

# Many Enemies, Much Honor? On the Competitiveness of Elections in Proportional Representation Systems.\*

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## Abstract

We develop a model of elections with an endogenous number of candidates and horizontal (policy) and vertical (quality) differentiation in proportional representation electoral systems. We show that in symmetric equilibria *(i)* the larger the number of candidates, the more intense is quality competition; *(ii)* changes in the cost of running for office or shifts in the cost of increasing the quality of the candidates induce a positive correlation between the equilibrium number of candidates running for office and their investment in quality; and *(iii)* a less ideologically focused electorate leads to more differentiation in the policy positions represented in the election and to a smaller number of candidates running for office. We also consider equilibria with limited asymmetry among candidates, and establish a positive relation between the quality of candidates and the effective number of parties.

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# 1 Introduction

Almost one third of all countries and more than thirty percent of all established democracies use a proportional representation (PR) electoral system. In its purest form, a proportional electoral system maps the share of votes obtained by each party in the election into an equal share of seats in the legislature. Since the seminal work of Duverger (1954), PR has been held responsible (at least partially) for the proliferation of political parties in PR democracies. More recently, the political debate shifted its focus to the relation between an increase in the number of legislative parties and a deterioration in the quality of the political environment in terms of lack of competence, or corruption of elected politicians. While the existing literature contains numerous studies supporting Duverger’s hypothesis, the connection between the number of legislative parties and their investment in “quality” has been overlooked both theoretically and empirically.<sup>1</sup> In this paper, we build on Iaryczower and Mattozzi (2008a) to develop a simple theoretical framework in which the quality and the number of candidates running for office are endogenous equilibrium outcomes, and provide conditions under which elections in PR would result in a positive association between the quality and number of candidates running for office.

The essential features of the model are the following. Potential candidates are horizontally differentiated according to a policy position they represent. In particular, they are endowed with a policy position they can champion in government if they choose to run for office and get elected.<sup>2</sup> With their given policy positions, candidates who choose to run for office then compete along a bounded vertical dimension, which we represent as costly activities (investment of money, time or effort) that increase voters’ perception of the quality of a candidate’s platform.<sup>3</sup> Politicians derive utility exclusively from rents they

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<sup>1</sup>Regarding the relation between number of candidates/parties and electoral systems see Osborne and Slivinski (1996), Persson, Roland, and Tabellini (2003), Morelli (2004) and Iaryczower and Mattozzi (2008b) among others. Regarding the relation between electoral systems and corruption see Myerson (1993) and Persson, Tabellini, and Trebbi (2006).

<sup>2</sup>For models of differentiation and entry in industrial organization see Anderson and de Palma (1988), Anderson, de Palma, and Thisse (1989), Caplin and Nalebuff (1986), Perloff and Salop (1985), Shaked and Sutton (1982), Shaked and Sutton (1987), and d’Aspremont, Gabszewicz, and Thisse (1979).

<sup>3</sup>This is in the line of Stokes (1963)’s early criticism to the spatial model and the recent literature incorporating vertical differentiation in majoritarian elections. See Groseclose (2001), Aragonés and Palfrey (2002), Schofield (2004), Herrera, Levine, and Martinelli (2008), Carrillo and Castanheira (2008), Meirowitz (2007), and Ashworth and Bueno de Mesquita (2007).

can appropriate while in office. We assume that there is a large finite number of risk averse and fully rational voters.

The mapping of votes' shares into seats' shares is given by the electoral system. In this paper we consider the case of a perfect PR system, where vote shares are transformed into seat shares one to one.<sup>4</sup> Regarding the mapping from seats to the distribution of rents, we assume that candidates participate in the distribution of rents proportionally to the share of seats obtained in the election (see for example Lizzeri and Persico (2001)). As for policy outcomes, we adopt the simplifying assumption that the policy outcome is the realization of a lottery between the policies represented by the candidates participating in the election, with weights equal to their vote shares (or seat share in the assembly). While it is by no means obvious which is the most appropriate model of policy determination under PR systems, this assumption captures in a stylized manner the additional uncertainty faced by voters that is introduced by the process of post-election bargaining in PR.<sup>5</sup> An *electoral equilibrium* is a Subgame Perfect Nash Equilibrium in pure strategies of the game of electoral competition, i.e., a strategy profile such that (i) voters cannot obtain a preferred policy outcome by voting for a different candidate in any voting game (on and off the equilibrium path), (ii) given the location and quality decisions of other candidates, and given voters' strategy, candidates cannot increase their expected rents by modifying their investment in quality, (iii) candidates running for office collect non-negative rents, and (iv) candidates not running for office prefer not to enter: they would make negative rents in an equilibrium of the continuation game.

We start our analysis by focusing on electoral equilibria with two candidates running for office, and we construct an equilibrium where candidates collect no rents. In this equilibrium, as well as in *any* electoral equilibrium with two candidates running for office and non-maximal (strictly interior) quality, candidates become more aggressive in quality competition the less differentiated they are in the policy space and, given differentiation,

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<sup>4</sup>Iaryczower and Mattozzi (2008a) use a similar model to compare equilibrium outcomes between different electoral systems.

