Simple Demonstrator of a Power Boost Control System

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Abstract

This paper discusses a simple device that is useful in demonstrating key issues related to reversible control systems. The demonstration device, a power assisted pulley, is described and analyzed. Evidence is presented in the form of experimental data and simulation highlighting a variety of aspects of reversible systems including the ability to boost power subject to dynamical limitations imposed by the system components (e.g. sensors and actuators). In addition effects such as the influence of power-assist gain on closed-loop system's dynamics are observed and analyzed.

Introduction

Power boost control systems are widely used in a variety of engineering applications [1]-[5]. In this paper we describe a power boost system in simple form that is amenable to easy analysis by control engineering students.

The power-assisted pulley that has been developed, is a reversible power boost system with two major features: (1) it amplifies or boosts an applied force, and (2) it maintains a 1:1 relationship between input and output displacements. The power-assisted pulley can be analogized to the power steering mechanism in a conventional automobile, in which the steering wheel torque is boosted in order to make the automobile easier to steer. The greater the torque applied to the steering wheel, the greater the torque applied to the front wheel assembly. Power steering in automobiles is reversible in that when wheel twist is resisted (eg. by a curb), torque resistance is sensed in the steering wheel.

Key properties of power boost control systems are easily demonstrated with a device that we refer to as a power pulley. It consists of a conventional mechanical pulley and a motor that serves to assist the pulley's rotation. The power pulley provides a mechanical advantage between input and output force

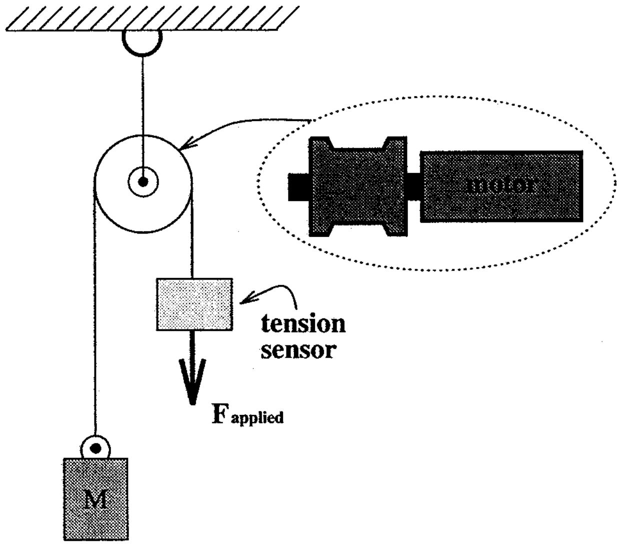


Figure 1: Power Pulley

while maintaining a 1:1 ratio between input and output position. The electromechanical components of a power pulley are shown in figure 1. The sensor in this figure monitors the cable tension. The motor serves as a bearing for the pulley and provides a means to introduce additional torque into the system. The cable does not slip on the pulley spool.

Passive pulley systems also are capable of providing a mechanical force advantage, but they do so with input displacements that vary with respect to output displacements by a factor equal to the mechanical advantage. The relationship between force advantage and displacement in the passive system stems from the fact that the work done is the product of force times displacement and that for a lossless pulley system the work in equals the work out. Two passive pulleys are shown in figure 2 and 3. In figure 3 the mechanical advantage is two. The mass in figure 3 is lifted by half the force required to lift the same mass in figure 2, but to lift this mass by 1 unit of length a force stroke of 2 units of length on the pulley's input side is required.

The Power Pulley

The power pulley system shown in figure 4 has only a few simple components- a direct drive motor, a non-slip cable, a cable tension sensor and control electronics. A weight, M, is suspended from a cable that wraps around the pulley and is attached to a cable tension sensor. This sensor is a damped linear spring coupled to a linear potentiometer. To the lowest order, the additional mass and

