Lecture 6: Income and Wealth Distribution

ECO 521: Advanced Macroeconomics I

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Fall 2012
(1) Some empirical evidence

(2) Benhabib, Bisin and Zhu (2012)

(3) Benhabib, Bisin and Zhu (2011)

(4) Other related literature
Empirical Evidence

- Will focus on inequality at top of income and wealth distribution
- Coined the term “the 1 percent”
- Good example of Keynes’ quote: “The ideas of economists and political philosophers, both when they are right and when they are wrong, are more powerful than is commonly understood. Indeed the world is ruled by little else. Practical men, who believe themselves to be quite exempt from any intellectual influence, are usually the slaves of some defunct economist”
**Evolution of Top Incomes**

*Figure 1. The Top Decile Income Share in the United States, 1917–2007.*

*Notes:* Income is defined as market income including realized capital gains (excludes government transfers). In 2007, top decile includes all families with annual income above $109,600.

Evolution of Top Incomes

Figure 2. Decomposing the Top Decile US Income Share into three Groups, 1913–2007

Notes: Income is defined as market income including capital gains (excludes all government transfers). Top 1 percent denotes the top percentile (families with annual income above $398,900 in 2007). Top 5–1 percent denotes the next 4 percent (families with annual income between $155,400 and $398,900 in 2007). Top 10–5 percent denotes the next 5 percent (bottom half of the top decile, families with annual income between $109,600 and $155,400 in 2007).
Figure 3. The Top 0.1 Percent Income Share and Composition, 1916–2007

Notes: The figure displays the top 0.1 percent income share and its composition. Income is defined as market income including capital gains (excludes all government transfers). Salaries include wages and salaries, bonus, exercised stock-options, and pensions. Business income includes profits from sole proprietorships, partnerships, and S-corporations. Capital income includes interest income, dividends, rents, royalties, and fiduciary income. Capital gains includes realized capital gains net of losses.
Cross-Country Evidence

- In practice, quite often don’t have data on share of top 1%
- Use Pareto interpolation. Assume income has CDF

\[ F(y) = 1 - \left( \frac{y}{k} \right)^{-\alpha} \]

- Useful property of Pareto distribution: average above threshold proportional to threshold

\[ \mathbb{E} \left[ \tilde{y} \mid \tilde{y} \geq y \right] = \frac{\int_y^\infty z f(z) dz}{1 - F(y)} = \frac{\alpha}{\alpha - 1} y \]

- Estimate and report \( \beta \equiv \alpha / (\alpha - 1) \).
- Example: \( \beta = 2 \) means average income of individuals with income above $100,000 is $200,000 and average income of individuals with income above $1 million is $2 million.
- Obviously imperfect, but useful because \( \alpha \) or \( \beta \) is exactly what our theories generate.
TABLE 6
COMPARATIVE TOP INCOME SHARES

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Cross-Country Evidence

*Figure 12. Inverted-Pareto β Coefficients: English-Speaking Countries, 1910–2005*
Cross-Country Evidence

Figure 13. Inverted-Pareto $\beta$ Coefficients, Middle Europe and Japan, 1900–2005
Cross-Country Evidence

Figure 14. Inverted-Pareto \( \beta \) Coefficients, Nordic and Southern Europe, 1900–2006
Figure 15. Inverted-Pareto $\beta$ Coefficients, Developing Countries: 1920–2005
Income Inequality and Growth

- Growth of average real incomes 1975-2006 in US vs. France:
  - US: 32.2 %
  - France: 27.1 %

- Growth of average real incomes 1975-2006 in US vs. France excluding top 1%:
Income Inequality and Growth

- Growth of average real incomes 1975-2006 in US vs. France:
  - US: 32.2 %
  - France: 27.1 %

- Growth of average real incomes 1975-2006 in US vs. France excluding top 1%:
  - US: 17.9 %
  - France: 26.4 %
Income Inequality and Growth

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  - US: 32.2 %
  - France: 27.1 %

- Growth of average real incomes 1975-2006 in US vs. France excluding top 1%:
  - US: 17.9 %
  - France: 26.4 %

- Footnote: “It is important to note that such international growth comparisons are sensitive to the exact choice of years compared, the price deflator used, the exact definition of income in each country, and hence are primarily illustrative.”
### TABLE 1