<sup>5</sup>Austen-Smith and Banks (1988), Baron and Diermeier (2001), and Persson, Roland, and Tabellini (2003) study strategic voting induced by the process of government and coalition formation among elected representatives in PR for a given number of parties (three for Austen-Smith and Banks (1988), Baron and Diermeier (2001), four for Persson, Roland, and Tabellini (2003)). Iaryczower and Mattozzi (2008a) explore alternative specifications of the policy function mapping elected representatives into policy outcomes.

the weaker is voters' *ideological focus* (Stokes (1963)). In an equilibrium with no rents, however, a heightened responsiveness of voters to candidates' quality must result in a larger ideological differentiation between candidates running for office, and does not (directly) influence the equilibrium choice of quality.

We then extend the analysis to symmetric electoral equilibria with three or more candidates running for office. In symmetric equilibria, the larger the number of candidates, the closer substitutes they are to each other in the ideological dimension, and therefore the more intense quality competition is. Within the class of symmetric electoral equilibria, we also explore how changes in the “supply side” parameters of the model can affect the number and quality of candidates running for office. In particular, we show that changes in the fixed cost of running for office, or shifts in the cost function of quality, may induce a positive correlation between the equilibrium number of candidates running for office and their quality. We also explore the role of “demand-side” factors, such as the responsiveness of voters to quality differentiation among candidates. We show that in symmetric equilibria where candidates collect no rents, a less ideologically focused electorate leads to more differentiation in the policy positions represented in the election, and to a smaller number of candidates. If instead candidates are collecting positive rents in equilibrium, the impact of demand-side factors can be absorbed by the expected level of rents without affecting the number of candidates running for office. Finally, we also show that the positive relation between quality and number of parties extends to the case of limited asymmetry among equilibrium candidates. In particular, we show that in this case there is a positive equilibrium relation between the quality of candidates and the *effective number of parties* (Laakso and Taagepera (1979)). This is consistent with anecdotal evidence linking (perceived) corruption among public officials and politicians and the effective number of parties.

## 2 The Basic Model

Let  $X = [0, 1]$  be the ideological policy space. In any  $x \in [0, 1]$  there is a potential candidate who can perfectly represent policy  $x$  if elected. There are three stages. In the first stage, all potential candidates simultaneously decide whether or not to run in the election. In

order to run, a candidate must pay a fixed cost  $F > 0$ . We denote the set of candidates running for office at the end of the first stage by  $\mathcal{K} = \{1, \dots, K\}$ . In the second stage, all candidates running for office simultaneously choose a level of quality  $\theta_k \in [0, 1]$  at a cost  $C(\theta_k)$ . We assume that  $C(\cdot)$  is increasing and convex, and let  $C(1) \equiv \bar{c}$ . In the third stage, a large finite number  $n$  of fully strategic voters vote in an election.

A voter  $i$  with ideal point  $z^i \in X$  ranks candidates according to utility function  $u(\cdot; z^i)$ , which assigns the payoff  $u(\theta_k, x_k; z^i) \equiv 2\alpha v(\theta_k) - (x_k - z^i)^2$  to candidate  $k$  with characteristics  $(\theta_k, x_k)$ . We assume that  $v(\cdot)$  is increasing and concave, and define the function  $\Psi(\cdot) \equiv \frac{v'(\cdot)}{C'(\cdot)}$ . The parameter  $\alpha > 0$  measures voters' responsiveness to candidates' quality. Voters' ideal policies are uniformly distributed in  $X$ . Letting  $s_k$  denote the proportion of voters voting for  $k$ , and  $m_k$  denote  $k$ 's proportion of seats in government after the election, we assume that  $m_k = s_k$  (perfect PR). The final policy outcome is a lottery among the candidates participating in the election, with weights equal to their vote shares in the election (or seat share in the assembly). The expected share of rents captured by each candidate is proportional to his vote share in the election. Letting  $\theta_{\mathcal{K}}$  and  $x_{\mathcal{K}}$  denote the vector of quality and ideological positions of candidates running for office, and normalizing the payoff of potential candidates not running for office to zero, the payoff of candidate  $k \in \mathcal{K}$  is given by

$$\Pi_k(\theta_{\mathcal{K}}, x_{\mathcal{K}}, \mathcal{K}) = m_k(\theta_{\mathcal{K}}, x_{\mathcal{K}}) - C(\theta_k) - F.$$