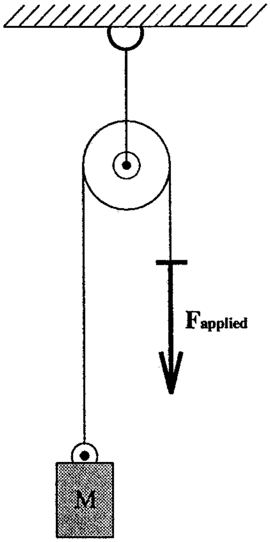


Figure 2: Mechanical Advantage=1

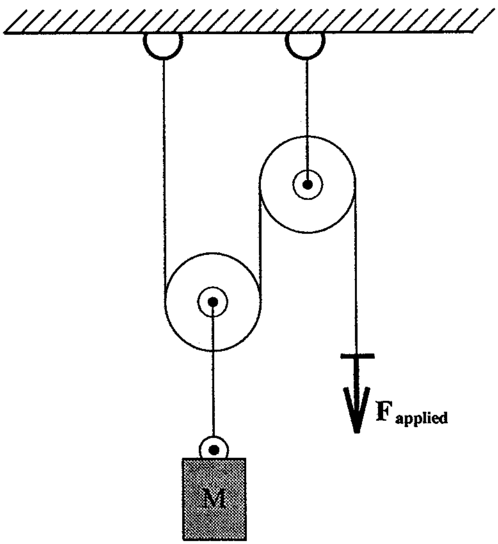


Figure 3: Mechanical Advantage=2

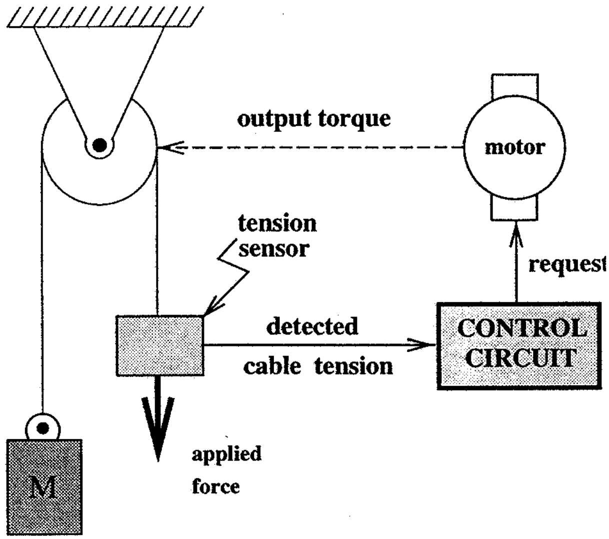


Figure 4: Power Pulley System

damping associated with the sensor assembly are negligable with respect to the other system components. In the analysis presented later however, these factors will be taken into account. In steady state the sensor measures the tension in the cable on the right side of the pulley.

When the motor is not powered, and is in steady state equiliribium the tension in the cable on the right side of the pulley equals the tension on the left side. The pulley thus functions as a simple 1:1 lifting device. When an appropriately designed electronic circuit is used to drive the motor, such that the motor torque is made proportional to the measurement of the cable tension on the right side of the pulley, a mechanical advantage is realized. This mechanical advantage scales with the constant of proportionality between cable tension and motor current. The concept of true mechanical advantage holds only when the sensor and the motor are ideal. An ideal motor is a device that adds torque but has no inertia or damping. An ideal tension sensor is a device that measures tension but is massless, displacementless, and adds no damping. Internal system dynamics of the power pulley arise due to factors such as motor armature inertia and mass spring damper properties of the tension sensor. In general these need to be considered in the full analysis of the power pulley.

Calculations:

An analysis is presented considering both the static and dynamic properties of the power-assisted pulley system as shown below:

Forces present in the system:

The forces present in the system are shown on figure 6 and are the following:
T_1 - tension of the cable on the left side of the pulley
Mg - weight of the mass being lifted
mg - weight of the mass associated with the cable tension sensor
T_2 - tension of the cable on the right side of the pulley
F_s - Force exerted by the spring, which according to Hooke's law,

$$F_s = K_1(y - r_i) \quad \text{, where} \tag{1}$$

y is the position of mass M, and r_i is the position of the tension sensor. These two quantities are shown on figure figure 5 Note that K_1 is in units of [force/distance]
F_d - Damping in the sensor assembly, which is modelled as proportional to the speed of the mass m, that is,

$$F_d = \alpha \frac{d}{dt}(y - r_i) \tag{2}$$

τ_f - Damping torque in the pulley assembly, which is modelled as proportional to the angular speed of the pulley, that is,

$$\tau_f = \beta\dot{\theta} \tag{3}$$

τ_m - Torque provided by the motor. Through the action of an electronic circuit that is described later, this torque is made to be proportional to the displacement of the spring, that is,

$$\tau_m = K_2(y - r_i) \tag{4}$$

Note that K_2 is in units of [force].

