillovers/contagion – externalities
  - Direct contractual: domino effect (interconnectedness)
  - Indirect: price effect (fire-sale externalities) credit crunch, liquidity spirals

- Adverse GE response amplification, persistence
Minsky moment – Wile E. Coyote Effect
Instruments

- Price stability
  Monetary policy
  - Short-term interest
  - Policy rule (terms structure)

- Financial stability
  Macroprudential policy
  - Reserve requirements
  - Capital/liquidity requirements
  - Collateral policy
  - Margins/haircuts
  - Capital controls

- Output (gap)
Methodology

- **Verbal Reasoning** *(qualitative)*
  
  Fisher, Keynes, ...

- **Macro**
  
  Growth theory
  
  - Dynamic *(cts. time)*
  
  - Deterministic

  Introduce stochastic
  
  - Discrete time
    
    - Brock-Mirman, Stokey-Lucas
    
    - DSGE models

- **Finance**

  Portfolio theory
  
  - Static
  
  - Stochastic

  Introduce dynamics
  
  - Cts. time
    
    - Option Black Scholes
    
    - Term structure CIR
    
    - Agency theory Sannikov

- **Cts. time macro with financial frictions**
Heterogeneous agents + frictions

- Lending-borrowing/insuring since agents are different
  - Poor-rich
  - Productive
  - Less patient
  - Less risk averse
  - More optimistic

- Rich-poor
  - Less productive
  - More patient
  - More risk averse
  - More pessimistic

- Limited direct lending due to frictions

- Friction → \( p_s \)MRS\(_s\) different even after transactions

- Wealth distribution matters! (net worth of subgroups)

- Financial sector is not a veil
Liquidity Concepts

- Financial instability arises from the fragility of liquidity

Technological liquidity
- Reversibility of investment

Market liquidity
- Specificity of capital
- Price impact of capital sale

Funding liquidity
- Maturity structure of debt
- Can’t roll over short term debt
- Sensitivity of margins
- Margin-funding is recalled

- The liquidity mismatch between assets and liabilities determines the severity of the amplification effects
Types of Funding Constraints

- **Equity constraint**
  - Skin in the game constraint

- **Debt constraints**
  - Costly state verification a la Townsend
  - Commitment problems/collateral constraints
    - Incomplete contracts a la Hart-Moore
    - Credit rationing a la Stiglitz-Weiss

- Affected by
  - Price of collateral
  - Volatility of collateral

BruSan

CF, BGG

KM, BP
1. **Net worth effects:**
   a. **Persistence:** Carlstrom, Fuerst
   b. **Amplification:** Bernanke, Gertler, Gilchrist
   c. **Instability:** Brunnermeier, Sannikov

2. **Volatility effects:** Credit quantity constraints
   a. **Credit rationing:** Stiglitz, Weiss
   b. **Margin spirals:** Brunnermeier, Pederson
   c. **Endogenous constraints:** Geanakoplos

3. **Demand for liquid assets & Bubbles – “self insurance”**
   a. OLG, Aiyagari, Bewley, Krusell-Smith, Holmstroem Tirole,...

4. **Financial intermediaries & Theory of Money**
Amplification & Instability - Overview

  - Perfect (technological) liquidity, but persistence
  - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period

  - Technological/market illiquidity
  - KM: Leverage bounded by margins; BGG: Verification cost (CSV)
  - Stronger amplification effects through prices (low net worth reduces leveraged institutions’ demand for assets, lowering prices and further depressing net worth)

- Brunnermeier & Sannikov (2010)
  - Instability and volatility dynamics, volatility paradox

- Brunnermeier & Pedersen (2009), Geanakoplos
  - Volatility interaction with margins/haircuts (leverage)
Amplification & Instability - Overview

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Persistence

- Even in standard real business cycle models, temporary adverse shocks can have long-lasting effects.
- Due to feedback effects, persistence is much stronger in models with financial frictions.
  - Bernanke & Gertler (1989)
  - Carlstrom & Fuerst (1997)
- Negative shocks to net worth exacerbate frictions and lead to lower capital, investment and net worth in future periods.
Costly State Verification

- Key friction in previous models is costly state verification, i.e. CSV, a la Townsend (1979)
- Borrowers are subject to an idiosyncratic shock
  - Unobservable to lenders, but can be verified at a cost
- Optimal solution is given by a contract that resembles standard debt
CSV: Contracting

- Competitive market for capital
  - Lender’s expected profit is equal to zero
  - Borrower’s optimization is equivalent to minimizing expected verification cost

- Financial contract specifies:
  - Debt repayment for each reported outcome
  - Reported outcomes that should be verified
CSV: Optimal Contract

