Lecture 5: Competitive Equilibrium in the Growth Model

ECO 503: Macroeconomic Theory I

Benjamin Moll

Princeton University

Fall 2014

Competitive Eqm in the Growth Model

- Recall two issues we are interested in regarding resource allocation problems
  1. efficient allocations
  2. decentralized equilibrium allocations
- So far did (1). Now consider (2).
- Focus on particular decentralized equilibrium concept: **competitive equilibrium**
  - see Lecture 1
  - benchmark notion of decentralized eqm, but not only one

Competitive Eqm in the Growth Model

- Static economies: one way to formulate CE
- Dynamic economies: **three** ways to formulate CE
  1. "Arrow-Debreu CE" (ADCE)
  2. "Sequence of Markets CE" (SOMCE)
  3. "Recursive CE" (RCE)
- Outcomes same for all three. Just different representations.
- Begin with ADCE
  - extension of static CE
  - but defining commodities as pairs of goods $\times$ time

Preliminaries

- Detail to consider in economy with capital: who owns capital?
  - households who then rent it to firms?
  - firms who own capital who are in turn owned by households?
  - reality: see some of each
- Turns out this is of no substantive importance in this setting
  - lecture: assume capital owned by HH and rented to firms
  - homework: other extreme
- Also assume single "stand-in" firm
  - homework: show that this is harmless
- We also go back to discrete-time formulation

ADCE

- **Definition**: An ADCE for the growth model are sequences $\{c_t^h, h_t^h, k_t^h, k_t^f, h_t^f, p_t, w_t, R_t\}_{t=0}^{\infty}$ s.t.
  1. (HH max) Taking $\{p_t, w_t, R_t\}$ as given, $\{c_t^h, h_t^h, k_t^h\}$ solves

$$\max_{\{c_t^h, h_t^h, k_t^h\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.}$$

$$\sum_{t=0}^{\infty} p_t(c_t^h + k_{t+1}^h - (1-\delta)k_t^h) \leq \sum_{t=0}^{\infty} (R_t k_t + w_t h_t)$$

$$c_t^h \geq 0, \quad 0 \leq h_t^h \leq 1, \quad k_{t+1}^h \geq 0, \quad k_0^h = \bar{k}_0$$

  2. (Firm max) Taking $\{p_t, w_t, R_t\}$ as given, $\{k_t^f, h_t^f\}$ solves

$$\max_{\{k_t^f, h_t^f\}} \sum_{t=0}^{\infty} (p_t F(k_t^f, h_t^f) - w_t h_t^f - R_t k_t^f) \quad k_t^f \geq 0, \quad h_t^f \geq 0.$$

  3. (Market clearing) For each t:

$$k_t^h = k_t^f, \quad h_t^h = h_t^f, \quad c_t^h + k_{t+1}^h - (1-\delta)k_t^h = F(k_t^f, h_t^f)$$

Comments

- Single budget constraint for HH
- Prices take care of discounting implicitly
- Everything happens at $t = 0$

Simplifying

- An ADCE for the growth model are sequences $\{c_t, h_t, k_t, p_t, w_t, R_t\}_{t=0}^{\infty}$ s.t.
  1. (HH max) Taking $\{p_t, w_t, R_t\}$ as given, $\{c_t, h_t, k_t\}$ solves
  $$\max_{\{c_t, h_t, k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.}$$
  $$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1} - (1-\delta)k_t) \leq \sum_{t=0}^{\infty} (R_t k_t + w_t h_t)$$
  $$c_t \geq 0, \quad 0 \leq h_t \leq 1, \quad k_{t+1} \geq 0, \quad k_0 = \bar{k}_0$$
  2. (Firm max) Taking $\{p_t, w_t, R_t\}$ as given, $\{k_t, h_t\}$ solves
  $$\max_{\{k_t, h_t\}} \sum_{t=0}^{\infty} (p_t F(k_t, h_t) - w_t h_t - R_t k_t) \quad k_t \geq 0, \quad h_t \geq 0.$$
  3. (Market clearing) For each t:
  $$c_t + k_{t+1} - (1-\delta)k_t = F(k_t, h_t)$$
  (k and h markets clear implicitly)