Equations of motion:

Equations describing this system are as follows:

$$\begin{aligned} M\ddot{y} &= Mg - T_1 & (5)\\ m\ddot{y} &= T_2 - mg - F_s - F_d & (6)\\ I\ddot{\theta} &= T_1 r - T_2 r - \tau_f - \tau_m & (7) \end{aligned}$$

All of these equations were derived from the free body diagrams illustrated in figure 6. I denotes moment of inertia associated with the pulley wheel, θ denotes the angular deflection of the pulley, and r is the radius of the pulley. Note that I is determined relative to the pulley axis and it includes the moment of inertia due to pulley wheel, I_p, and the motor armature, I_m, such that:

$$I_{system} = I_m + I_p \quad .$$

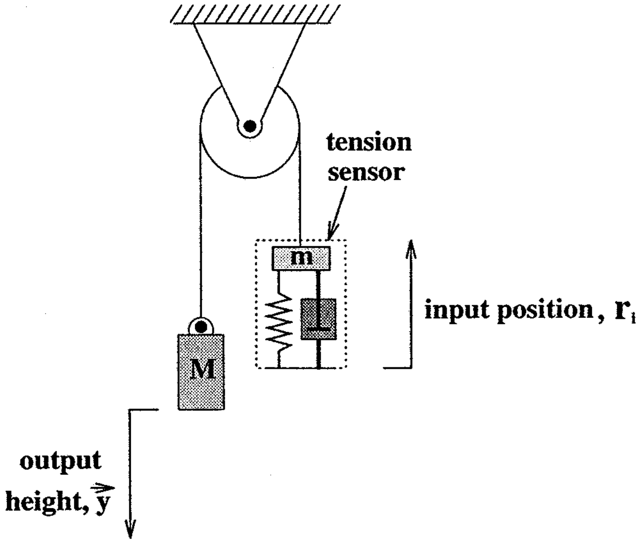


Figure 5: Power Pulley

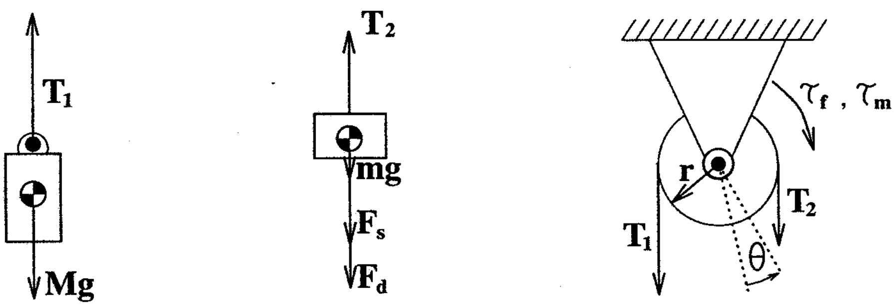


Figure 6: Forces in the system

Mechanical Advantage

The mechanical advantage is determined in equiliribium, which means $\dot{y} = 0$, $\ddot{y} = 0$ and therefore all the frictional forces drop out. In equiliribium the spring pulls with force T_2, so the mechanical advantage is:

$$G = \frac{Mg}{T_2}$$

An expression for T_2 is obtained by combining the equations of motion (5) and (7) given equation (4) which describes the torque provided by the motor: This gives,

$$T_2 = \frac{MgK_1r}{K_1r + K_2}$$

It follows thus that the mechanical advantage, G, is:

$$G = 1 + \frac{K_2}{K_1r} \tag{8}$$

If equiliribium is maintained, for example by varying r_i slowly compared to the time constants connected with the sensor and actuator, a force advantage is provided. If $G = 2$ (ie. $K_2 = K_1r$) the power pulley will function as the force equivalent of All of these equations were derived from the free body diagrams illustrated in figure 3 with the new feature that input displacement will equal output displacement. Note that this ideal condition ignores sensor and actuator dynamics, a topic which we now consider.