A strategy for candidate  $k$  is a decision of whether to run ( $e_k = 1$ ) or not for office, and a plan of investment in quality  $\theta_k(\mathcal{K}, x_{\mathcal{K}}) \in [0, 1]$ . A strategy for a voter  $i$  is a function  $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) \in \mathcal{K}$ , where  $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = k$  indicates that the choice of voting for candidate  $k$ , and  $\sigma = \{\sigma_1(\cdot), \dots, \sigma_n(\cdot)\}$  denotes a voting strategy profile. An *electoral equilibrium* is a subgame perfect Nash equilibrium of the game of electoral competition; i.e., a set of candidates running for office  $\mathcal{K}^*$ , policy positions  $x_{\mathcal{K}^*}^*$ , quality choices  $\theta_{\mathcal{K}^*}^*$ , and a voting profile  $\sigma^*$  such that: (i)  $\theta_k^*$  is optimal for  $k$  given  $\{\theta_{\mathcal{K}^* \setminus k}^*(\mathcal{K}^*, x_{\mathcal{K}^*}^*), x_{\mathcal{K}^*}^*, \sigma(\mathcal{K}^*, x_{\mathcal{K}^*}^*, \theta_{\mathcal{K}^* \setminus k}^*, \theta_k^*)\}$ ; i.e.,  $\theta_{\mathcal{K}^*}^*$  is a (pure Nash) equilibrium of the continuation game  $\Gamma_{\mathcal{K}^*}$ ; (ii) if  $k \in \mathcal{K}^*$ , then  $\Pi_k(\mathcal{K}^*, x_{\mathcal{K}^*}^*, \theta_{\mathcal{K}^*}^*, \sigma^*(\mathcal{K}^*, x_{\mathcal{K}^*}^*, \theta_{\mathcal{K}^*}^*)) \geq 0$  (no exit condition); (iii) if  $k \notin \mathcal{K}^*$ , then  $\Pi_k(\mathcal{K}^* \cup k, x_k, x_{\mathcal{K}^*}^*, \theta_k^*, \theta_{\mathcal{K}^*}^*, \sigma^*(\mathcal{K}^* \cup k, x_k, x_{\mathcal{K}^*}^*, \theta_k^*, \theta_{\mathcal{K}^*}^*)) < 0$  in an equilibrium of the continuation game (non-profitable entry). An *outcome* of the game is a set of candidates running for office  $\mathcal{K}$ , policy positions  $x_{\mathcal{K}}$ , and quality choices  $\theta_{\mathcal{K}}$ . A *polity* is a triplet  $(\alpha, \bar{c}, F) \in \mathfrak{R}_+^3$ .

We say that Proportional Representation *admits* an electoral equilibrium with outcome  $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$  if there exist a set of polities  $P \subseteq \mathfrak{R}_+^3$  with positive measure such that if a polity  $p \in P$  then there exists an electoral equilibrium with outcome  $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$ .

### 3 Results

We start by characterizing the properties of electoral equilibria with two candidates running for office. First note that, in the absence of investment in quality, equilibrium imposes only relatively weak constraints on the composition of the field of candidates. In particular, the equilibrium requirement of non-negative rents for candidates running for office implies a lower bound on ideological differentiation, while the no-entry condition imposes an upper bound on ideological differentiation. Consider next two candidates 1 and 2 representing policy positions  $x_1 = \Delta_0$  and  $x_2 = x_1 + \Delta$ , with quality  $\theta_1$  and  $\theta_2$ , and let  $\tilde{x}_{12} \in \mathcal{R}$  denote the (unique) value of  $x$  such that  $u(\theta_1, x_1; x) = u(\theta_2, x_2; x)$ , so that  $u(\theta_1, x_1; z^i) > u(\theta_2, x_2; z^i)$  if and only if  $z^i > \tilde{x}_{12}$

$$\tilde{x}_{12} = \frac{x_1 + x_2}{2} + \alpha \frac{[v(\theta_1) - v(\theta_2)]}{\Delta}. \quad (1)$$

Note next that in our model strategic voting is in fact equivalent to sincere voting on and off the equilibrium path. Since the probability that each candidate running for office is elected and implements his ideology is proportional to the share of votes received in the election, voting for a candidate who is not the most preferred one is always a strictly dominated strategy. In fact, by switching her vote to her most preferred candidate, a voter only affects the lottery's weights of exactly two candidates and, with two alternatives, strategic voting and sincere voting coincide.<sup>6</sup> Thus candidate 1's vote share given  $(x, \theta)$  is  $m_1(\theta, x) = \min\{0, \tilde{x}_{12}\}$ . Note that if  $\theta_1 \geq \theta_1(\theta_2, x)$ , where  $m_1(\theta_1(\theta_2, x); \theta_2, x) \equiv 0$ , the vote share mapping  $m_k(\theta_k; \theta_{-k}, x)$  is differentiable and the marginal vote share is given by

$$\frac{\partial m_1}{\partial \theta_1} = \frac{\alpha v'(\theta_1)}{\Delta}, \quad (2)$$

that is, the marginal impact of quality on vote share given the identity of  $k$ 's relevant competitors is well-defined, increases with  $\alpha$ , and decreases with  $\Delta$ . In the next proposition we focus on equilibria in which exactly two candidates run for office.

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<sup>6</sup>See Iaryczower and Mattozzi (2008a) for a formal argument.