- Incentive compatibility implies that
  - Repayment outside of VR is constant
  - Repayment outside of VR is weakly greater than inside
- Maximizing repayment in VR reduces the size and thus the expected verification cost
Output is produced according to $Y_t = A_t f(K_t)$

Fraction $\eta$ of entrepreneurs and $1 - \eta$ of households
- Only entrepreneurs can create new capital from consumption goods

Individual investment yields $\omega i_t$ of capital
- Shock is given by $\omega \sim G$ with $E[\omega] = 1$
- This implies consumption goods are converted to capital one-to-one in the aggregate
- *No technological illiquidity!*
Households can verify $\omega$ at cost $\mu i_t$

- Optimal contract is debt with audit threshold $\bar{\omega}$
- Entrepreneur with net worth $n_t$ borrows $i_t - n_t$ and repays $\min\{\omega_t, \bar{\omega}\} \times i_t$

Auditing threshold is set by HH breakeven condition

$$\left[ \int_0^{\bar{\omega}} (\omega - \mu)dg(\omega) + (1 - G(\bar{\omega}))\bar{\omega} \right] i_t q_t = i_t - n_t$$

- Here, $q_t$ is the price of capital

No positive interest (within period borrowing) and no risk premium (no aggregate investment risk)
Entrepreneur’s optimization:

\[ \max_{i_t} \int_{\omega_t}^{\infty} (\omega - \bar{\omega}_t) dG(\omega) i_t q_t \]

Subject to HH breakeven constraint

Linear investment rule \( i_t = \psi(q_t) n_t \)

Leverage \( \psi(q_t) \) is increasing in \( q_t \)

Aggregate supply of capital is increasing in

- Price of capital \( q_t \)
- Aggregate net worth \( N_t \)
Return to holding capital:

\[ R_{t+1}^k = \frac{A_{t+1}f'(K_{t+1})+(1-\delta)q_{t+1}}{q_t} \]

Risk averse HH have discount factor \( \beta \)

- Standard utility maximization
- Budget constraint:
  \[ c_t \leq A_t f'(K_t)k_t + q_t[(1-\delta)k_t - k_{t+1}] \]
- Euler equation:
  \[ u'(c_t) = \beta E_t[R_{t+1}^k u'(c_{t+1})] \]
CF: Demand for Capital

- Risk-neutral entrepreneurs are less patient, \( \beta < \bar{\beta} \)
  - Euler equation: \( 1 = \beta E_t[R_{t+1}^{k} \rho(q_t)] \)
  - Return on internal funds:
    \[ \rho(q_t) = \int_{\bar{\omega}_t}^{\infty} (\omega - \bar{\omega}_t) dG(\omega) \psi(q_t) q_t \]
- Aggregate demand for capital is decreasing in \( q_t \)
CF: Persistence & Dampening

- Negative shock in period $t$ decreases $N_t$
  - This increases financial friction and decreases $I_t$

- Decrease in capital supply leads to
  - Lower capital: $K_{t+1}$
  - Lower output: $Y_{t+1}$
  - Lower net worth: $N_{t+1}$
  - Feedback effects in future periods $t + 2$, ...

- Decrease in capital supply also leads to
  - Increased price of capital $q_t$
  - Dampening effect on propagation of net worth shock
Bernanke, Gertler and Gilchrist (1999) introduce *technological illiquidity* in the form of nonlinear adjustment costs to capital.

- Negative shock in period $t$ decreases $N_t$
  - This increases financial friction and decreases $I_t$

- In contrast to the dampening mechanism present in CF, decrease in capital supply leads to
  - Decreased price of capital due to adjustment costs
  - *Amplification* effect on propagation of net worth shock
BGG assume separate investment sector

- This separates entrepreneurs’ capital decisions from adjustment costs

Φ(·) represents *technological illiquidity*

- Increasing and concave with Φ(0) = 0

  \[ K_{t+1} = \Phi \left( \frac{I_t}{K_t} \right) K_t + (1 - \delta)K_t \]

FOC of investment sector

- \( \max_{I_t} \{ q_t K_{t+1} - I_t \} \Rightarrow q_t = \Phi' \left( \frac{I_t}{K_t} \right)^{-1} \)

jump to KM97
Entrepreneurs alone can hold capital used in production

At time $t$, entrepreneurs purchase capital for $t + 1$
  - To purchase $k_{t+1}$, an entrepreneur borrows $q_t k_{t+1} - n_t$
  - Here, $n_t$ represents entrepreneur net worth