Dynamics of the System

To understand the dynamic properties of the system let's determine the relationship between r_i, the input position of sensor assembly and y, the output position of mass M. Combining the three equations of motion (equations 5-7) and equations (1) through (4), which describe the forces present in the system, one obtains:

$$a\ddot{y} + b\dot{y} + cy = d\dot{R}_i + cR_i \tag{9}$$

(10)

This yields the following transfer function between r_i and y, namely,

$$\frac{Y(s)}{R_i(s)} = \frac{ds + c}{as^2 + bs + c}, \tag{11}$$

where:

$$a = m + M + \frac{I}{r^2} \quad ,$$

$$b = \frac{\beta}{r^2} + \alpha \quad ,$$

$$c = \frac{K_2}{r} + K_1 \quad ,$$

$$d = \alpha \quad ,$$

and $Y(s)$=Laplace transform of $y(t)$
$R_i(s)$=Laplace transform of $r_i(t)$.

From this tranfer function it follows that the output displacement y of mass M in response to a step input r_i is

$$y = A_1 e^{x_1 t} + A_2 e^{x_2 t} \tag{12}$$

where t denotes time, and A_1 and A_2 are constants that depend on the initial conditions.
x_1 and x_2 are the roots of the denominator polynomial,

$$as^2 + bs + c \tag{13}$$

and they are equal to:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{14}$$

Depending on the sign of the discriminant of equation (11) the roots $x_{1,2}$ can be either complex or real. From equation (10) one can see immediately that this will influence the behavior of the system. We now consider the two cases in which the discriminant is non-negative and the case when it is negative.

discriminant is non-negative

This condition occurs when the friction in the system is large, that is,

$$\beta + \alpha \geq \sqrt{4ac}$$

or

$$\beta + \alpha \geq 2\sqrt{(m + Mr^2 + \frac{I}{r^2})(K_2 r + K_1)}$$

Under this condition the roots $x_{1,2}$ will be real, and therefore the system does not oscillate.

discriminant is negative

This occurs when the friction is low, that is,

$$\beta + \alpha < \sqrt{4ac}$$

or

$$\beta + \alpha < 2\sqrt{(m + Mr^2 + \frac{I}{r^2})(K_2 r + K_1)}$$

Given that,

$$e^{(a+bi)t} = e^{at}cosbt + e^{at}sinbt,$$

therefore one can rewrite eq. 11 as:

$$y = A_1 e^{\frac{-b}{2a}} cos(\frac{\sqrt{4ac - b^2}}{2a}t) + A_2 e^{\frac{-b}{2a}} sin(\frac{\sqrt{4ac - b^2}}{2a}t)$$

From this it follows that the system oscillates and the frequency of oscillation is determined by the coefficient of the sine and cosine terms, namely:

$$f = \frac{1}{2\pi}\frac{\sqrt{4ac - b^2}}{2a} \quad ,\text{or}$$

$$f = \frac{1}{2\pi}\frac{\sqrt{4a(\frac{K_2}{r} + K_1) - b^2}}{2a}$$

One notes from the above expression that the frequency of oscillation increases with K_2. Given that a variation in K_2 is equivalent to a change in G, it is expected that as the gain increases the transient oscillation frequency will also increase. Evidence of this is presented later in the Results section.

The Electronic Circuit

It was assumed earlier that the motor torque, τ_m, would be proportional to the deflection of the spring in the tension sensor assembly. To achieve this condition we consider two factors: (1) the torque of a permanent magnet DC motor is proportional to armature current and (2) the linear potentiometer provides a voltage output that is proportional to the spring deflection in the tension sensor assembly. Given these factors, a current regulator can accomplish the task provided that the reference current value derives from the linear potentiometer, that is paired with the spring in the qtension sensor, and that the circuit output produces a motor armature current that matches the specified reference value of current. Such a circuit is presented in figure 8.