Characterizing ADCE

- First Welfare Theorem applies, so could simply use fact that ADCE allocation = planner's allocation
- But will later consider environments with various distortions (taxes, monopoly power, financial frictions) in which this fails
- $\Rightarrow$ want to know how to solve for ADCE even when First Welfare Theorem fails. Consider such method now.
- General idea:
  - for each piece of ADCE: max problem $\Rightarrow$ necessary conditions
  - $+$ market clearing

Characterizing ADCE

- Necessary conditions for **consumer problem** ($h_t = 1$ wlog)

$$c_t : \quad \beta^t u'(c_t) = \lambda p_t, \qquad \lambda = \text{multiplier on b.c.} \tag{1}$$

$$k_{t+1} : \quad \lambda p_t + \lambda[-p_{t+1}(1-\delta) - R_{t+1}] = 0 \tag{2}$$

$$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1} - (1-\delta)k_t) \leq \sum_{t=0}^{\infty}(R_t k_t + w_t) \tag{3}$$

$$\text{TVC} : \quad \lim_{T\to\infty} \beta^T u'(c_T) k_{T+1} = 0 \tag{4}$$

$$\text{initial} : \quad k_0 = \bar{k}_0 \tag{5}$$

- Necessary conditions for **firm problem**

$$p_t F_k(k_t, h_t) = R_t \tag{6}$$

$$p_t F_h(k_t, h_t) = w_t \tag{7}$$

- **Market clearing**

$$c_t + k_{t+1} - (1-\delta)k_t = F(k_t, h_t) \tag{8}$$

Characterizing ADCE: TVC?

- As before, can think of TVC (4) as coming from finite horizon problem

$$\max_{\{c_t, h_t, k_t\}} \sum_{t=0}^{T} \beta^t u(c_t) \quad \text{s.t.}$$

$$\sum_{t=0}^{T} p_t(c_t + k_{t+1} - (1-\delta)k_t) \leq \sum_{t=0}^{T} (R_t k_t + w_t h_t), \quad k_{t+1} \geq 0$$

- Denote multipliers by λ, μ_t, necessary conditions at $t = T$ are

$$\begin{aligned} \beta^T u'(c_T) &= \lambda p_T \\ \lambda p_T &= \mu_T \qquad \Rightarrow \quad \beta^T u'(c_T) k_{T+1} = 0 \\ \mu_T k_{T+1} &= 0 \end{aligned}$$

Characterizing ADCE

- Use (1) at t and $t+1$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{p_t}{p_{t+1}} \tag{9}$$

- From (2)

$$\frac{p_t}{p_{t+1}} = \frac{R_{t+1}}{p_{t+1}} + 1 - \delta$$

$$\Rightarrow \frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{R_{t+1}}{p_{t+1}} + 1 - \delta$$

- From (6)

$$\frac{R_t}{p_t} = F_k(k_t, 1) = f'(k_t)$$

$$\Rightarrow \quad \frac{u'(c_t)}{\beta u'(c_{t+1})} = f'(k_{t+1}) + 1 - \delta \tag{10}$$

Characterizing ADCE

- Recall

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = f'(k_{t+1}) + 1 - \delta \tag{10}$$

- (10) + TVC (4) + initial condition (5) + market clearing (8) = same set of equations as for SP problem
- Hence: ADCE allocation is the same for the SP problem
- How get prices?
  - can always normalize one price to unity: wlog set $p_0 = 1$
  - get R_0, w_0 from (6) and (7) at $t = 0$
  - get p_1 from (9) given c_0, c_1, p_0
  - get R_1, w_1 from (6) and (7) at $t = 1$ given p_1
  - ...