Assume gross return to capital is given by $\omega R^k_{t+1}$
  - Here $\omega \sim G$ with $E[\omega] = 1$ and $\omega$ i.i.d.
  - $R^k_{t+1}$ is the endogenous aggregate equilibrium return
Entrepreneurs borrow from HH in a CSV framework

If \( R_{t+1}^k \) is deterministic, then threshold satisfies:

\[
\left[ (1 - \mu) \int_0^{\bar{\omega}} \omega dG(\omega) + (1 - G(\bar{\omega}))(\bar{\omega}) \right] R_{t+1}^k q_t k_{t+1} = R_{t+1}(q_t k_{t+1} - n_t)
\]

Here, \( R_{t+1} \) is the risk-free rate

If there is aggregate risk in \( R_{t+1}^k \) then BGG argue that entrepreneurs will insure HH against risk

This amounts to setting \( \bar{\omega} \) as a function of \( R_{t+1}^k \)

As in CF, HH perfectly diversify against entrepreneur idiosyncratic risk
Entrepreneurs solve the following problem:

\[ \max_{k_{t+1}} E \left[ \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}) dG(\omega) R_{t+1}^k q_t k_{t+1} \right] \]

Subject to HH breakeven condition (state-by-state)

Optimal leverage is again given by a linear rule

\[ q_t k_{t+1} = \psi \left( \frac{E[R_{t+1}^k]}{R_{t+1}} \right) n_t \]

In a log-linearized solution, the remaining moments are insignificant

Aggregate capital supply is increasing in \( E[R_{t+1}^k] \) and aggregate net worth \( N_t \)
Return on capital is determined in a general equilibrium framework

- Gross return to holding a unit of capital

\[ E[R_{t+1}^k] = E \left[ \frac{A_{t+1}f'(K_{t+1}) + q_{t+1}(1-\delta) + q_{t+1}\Phi \left( \frac{I_{t+1}}{K_{t+1}} \right) - I_{t+1}}{q_t} \right] \]

- Capital demand is decreasing in expected return

\[ E[R_{t+1}^k] \]
BGG: Persistence & Amplification

- Shocks to net worth $N_t$ are persistent
  - They affect capital holdings, and thus $N_{t+1}$, ...

- *Technological illiquidity* introduces amplification effect
  - Decrease in capital leads to reduced price of capital from
    $$q_t = \Phi' \left( \frac{I_t}{K_t} \right)^{-1}$$
  - Lower price of capital further decreases net worth
Kiyotaki & Moore 97

- Kiyotaki, Moore (1997) adopt a collateral constraint instead of CSV market illiquidity – second best use of capital
- Durable asset has two roles:
  - Collateral for borrowing
  - Input for production
- Output is produced in two sectors, differ in productivity
- Aggregate capital is fixed, resulting in extreme technological illiquidity
  - Investment is completely irreversible
KM: Amplification

- **Static** amplification occurs because fire-sales of capital from productive sector to less productive sector depress asset prices
  - Importance of *market liquidity* of physical capital
- **Dynamic** amplification occurs because a temporary shock translates into a persistent decline in output and asset prices
KM: Agents

- Two types of infinitely-lived risk neutral agents
- Mass $\eta$ of productive agents
  - Constant-returns-to-scale production technology yielding $y_{t+1} = ak_t$
  - Discount factor $\beta < 1$
- Mass $1 - \eta$ of unproductive agents
  - Decreasing-returns-to-scale production $y_{t+1} = F(k_t)$
  - Discount factor $\beta \in (\beta, 1)$
Since productive agents are less patient, they will want to borrow $b_t$ from unproductive agents

- However, friction arises in that each productive agent’s technology requires his individual human capital
- Productive agents cannot pre-commit human capital

This results in a collateral constraint $Rb_t \leq q_{t+1}k_t$

- Productive agent will never repay more than the value of his asset holdings, i.e. collateral
Since there is no uncertainty, a productive agent will borrow the maximum quantity and will not consume any of the output.

- Budget constraint: \( q_t k_t + b_t \leq (a + q_t)k_{t-1} - Rb_{t-1} \)
- Demand for assets: \( k_t = \frac{1}{q_t - \frac{a + q_t}{R}} [(a + q_t)k_{t-1} - Rb_{t-1}] \)

Unproductive agents are not borrowing constrained.