**Proposition 1** *Proportional Representation admits an electoral equilibrium in which exactly two candidates run for office. In any two-candidates equilibrium, candidates choose the same quality,*

$$\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1} \left( \frac{\Delta}{\alpha} \right) \leq C^{-1} \left( \frac{1}{2} - F \right). \quad (3)$$

*Furthermore, the more responsive are voters to differences in quality between candidates (the higher is  $\alpha$ ), the higher is candidates' investment in quality, and if candidates do not capture positive rents, also the higher is the degree of ideological polarization between candidates ( $\Delta$ ).*

**Proof of Proposition 1.** To prove this result, we show that if  $\bar{c} \leq \frac{1}{4}$ ,  $\bar{c} + F > \frac{1}{2}$ , and  $\frac{2\bar{c}}{\Psi(C^{-1}(\frac{1}{2}-F))} \leq \alpha \leq \frac{1-2\bar{c}}{\Psi(C^{-1}(\frac{1}{2}-F))}$ , there exists an electoral equilibrium in which two symmetrically located candidates run for office with non-maximal quality, and capture zero rents (showing that PR admits an equilibrium with two candidates collect positive rents follows a similar logic and is therefore omitted).

Suppose that candidates 1 and 2 run for office, and that  $\max\{\theta_1^*, \theta_2^*\} < 1$ . This implies that the FOCs must be satisfied with equality and, in particular, that  $\frac{\alpha}{\Delta} v'(\theta_k) = C'(\theta_k)$  for  $k = 1, 2$ , and hence that  $\Delta \geq \alpha\Psi(1)$ . Then,

$$\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1} \left( \frac{\Delta}{\alpha} \right). \quad (4)$$

Note that when  $\theta_2^* = \theta^* = \Psi^{-1} \left( \frac{\Delta}{\alpha} \right)$ , 1's marginal profit is well-defined, continuous and decreasing at all points  $\theta_1 > \underline{\theta}(\theta^*)$ . Since the condition for non-negative rents is part of the equilibrium definition, it follows that  $\theta_1^* = \theta^*$  is indeed a best response. Furthermore, since  $\theta_1^* = \theta_2^*$ , we have that  $\tilde{x}_{12} = \Delta_0 + \frac{\Delta}{2}$ . Given that in equilibrium candidates must collect nonnegative rents, then it must be true that  $\Pi_1^* = \Delta_0 + \frac{\Delta}{2} - C(\theta_1^*) - F \geq 0$  and  $\Pi_2^* = 1 - \Delta_0 - \frac{\Delta_1}{2} - C(\theta_1^*) - F \geq 0$ , or equivalently,

$$F + C(\theta^*) - \frac{\Delta}{2} \leq \Delta_0 \leq 1 - \frac{\Delta}{2} - C(\theta^*) - F. \quad (5)$$

There exists  $\Delta_0$  satisfying (5) if and only if  $\theta^* \leq C^{-1}(\frac{1}{2}-F)$  or, substituting from (4), if and only if  $\Delta \geq \alpha\Psi(C^{-1}(\frac{1}{2}-F))$ . Since  $\bar{c} + F > \frac{1}{2}$ , it follows that  $\alpha\Psi(C^{-1}(\frac{1}{2}-F)) \geq \alpha\Psi(1)$ . Therefore, if  $\Delta \geq \alpha\Psi(C^{-1}(\frac{1}{2}-F))$  also  $\Delta \geq \alpha\Psi(1)$ . Choose then  $\Delta = \alpha\Psi(C^{-1}(\frac{1}{2}-F))$ .

From (4),  $\theta^* = C^{-1}(\frac{1}{2} - F)$  or  $C(\theta^*) = \frac{1}{2} - F$ . Also since inequalities in (5) hold as equalities,  $\Delta_0 = F + C(\theta^*) - \frac{\Delta}{2} = \frac{1-\Delta}{2}$  and therefore  $\Delta_0 = \frac{1}{2} - \frac{\alpha}{2}\Psi(C^{-1}(\frac{1}{2} - F))$ . Hence,  $\Pi_1^* = \Pi_2^* = 0$ .

Consider next entry of a third candidate  $j$  with  $x_j \in (x_1, x_2)$  and assume the following continuation play:  $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = 1$ . The optimality conditions for  $k = 1$  and  $k = 2$  are  $\frac{\alpha}{(1-\delta_j)\Delta}\Psi(1) \geq 1$  and  $\frac{\alpha}{\delta_j\Delta}\Psi(1) \geq 1$ , where  $\delta_j = \frac{x_2 - x_j}{\Delta}$ . The necessary first order condition for  $j$  is  $\frac{\alpha}{\delta_j(1-\delta_j)\Delta}\Psi(1) \geq 1$  which is implied by the previous inequalities. These conditions are satisfied if and only if

$$\max\{\delta_j, 1 - \delta_j\}\Delta \leq \alpha\Psi(1). \quad (6)$$

Now suppose that  $\delta_j \leq \frac{1}{2}$ . Then (6) is  $(1 - \delta_j)\Delta \leq \alpha\Psi(1)$ , and thus we need  $\alpha\Psi(C^{-1}(\frac{1}{2} - F)) \leq \Delta \leq \frac{\alpha}{1-\delta_j}\Psi(1)$ . This is feasible if

$$\frac{\Psi(C^{-1}(\frac{1}{2} - F)) - \Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \leq \delta_j \leq \frac{1}{2}. \quad (7)$$