**Top Percentile Share and Average Income Growth in the United States**

<table>
<thead>
<tr>
<th>Period</th>
<th>Average income real annual growth</th>
<th>Top 1% incomes real annual growth</th>
<th>Bottom 99% incomes real annual growth</th>
<th>Fraction of total growth captured by top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976–2007</td>
<td>1.2%</td>
<td>4.4%</td>
<td>0.6%</td>
<td>58%</td>
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<td>Clinton expansion</td>
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<td>1993–2000</td>
<td>4.0%</td>
<td>10.3%</td>
<td>2.7%</td>
<td>45%</td>
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<td>Bush expansion</td>
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<tr>
<td>2002–2007</td>
<td>3.0%</td>
<td>10.1%</td>
<td>1.3%</td>
<td>65%</td>
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</table>

**Notes:** Computations based on family market income including realized capital gains (before individual taxes). Incomes are deflated using the Consumer Price Index (and using the CPI-U-RS before 1992). Column (4) reports the fraction of total real family income growth captured by the top 1 percent. For example, from 2002 to 2007, average real family incomes grew by 3.0 percent annually but 65 percent of that growth accrued to the top 1 percent while only 35 percent of that growth accrued to the bottom 99 percent of U.S. families.

**Source:** Piketty and Saez (2003), series updated to 2007 in August 2009 using final IRS tax statistics.
Benhabib, Bisin and Zhu (2012)

- About wealth rather than income inequality
- Some motivating facts
- Wolff (2006): in US, top 1% of households hold 33.4% of wealth.
U.S. Wealth Distribution

Density

Ratio of individual wealth to aggregate wealth
U.S. Wealth Distribution

- **Features:**
  - Right skewness
  - Heavy upper tail
  - Pareto distribution

- **Finding in previous literature:** models with labor income risk only (Aiyagari 1994 and most other Bewley models) cannot generate this. Intuition: precautionary savings motive tapers off for high wealth individuals.
Nice Collection of Modeling Tricks.

- Blanchard-Yaari perpetual youth: life-cycle but don’t have to keep track of age distribution.
- Portfolio choice under CRRA utility
- Double Pareto distribution
- In contrast to typical Bewley models: closed-form solution for wealth distribution (though no GE).
Model

- Continuum of agents
- Constant death rate $p$.
- When an agent dies, one child is born.
- Investment opportunity
  - riskless asset
    $$dQ(t) = Q(t)rdt$$
  - risky asset
    $$dS(t) = S(t)\alpha dt + S(t)\sigma dB(t)$$
    where $r < \alpha$. 
Bequests

• “Joy-of-giving” bequest motive

• $Z(s, t)$: bequest agent born at time $s$ leaves at time $t$ if the agent dies.

• Can purchase life insurance, $P(s, t)$ at price $\mu$.
  
  • = right to bequeath $P(s, t)/\mu$ at time of death
  
  • Purpose: eliminate “accidental bequests”

  • Negative life insurance can be interpreted as annuities

  • In a fair market $\mu = p$

• Bequests are:

\[
Z(s, t) = W(s, t) + \frac{P(s, t)}{p}
\]
Preferences

• “Joy-of-giving” bequest motive (Atkinson, 1971). Alternative?

• Utility of agent born at time $s$

$$\mathbb{E}_s \int_s^\infty e^{-(\theta+p)(v-s)} [u(C(s, v)) + p\phi(Z(s, v))] \, dv$$

• $p$: probability of death

• $\theta$: discount rate

• Functional forms: both CRRA

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad \phi(Z) = \chi \frac{Z^{1-\gamma}}{1-\gamma}$$

• $\chi$: strength of bequest motive
Agent’s Problem: Portfolio Choice

• The agent’s utility maximization problem is

$$\max_{C, \omega, P} \mathbb{E}_s \int_s^\infty e^{-(\theta+p)(v-s)} \left[ u(C(s, v)) + p\phi(Z(s, v)) \right] dv$$

subject to

$$dW(s, t) = [rW(s, t) + (\alpha - r)\omega(s, t)W(s, t) - C(s, t) - P(s, t)]dt + \sigma\omega(s, t)W(s, t)dB(s, t)$$

$$Z(s, t) = W(s, t) + \frac{P(s, t)}{p}$$

• $W(s, t) \equiv S(s, t) + Q(s, t)$: total wealth.