- \( R = \beta^{-1} \) and asset demand is set by equating margins.
- Demand for assets: \( R = \frac{F'(k_t) + q_t + 1}{q_t} \)
**KM: Equilibrium**

- With fixed supply of capital, market clearing requires $\eta K_t + (1 - \eta)K_t = \bar{K}$
  - This implies $M(K_t) \equiv \frac{1}{R} F' \left( \frac{\bar{K} - \eta K_t}{1 - \eta} \right) = q_t - \frac{1}{R} q_{t+1}$
  - Note that $M(\cdot)$ is increasing

- Iterating forward, we obtain: $q_t = \sum_{s=0}^{\infty} \frac{1}{R^s} M(K_{t+s})$
In steady state, productive agents use tradable output $a$ to pay interest on borrowing:

This implies that steady state price $q^*$ must satisfy:

$$q^* - \frac{1}{R}q^* = a$$

Further, steady state capital $K^*$ must satisfy:

$$\frac{1}{R} F'(\frac{\bar{K} - \eta K^*}{1-\eta}) = a$$

This reflects inefficiency since marginal products correspond only to *tradable* output.
KM: Productivity Shock

- Log-linearized deviations around steady state:
  - Unexpected one-time shock that reduces production of all agents by factor $1 - \Delta$

- Change in assets for given change in asset price:
  - $\hat{K}_t = -\frac{\xi}{1+\xi} \left( \Delta + \frac{R}{R-1} \hat{q}_t \right)$, $\hat{K}_{t+s} = \frac{\xi}{1+\xi} \hat{K}_{t+s-1}$
  - $\frac{1}{\xi} = \left. \frac{d \log M(K)}{d \log K} \right|_{K=K^*}$

- Reduction in assets comes from two shocks:
  - Lost output $\Delta$
  - Capital losses on previous assets $\frac{R}{R-1} \hat{q}_t$
**KM: Productivity Shock**

- Change in price for given change in assets:
  - Log-linearize the equation \( q_t = \sum_{s=0}^{\infty} \frac{1}{R^s} M(K_{t+s}) \)
  - This provides: \( \hat{q}_t = \frac{1}{\xi} \frac{R-1}{R} \sum_{s=0}^{\infty} \frac{1}{R^s} \hat{K}_{t+s} \)

- Combining equations:
  - \( \hat{K}_t = - \left( 1 + \frac{1}{(\xi+1)(R-1)} \right) \Delta \)
  - \( \hat{q}_t = - \frac{1}{\xi} \Delta \)
KM: Static vs. Dynamic Amplification

- We can decompose the previous equations into static and dynamic multiplier effects
  - Static effect results from assuming $q_{t+1} = q^*$
  - Static multiplier:
    - $\hat{K}_t^S = -\Delta$
    - $\hat{q}_t^S = -\frac{(R-1)}{R} \frac{1}{\xi} \Delta$

- Dynamic multiplier:
  - $\hat{K}_t^D = -\frac{1}{(\xi+1)(R-1)} \Delta$
  - $\hat{q}_t^D = -\frac{1}{R} \frac{1}{\xi} \Delta$
BruSan10: Instability & Non-Linear Effects

- Previous papers only considered log-linearized solutions around steady state
- Brunnermeier & Sannikov (2010) build a continuous time model to study full dynamics
  - Show that financial system exhibits inherent instability due to highly non-linear effects
  - These effects are asymmetric and only arise in the downturn
- Agents choose a *capital cushion*
  - Mitigates moderate shocks near steady state
  - High volatility away from steady state
Structuring the Macro-literature on Frictions

1. Net worth effects:
   a. Persistence: Carlstrom, Fuerst
   b. Amplification: Bernanke, Gertler, Gilchrist
   c. Instability: Brunnermeier, Sannikov

2. Volatility effects: Credit quantity constraints
   a. Credit rationing: Stiglitz, Weiss
   b. Margin spirals: Brunnermeier, Pederson
   c. Endogenous constraints: Geanakoplos

3. Demand for liquid assets & Bubbles – “self insurance”
   a. OLG, Aiyagari, Bewley, Krusell-Smith, Holmstroem Tirole,…

4. Financial intermediaries & Theory of Money
Credit Rationing

- Credit rationing refers to a failure of market clearing in credit
  - In particular, an excess demand for credit that fails to increase market interest rate
- Stiglitz, Weiss (1981) show how asymmetric information on risk can lead to credit rationing
For collateralized lending, debt constraints are directly linked to the volatility of collateral
- Constraints are more binding in volatile environments
- Feedback effect between volatility and constraints

These margin spirals force agents to delever in times of crisis
- Collateral runs
- Counterparty bank run
- Multiple equilibria
BP: Margins – Value at Risk (VaR)

- Margins give incentive to hold well diversified portfolio
- How are margins set by brokers/exchanges?
  - **Value at Risk**: \( \Pr(-(\mathit{p}_{t+1} - \mathit{p}_t) \geq m) = 1\% = \pi \)
Borrowers’ balance sheet