Suppose instead that  $\delta_j \geq \frac{1}{2}$ . Then (6) is  $\delta_j\Delta \leq \alpha\Psi(1)$ , and we need  $\alpha\Psi(C^{-1}(\frac{1}{2} - F)) \leq \Delta \leq \frac{\alpha}{\delta_j}\Psi(1)$ . This is feasible if

$$\frac{1}{2} \leq \delta_j \leq \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))}. \quad (8)$$

Combining (7) and (8) we obtain

$$\frac{\Psi(C^{-1}(\frac{1}{2} - F)) - \Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \leq \delta_j \leq \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))}. \quad (9)$$

When (9) holds, i.e., following the entry of a centrist candidate, and  $\Delta = \alpha\Psi(C^{-1}(\frac{1}{2} - F))$ , then  $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = 1$  is a joint best response provided that the incumbent candidates choose not to drop from the race.<sup>7</sup> A sufficient condition for the latter statement to be true (when (9) holds) is  $\alpha \leq \frac{1-2\bar{c}}{\Psi(1)}$ . When  $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_j = 1$  we have that  $\hat{\Pi}_j = \frac{\Delta_1}{2} - \bar{c} - F < 0$ , since  $\bar{c} + F > \frac{1}{2}$  and  $\Delta < 1$ . Now consider entries such that  $\delta_j > \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} = \frac{\Psi(1)}{\Delta}$ . In this case  $j$  enters relatively close to  $k = 1$ , and a strategy profile such that all three candidates

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<sup>7</sup>Note that when  $\delta_j \leq (\geq) \frac{1}{2}$ , we need  $\Delta \leq \frac{\alpha}{(1-\delta_j)}\Psi(1)$  ( $\Delta \geq \frac{\alpha}{\delta_j}\Psi(1)$ ). From (8) this holds for all “feasible”  $\delta_j$  if and only if  $\Delta \leq \alpha\Psi(C^{-1}(\frac{1}{2} - F))$ . But then, since we also need  $\Delta \geq \alpha\Psi(C^{-1}(\frac{1}{2} - F))$ , this must hold with equality. It is not surprising that, given zero profit in equilibrium, it must be the case that a unique  $\Delta$  is the one that covers all possible  $\delta_j$  in (9).

choose maximal quality cannot be an equilibrium of the continuation game. Consider instead  $\hat{\theta}_2 \in (0, 1)$ , and  $\hat{\theta}_1 = \hat{\theta}_j = 1$ . The FOC for  $k = 2$  is  $\frac{\alpha}{\delta_j \Delta} \Psi(\hat{\theta}_2) = 1$ , or equivalently  $\hat{\theta}_2 = \Psi^{-1}\left(\frac{\delta_j \Delta}{\alpha}\right) = \Psi^{-1}\left(\frac{x_2 - x_j}{\alpha}\right)$ . The FOC for  $k = 1$  is, as before,  $(1 - \delta_j)\Delta \leq \alpha\Psi(1)$ , and the FOC for  $j$  is not relevant. Therefore, we need  $\alpha\Psi(C^{-1}(\frac{1}{2} - F)) \leq \Delta \leq \frac{\alpha}{1 - \delta_j}\Psi(1)$ , which is feasible if

$$\frac{\Psi(C^{-1}(\frac{1}{2} - F)) - \Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))} \leq \delta_j, \quad (10)$$

and this always holds with  $\delta_j > \frac{\Psi(1)}{\Psi(C^{-1}(\frac{1}{2} - F))}$ .<sup>8</sup> We need to show now that  $\hat{\Pi}_j = \tilde{x}_{j2}(1, \hat{\theta}_2) - \frac{x_1 + x_j}{2} - \bar{c} - F < 0$ . If  $\tilde{x}_{1j}$  were fixed,  $j$  would be better off by choosing  $\tilde{\theta}_j = \Psi^{-1}\left(\frac{\delta_j \Delta}{\alpha}\right) = \hat{\theta}_2$ . But then  $\hat{\Pi}_j < \frac{\Delta}{2} - C(\hat{\theta}_2) - F < \frac{\Delta}{2} - C(\theta^*) - F < 0$ . Again, we need to make sure that the incumbent candidates choose not to drop from the electoral race in the continuation game. A sufficient condition for this is  $\alpha \leq \frac{1 - 2\bar{c}}{\Psi(C^{-1}(\frac{1}{2} - F))}$ . For no entry at the extremes it is sufficient that  $\max\{\Delta_0, 1 - \Delta - \Delta_0\} < F$  and  $\frac{\Delta}{2} > \bar{c}$  or  $\alpha \geq \frac{2\bar{c}}{\Psi(C^{-1}(\frac{1}{2} - F))}$ . Hence, if  $F + \bar{c} \geq \frac{1}{2}$ ,  $\bar{c} \leq \frac{1}{4}$ , and  $\alpha \in \left(\frac{2\bar{c}}{\Psi(C^{-1}(\frac{1}{2} - F))}, \frac{1 - 2\bar{c}}{\Psi(C^{-1}(\frac{1}{2} - F))}\right)$  then all the previous conditions hold and an equilibrium exists. The second part of the proposition follows from simple inspection of (4) and from noticing that, when candidates are collecting zero rents,  $\theta^* = C^{-1}\left(\frac{1}{2} - F\right)$ . ■