A simple analysis of the current regulating circuit in figure 8 shows that:

$$I_2 = \frac{V_p}{R_1}(1 + \frac{R_2}{R_3}) \quad . \tag{15}$$

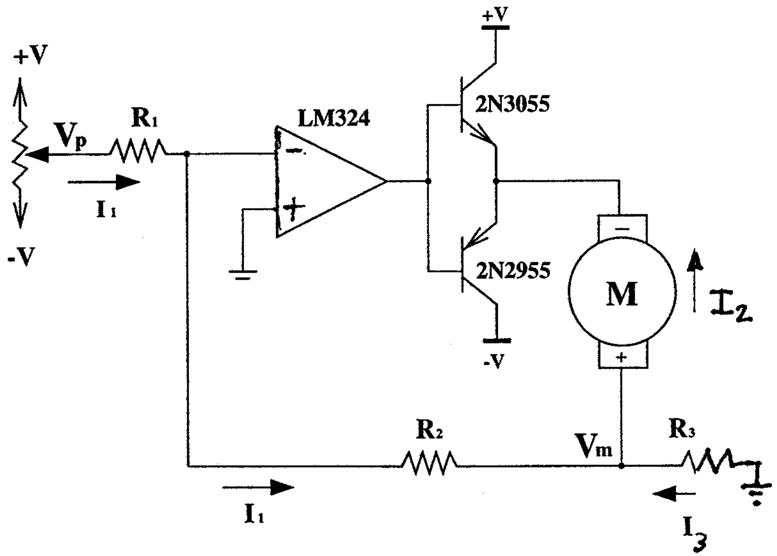


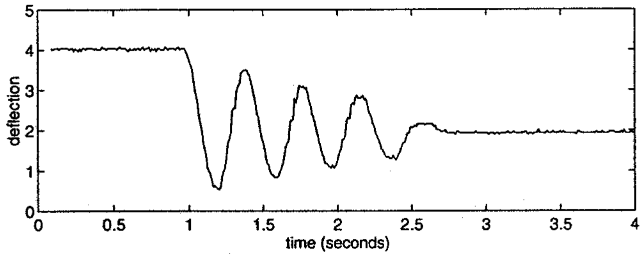
Figure 7: The Electronic Circuit

To determine the actual relationship between motor torque and spring deflection requires knowledge of additional factors such as linear travel excursion of the potentiometer, and the motor constant which relates torque to current. In our case values for these and other parameters are as follows,
V=10[V]; R_1=10[$K\Omega$]; R_2=variable between 1[$K\Omega$] and 10[$K\Omega$]; R_3=2[Ω](power); potentiometer's linear range (ie. the full stroke of the linear potentiometer) = 12[cm]; K_m(the motor constant) = 0.1 [Newton-meters/amp]; K_1(the spring constant)=10 [N/cm].

Results

The measurements in figure 8 demonstrate the change in spring deflection and the variation of frequency of oscillations as the gain is changed. Here the input position r_i is fixed, and the mass used is 4 kg. Four units of deflection are observed for both cases in figure 8 before the circuit is activated at t=1 second. In the upper case the steady state deflection changes from 4 to 1.9, which corresponds to mechanical advantage of $2.1 = \frac{4}{1.9}$. In the lower case the deflection changes from 4 to 2.8, which corresponds to a mechanical advantage of $1.4 = \frac{4}{2.8}$ The observed variation in oscillation frequency is consistent with our previous calculations.

Note that For a given gain and a given mass one would expect the spring deflectrion to remain constant as r_i is varied slowly. So therefore the earlier mentioned 1:1 relationship between input and output for the power pulley is



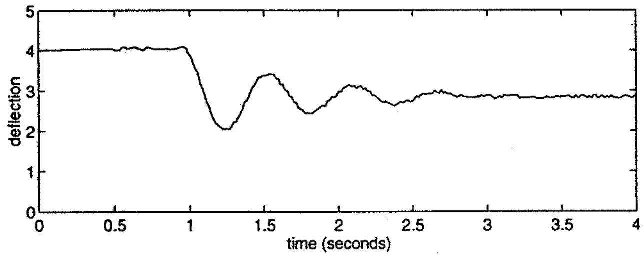


Figure 8: System responses to a unit step. Two different gains. One unit of deflection corresponds to 1 kg. load on the tension sensor.

only strictly true, in the limit when one ignores the travel in the spring-based tension sensor.

Conclusion

In summary, we have presented a device which boosts the effective power of a pulley so that one can exert a larger force on a load by an electronically controled factor. Subtle effects such that derive from sensor dynamics are observed and modelled. This device is presented as a tool for understanding reversible power boost systems.

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List of Figures