Aside: Walras' Law

- Q: Why didn't we use budget constraint (3)?
- A: because it is implied by the other equations, in particular firm's problem + market clearing (8) $\Rightarrow$ (3)
- Firm's problem and market clearing are

$$\sum_{t=0}^{\infty}(p_t F(k_t, h_t) - w_t h_t - R_t k_t) = 0 \tag{11}$$

$$c_t + k_{t+1} - (1-\delta)k_t = F(k_t, h_t) \tag{12}$$

Substituting (12) into (11) gives (3):

$$\sum_{t=0}^{\infty}[p_t(c_t + k_{t+1} - (1-\delta)k_t) - w_t h_t - R_t k_t] = 0$$

- This is "Walras' Law" `http://en.wikipedia.org/wiki/Walras'_law`
  - very general: all budget constraints $\Rightarrow$ resource constraint
  - useful check when writing models: if Walras' Law doesn't hold, you did something wrong (e.g. forgot term in mkt clearing)

Steady State ADCE

- **Definition**: A St.st. ADCE is a value of k^* and an ADCE for the economy with $\bar{k}_0 = k^*$ s.t. $k_t = k^*$ (and $c_t = c^*$) for all t.

- Clearly from (9)
$$\frac{1}{\beta} = f'(k^*) + 1 - \delta$$
- $\Rightarrow k^*$ same as in SP problem
- Question: what do you think prices look like in a st.st. ADCE?
  - $p_t =$ constant?
  - $R_t =$ constant?
  - $w_t =$ constant?

Steady State ADCE

- Let's work it out
- Have

$$\frac{u'(c^*)}{\beta u'(c^*)} = \frac{p_t}{p_{t+1}} \quad \Rightarrow \quad \frac{p_{t+1}}{p_t} = \beta$$

- normalizing $p_0 = 1$

$$p_t = \beta^t$$

- Also

$$\frac{R_t}{p_t} = F_k(k^*, 1), \quad \frac{w_t}{p_t} = F_h(k^*, 1)$$

- Summary:
  - $R_t/p_t, w_t/p_t$ constant
  - R_t, w_t, p_t decreasing at rate β
- So prices are not constant in st.st. ADCE
  - prices implicitly reflect discounting of future values
  - price of future output is lower
  - return to future work is lower

Alternative Pricing Convention

- Denote factor prices relative to output price in each period
- Write budget constraint as

$$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1} - (1-\delta)k_t) \leq \sum_{t=0}^{\infty} p_t(\tilde{R}_t k_t + \tilde{w}_t h_t)$$

- This formulation is useful for thinking about real rates of return and interest rates in ADCE
    - no explicit credit market in ADCE
    - but can infer implicit real interest rate on one-period ahead borrowing and lending

Alternative Pricing Convention

- Denote real interest rate by r_t
- Definition: $1 + r_{t+1} =$ amount of consumption you can get tomorrow by giving up unit of consumption today
    - giving up one unit today saves p_t
    - with this you buy p_t / p_{t+1} tomorrow

$$\Rightarrow 1 + r_{t+1} = \frac{p_t}{p_{t+1}}$$

- In steady state, $1 + r_t = 1/\beta$
- Real rate of return on capital: from HH max. w.r.t. k_{t+1}

$$p_t = p_{t+1}\tilde{R}_{t+1} + p_{t+1}(1 - \delta)$$

    - buy 1 unit of k today, get $p_{t+1}\tilde{R}_{t+1} + p_{t+1}(1 - \delta)$ tomorrow
    - must equal cost of doing so p_t

$$1 + r_{t+1} = \tilde{R}_{t+1} + 1 - \delta \quad \Rightarrow \quad \tilde{R}_t = r_t + \delta$$

- Terminology: rental rate $\tilde{R}_t =$ "user cost of capital" $r_t + \delta$