Note that, since in *any* electoral equilibrium with two candidates running for office and positive rents  $\theta_1^* = \theta_2^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{\alpha}\right)$ , candidates become more aggressive in quality competition the less differentiated they are in the policy space and, given  $\Delta$ , the weaker is voters' ideological focus (the larger is  $\alpha$ ). To achieve zero rents, however, it must be the case that  $\Delta = \alpha\Psi(C^{-1}(\frac{1}{2} - F))$ , and thus quality choice is invariant to  $\alpha$ . A heightened responsiveness of voters to candidates' quality results entirely in a larger ideological differentiation between candidates running for office. In other words, if we think of the equilibrium with zero rents as a plausible long run political configuration, candidates will be more centrist (less polarized) the *stronger* is voters' ideological focus. Note also that the no-rents condition uniquely pins down observable behavior on the equilibrium path. If instead some candidates are allowed to collect positive rents in equilibrium, other electoral equilibria with some limited asymmetry (in centrality and payoffs) can emerge.

<sup>8</sup>Note that before we were satisfying (7) with  $\delta_j < 1/2$ , and now we are satisfying (10), which is the first part of (7), with  $\delta_j > 1/2$ . The reason is that before we were forcing  $k = 2$  to keep choosing maximal quality even when  $j$  was entering relatively far away from him.

The technique we used to construct an equilibrium with two candidates easily extends to symmetric equilibria with an arbitrary number  $K$  of candidates running for office. In fact, even in the latter more general case, “local” changes in the quality choice by one candidate only lead to changes in “local” competition. This is due to the fact that small changes in  $k$ ’s quality choice only lead to changes in the distribution of votes between  $k$  and its closest competitors, one on each side of the policy spectrum. On the other hand, dealing with more than two candidates raises some technical issues. Indeed, the identity of the relevant competitors of each candidate will not generically remain fixed: since closer candidates in the issue space are better substitutes for each other, changes in candidate  $k$ ’s quality choice will have a stronger impact on how voters rank  $k$  relative to its closest competitors than to more differentiated candidates in the policy space. As a result, changes in candidates  $k$ ’s quality choice can in principle lead to changes in the identity of its relevant competitors, and thus to non differentiability in the mapping from quality choice to vote shares. A simple way to get around this problem is to focus on a particular class of symmetric equilibria, in which all candidates running for office are located at the same distance to their closest neighbors. We call equilibria of this class *location-symmetric electoral equilibria* (LSE), and we refer the interested reader to Iaryczower and Mattozzi (2008b) for an exhaustive analysis.<sup>9</sup> Within the class of LSE, best responses are accurately represented by first order conditions. Hence, in a LSE with  $K \geq 3$  candidates running for office such that  $\theta_k^* < 1$  for all  $k = 2, \dots, K - 1$ , we have that

$$\theta_k^* = \Psi^{-1} \left( \frac{\Delta}{2\alpha} \right) \forall k = 2, \dots, K - 1 \quad \text{and} \quad \theta_1^* = \theta_K^* = \Psi^{-1} \left( \frac{\Delta}{\alpha} \right).$$

In Iaryczower and Mattozzi (2008b) we establish sufficient conditions for the existence of LSE with  $K$  parties. These conditions are entirely *supply side* requirements involving  $\bar{c}$  and the entry costs  $F$ . In particular, if  $\bar{c} \leq F$  and  $F \in \left( \frac{1}{2K}, \frac{1}{K} - \bar{c} \right)$  we can construct a LSE with  $K$  candidates running for office. Moreover, if  $\alpha$  is relatively small, all candidates will choose interior quality in equilibrium. These sufficient conditions are rather intuitive. In fact, the upper bound on  $F$  is meant to assure that running for office is profitable for each candidate or, stated differently, it captures the obvious fact that for a given level of entry

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<sup>9</sup>Formally, an electoral equilibrium is location-symmetric (LSE) if the distance between any two neighboring candidates  $k$  and  $k + 1$  for  $k = 1, \dots, K - 1$  is  $x_{k+1} - x_k = \Delta$ , and  $x_1 = 1 - x_K = \Delta_0$ .

costs there is a maximal number of candidates running for office that can be supported in a LSE. The lower bound instead, coupled with the assumption that  $\bar{c} \leq F$ , makes it possible to deter the potential entry of additional candidates.