- Loss spiral – like in BGG/KM
  - Net wealth > \( \alpha \times \) for asym. info reasons
  - constant or increasing leverage ratio

- Margin/haircut spiral
  - Higher margins/haircuts
  - No rollover
  - redemptions
  - forces to delever

Mark-to-market vs. mark-to-model

- worsens loss spiral
- improves margin spiral

Source: Brunnermeier & Pedersen (2009)

- Both spirals reinforce each other
BP: Margin Spirals - Intuition

1. Volatility of collateral increases
   - Permanent price shock is accompanied by higher future volatility (e.g. ARCH)
     - Realization how difficult it is to value structured products
   - Value-at-Risk shoots up
   - Margins/haircuts increase = collateral value declines
   - Funding liquidity dries up
   - Note: all “expert buyers” are hit at the same time, SV 92

2. Adverse selection of collateral
   - As margins/ABCP rate increase, selection of collateral worsens
   - SIVs sell-off high quality assets first (empirical evidence)
   - Remaining collateral is of worse quality
BP: Model Setup

- Time: \( t=0,1,2 \)
- One asset with final asset payoff \( v \) (later: assets \( j=1,\ldots,J \))
- Market illiquidity measure: \( \Lambda_t = |E_t(v) - p_t| \)
  (deviation from “fair value” due to selling/buying pressure)
- Agents
  - Initial customers with supply \( S(z, E_t[v] - p_t) \) at \( t=1,2 \)
  - Complementary customers’ demand \( D(z, E_2[v] - p_2) \) at \( t=2 \)
  - Risk-neutral dealers provide *immediacy* and
    - face capital constraint
  - \( x_m(\sigma, \Lambda) \leq W(\Lambda) := \max\{0, B + x_0(E_1[v] - \Lambda)\} \)
    - cash “price” of stock holding
BP: Financiers’ Margin Setting

- Margins are set based on Value-at-Risk
- Financiers do not know whether price move is due to
  - *Likely*, movement in fundamental
  - *Rare*, Selling/buying pressure by customers who suffered asynchronous endowment shocks.

\[
m_1^j^+ = \Phi^{-1}(1 - \pi)\sigma_2 = \bar{\sigma}^j + \bar{\theta}|\Delta p_1^j| = m_1^j^-
\]
BP: Margin Spiral – Increased Vol.

\[ v_t = v_{t-1} + \Delta v_t = v_{t-1} + \sigma_t \varepsilon_t \]

\[ \sigma_{t+1} = \sigma + \theta |\Delta v_t| \]

Selling pressure
initial customers

complementary
customers
BP: Model Setup in a Figure

\[ v_t = v_{t-1} + \Delta v_t = v_{t-1} + \frac{3}{4} \]

\[ \frac{3}{4} + \mu_j \Delta v_t \]

\[ \Lambda \]

Selling pressure
Initial customers

Complementary
Customers

\[ \gamma = 0.01 \quad \sigma^2 = 16 \quad z_0 = 20 \quad z_1 = 20 \quad v_0 = 140 \quad v_1 = 120 \]

\[ p_0 = 130 \quad k = 10 \quad \theta = 0.3 \quad \eta_1 = 0 \quad W_0 = 700 \quad x_0 = 0 \]

\[ x_1 < \frac{W_1}{m_1} = \frac{W_1}{(\sigma + \tilde{\theta}|\Delta p_1|)} \]

customers’ supply

\[ \gamma = 0.025 \quad \sigma^2 = 11 \quad z_0 = 20 \quad z_1 = 20 \quad \nu_0 = 140 \quad \nu_1 = 120 \]

\[ p_0 = 130 \quad k = 5 \quad \vartheta = 0.3 \quad \eta_1 = 0 \quad W_0 = 750 \quad x_0 = 0 \]

\[ x_1 < \frac{W_1}{m_1} = \frac{W_1}{(\sigma + \bar{\theta}|\Delta p_1|)} \]

Customers’ supply

\[ x_1 < W_1/m_1 = W_1/(\sigma + \bar{\theta}|\Delta p_1|) \]

customers’
supply
Data Gorton and Metrick (2011)

Haircut Index

“The Run on Repo”
Margins **stable** in tri-party repo market
- contrasts Gorton and Metrick
- no general run on certain collateral

Run (non-renewed financing) only on select **counterparties**
- Bear Stearns (anecdotally)
- Lehman (in the data)

Like 100% haircut...
**counterparty specific!**
Bilateral and Tri-party Haircuts?

Differences in Median Haircuts

Source: FRBNY Calculations
Overview

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