We can use these conditions and the FOCs to investigate the effect of changes in the budget constraint on the equilibrium number of candidates running for office and their quality. First, notice that as  $F$  decreases, the upper bound defined above becomes less binding. As a consequence, it will be possible to support LSE with more candidates (of higher quality) running for office. To see why this is the case, note that a necessary condition for existence of a LSE with interior quality is  $\Delta \geq C(\Psi^{-1}(\frac{\Delta}{2\alpha})) + F$ , i.e., candidates collect non-negative rents in equilibrium. If we consider a decrease in  $F$ , the latter inequality becomes less binding and can hold for a smaller value of  $\Delta$ , which implies a higher quality and a larger number of candidates running for office. A similar logic applies in the case of downward parallel or proportional shifts of the cost function. Hence, changes in the “supply side” of the political environment induce a positive correlation between the number of candidates running for office and their equilibrium quality. We summarize this conclusion informally in the following remark:

**Remark 1** *Everything else constant, reductions in the fixed cost of running for office  $F$ , and/or downward (parallel or proportional) shifts in the cost of quality function  $C(\cdot)$ , increase both the number of candidates running for office and their quality in PR elections.*

If we focus on changes on the *demand side* of the political environment, however, the comparison is less clear. Consider changes in the responsiveness of voters to candidates’ quality ( $\alpha$ ). Increasing  $\alpha$  has the direct effect of making a given field of candidates “more aggressive” in quality competition. This has the effect of reducing the expected rents of all participants in the election. In a LSE where candidates running for office collect positive rents, the system has enough *flexibility* so that as voters become more responsive to the candidates’ quality, quality competition can become tighter without affecting the equilibrium number of candidates. As candidates “compete away” their rents, however, increased voters’ responsiveness to candidates’ quality *must* lead to changes in the level of ideological differentiation and, eventually, in the number of candidates deciding to run for office. In fact, when candidates collect no rents in equilibrium, optimal quality depends

on  $\alpha$  only indirectly, through the equilibrium level of differentiation  $\Delta$ , which is increasing in  $\alpha$ .<sup>10</sup> In this case it follows that a less ideologically focused electorate must lead to a smaller number of candidates running for office. The overall effect on quality, however, is ambiguous.

So far we focused on location-symmetric equilibria. Note that in the class of LSE, it follows immediately that the number of candidates (inversely related to the degree of ideological differentiation between candidates) is directly related to the level of quality competition; i.e. the larger the number of candidates, the closer substitutes candidates are to each other, and therefore the more intense quality competition is. This result generalizes with some caveats to configurations of candidates with limited asymmetry.<sup>11</sup> Our first objective is to find a proper way to measure the number of candidates in an asymmetric environment. Consider, for example, comparing an outcome with four minority candidates each obtaining one percent of the vote and a fifth one capturing the remaining ninety six percent, with a second outcome where three candidates each obtains a third of the votes. As this example suggests, looking at the raw number of candidates in the context of asymmetric political configurations can be misleading, since the number of *relevant* candidates can be said to be larger in the latter outcome than in the former. One measure that overcomes this problem, and it is largely used in the political science literature, is the *effective number of parties* introduced by Laakso and Taagepera (1979). The Laakso-Taagepera effective number of parties (or candidates for our purposes) is defined as  $e = 1/H$ , where in turn  $H = \sum_{k=1}^K m_k^2$  is the Herfindahl index, which is commonly employed to measure concentration of industries in industrial organization. The popularity of the effective number of candidates is due to a number of attractive properties (see Encaoua and Jacquemin (1980)). First, it is symmetric, or invariant to permutations of vote shares, between candidates. Second, it satisfies the *transfer principle*: the transfer of a part of a candidate's vote share to a candidate with a bigger vote share must not increase the

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<sup>10</sup>When equilibrium rents are equal to zero, it follows that  $\Delta$  is the unique solution to  $\Delta = C(\Psi^{-1}(\frac{\Delta}{2\alpha})) + F$ , which is increasing in  $\alpha$ .

<sup>11</sup>Note that we can easily extend our previous analysis and results to accommodate *some* limited asymmetry in location. While a full characterization of asymmetric equilibria is beyond the scope of this paper, note however that electoral equilibria in PR can never be too asymmetric within our framework, as the joint equilibrium requirements of non-negative rents and no profitable entry imply that the amount of asymmetry which is possible to support in an electoral equilibrium must be rather limited.

effective number of candidates. For a given number of candidates, this condition implies that  $e$  attains its maximum value when the candidates have equal vote shares, and its minimum value when a single candidate captures (almost) the entire electorate. Third, the value of  $e$  for symmetric candidates must increase when the number of candidates grows from  $K$  to  $K + 1$ . In particular, the effective number of candidates (weakly) decreases when we transfer vote share from one candidate to another one with a higher initial vote share. Given the definition of effective number of candidates, we can show the following result:

**Proposition 2** *Consider an electoral equilibrium with three candidates running for office such that  $\Delta_1 > \Delta_2$ . Then  $\theta_2^* \geq \theta_3^* \geq \theta_1^*$ , with the inequalities strict if quality is non-maximal in equilibrium. Consider an alternative electoral equilibrium with  $x'_2 > x_2$ . Then  $\theta_2^{**} > \theta_2^*$ ,  $\theta_3^{**} > \theta_3^*$ , and  $\theta_1^{**} < \theta_1^*$ . Furthermore, if  $\Psi$  is convex, then the new equilibrium has both a smaller effective number of candidates and a lower average quality of candidates. Similarly, consider a LSE with  $K$  parties, and an alternative electoral equilibrium with  $x'_{-k} = x_{-k}$  and  $x'_k \in (x_k, x_{k+1})$ . Then if  $\Psi$  is convex, the new equilibrium has both a smaller effective number of candidates and a lower average quality of candidates.*

**Proof of Proposition 2.** In order to prove this result, first we need to introduce some additional notation, which will prove useful to handle non-symmetric configurations of candidates. Provided that  $\theta_k \geq \underline{\theta}_k(\theta_{-k}, x)$ ,  $k$ 's vote share  $m_k(\theta_k; \theta_{-k}, x)$  can be expressed as

$$m_k(\theta_k; \theta_{-k}, x) = \frac{\Delta_k^T}{2} + \alpha \left[ \frac{v(\theta_k) - v(\theta_{r(k)})}{\Delta_k^r} + \frac{v(\theta_k) - v(\theta_{\ell(k)})}{\Delta_k^\ell} \right],$$

where  $\Delta_k^r$  and  $\Delta_k^\ell$  denote the distance between the policy represented by  $k$  and that of its neighbors,  $\theta_{r(k)}$  and  $\theta_{\ell(k)}$  denote the campaign effort of  $k$ 's neighbors, and  $\Delta_k^T \equiv \Delta_k^\ell + \Delta_k^r$ . Letting  $\delta_k \equiv \frac{\Delta_k^r}{\Delta_k^T}$ , it follows that  $k$ 's FOC is given by

$$\theta_k = \Psi^{-1} \left( \frac{\delta_k(1 - \delta_k)\Delta_k^T}{\alpha} \right),$$

for  $k \in (2, K - 1)$ . In the case of three candidates running for office with  $\Delta_1 > \Delta_2$ , FOCs deliver  $\theta_2^* = \Psi^{-1}(\frac{\delta_2(1-\delta_2)\Delta_2^T}{\alpha})$ ,  $\theta_3^* = \Psi^{-1}(\frac{\delta_2\Delta_2^T}{\alpha})$ , and  $\theta_1^* = \Psi^{-1}(\frac{(1-\delta_2)\Delta_2^T}{\alpha})$ . It follows immediately that  $\theta_2^* \geq \theta_3^* \geq \theta_1^*$  since  $\Psi(\cdot)$  is decreasing and  $\delta_2 < \frac{1}{2}$ . Next, note that since  $\delta_2(1 - \delta_2)$  is monotonically increasing in  $\delta_2$  for  $\delta_2 < \frac{1}{2}$ , then  $\delta_2' < \delta_2$  implies that  $\theta_2^{**} > \theta_2^*$ ,

$\theta_3^{**} > \theta_3^*$ , and  $\theta_1^{**} < \theta_1^*$ . Now  $\theta_m^* = \frac{1}{3} \sum_k \theta_k^*$ , and since  $\theta_2^{**} > \theta_2^*$  it is enough to show that  $\theta_3^{**} - \theta_3^* > \theta_1^* - \theta_1^{**}$ . This can be written as

$$\Psi^{-1} \left( \frac{\delta_2 \Delta_2^T}{\alpha} - \frac{\varepsilon \Delta_2^T}{\alpha} \right) - \Psi^{-1} \left( \frac{\delta_2 \Delta_2^T}{\alpha} \right) > \Psi^{-1} \left( \frac{(1 - \delta_2) \Delta_2^T}{\alpha} + \frac{\varepsilon \Delta_2^T}{\alpha} \right) - \Psi^{-1} \left( \frac{(1 - \delta_2) \Delta_2^T}{\alpha} \right),$$

which follows from convexity of  $\Psi$ . In fact, if  $\Psi$  is decreasing and convex then  $\Psi^{-1}$  is also convex. The last part of the proposition can be proved in a similar way. ■

Note that the result of Proposition 2 holds when  $\Psi$  is convex, which is not implied by the assumptions of  $v$  concave and  $C$  convex. Convexity of  $\Psi$ , however, is satisfied in the case of many commonly used parametric specifications. For example, when  $C(\theta) = A\theta^B$ , with  $A > 0$  and  $B > 1$ , and  $v$  belongs to the class of hyperbolic absolute risk aversion (HARA) utility functions, i.e.,  $v(\theta) = \frac{1-d}{d} \left( \frac{a\theta}{1-d} \right)^d$  with  $a > 0$  and  $d < 1$ . The HARA class includes as special cases the constant absolute risk aversion (with  $b = 1$  and  $d \rightarrow -\infty$ ), constant relative risk aversion (with  $b \rightarrow 0$  and  $a = 1 - d$ ), logarithmic (with  $b \rightarrow 0$  and  $d \rightarrow 0$ ), as well as power and exponential utility functions.

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