• $\omega(s, t) \equiv \frac{S(s, t)}{W(s, t)}$: share of wealth invested in risky asset.
Agent’s Problem: Portfolio Choice

Proposition

*The agent’s optimal policies are characterized by*

\[
C(s, t) = A^{-\frac{1}{\gamma}} W(s, t), \quad \omega(s, t) = \frac{\alpha - r}{\gamma \sigma^2},
\]

\[
Z(s, t) = \rho W(s, t)
\]

*where A and \(\rho\) are constants (see paper). Furthermore*

\[
dW(s, t) = gW(s, t)dt + \kappa W(s, t)dB(s, t).
\]

*where \(g\) and \(\kappa\) are constants.*
Idea of Proof

- HJB equation

\[(\theta + p)J(W) = \max_{C,\omega,P} \frac{C^{1-\gamma}}{1 - \gamma} + p\chi \frac{(W + P/p)^{1-\gamma}}{1 - \gamma}
\]

\[+ J'(W)[rW + (\alpha - r)\omega W - C - P] + \frac{1}{2}J''(W)\sigma^2\omega^2W^2\]

(aside: please **never** write recursive problems with t’s in them)

- Guess

\[J(W) = \frac{A}{1 - \gamma}W^{1-\gamma}\]

and verify.
Aggregation

- This is the great beauty of the Blanchard-Yaari model.

- Size of cohort born at $s$: $pe^{p(s-t)}$.

- Everyone’s wealth grows at rate $g$ on average.

- Mean wealth of cohort $s$ at time $t$

$$E_s W(s, t) = E_s W(s, s)e^{g(t-s)}$$

- Aggregate wealth

$$W(t) = \int_{-\infty}^{t} E_s W(s, t)pe^{p(s-t)}ds$$

- Differentiating

$$\dot{W}(t) = gW(t) - pW(t) + pE_t W(t, t)$$
Redistributive Policies

- Government subsidy:
  - If a newborn’s inheritance is lower than a threshold level, \( x^* W(t) \), proportional to aggregate wealth, the government gives the newborn a subsidy that brings her starting wealth to threshold level.
  - If the newborn’s inheritance is higher than the threshold, the newborn does not receive a government wealth subsidy.
- Government budget is balanced at all times.
Redistributive Policies

- Government subsidies financed by capital income tax, $\tau$.
  - Before-tax returns: $\tilde{r}$, $\tilde{\alpha}$
  - After-tax returns: $r = \tilde{r} - \tau$, $\alpha = \tilde{\alpha} - \tau$.

- Budget balance

$$
\tau W(t) = p \int_0^{\frac{x^*}{\rho}} W(t) (x^* W(t) - \rho W) h(W, t) dW
$$

- Pins down $x^*$ which is endogenous

- Aggregate starting wealth of new borns is then

$$
pE_t W(t, t) = (p\rho + \tau) W(t)
$$

$$
\Rightarrow \quad dW(t) = \tilde{g} W(t) dt, \quad \tilde{g} \equiv g + p\rho - \tau - p
$$
Wealth Distribution

• Growing economy. Work with

\[ X(x, t) \equiv \frac{W(s, t)}{W(t)} \]

• \( X(s, t) \) is also a Geometric Brownian Motion.

\[ dX(s, t) = (g - \tilde{g})X(s, t)dt + \kappa X(s, t)dB(s, t) \]

where we assume that \( g - \tilde{g} - \frac{1}{2} \kappa^2 \geq 0. \)

• Easy to characterize distribution of \( X(s, t) \) using Kolmogorov Forward Equation
Kolmogorov Forward Equation

- For $x < x^*$:
  \[
  \frac{\partial f(x, t)}{\partial t} = -\frac{\partial}{\partial x}((g - \tilde{g})xf(x, t)) + \frac{1}{2} \frac{\partial^2}{\partial x^2}(\kappa^2 x^2 f(x, t)) - pf(x, t)
  \]

- For $x > x^*$:
  \[
  \frac{\partial f(x, t)}{\partial t} = -\frac{\partial}{\partial x}((g - \tilde{g})xf(x, t)) + \frac{1}{2} \frac{\partial^2}{\partial x^2}(\kappa^2 x^2 f(x, t)) - pf(x, t) + pf \left( \frac{x}{\rho}, t \right) \frac{1}{\rho}
  \]
Kolmogorov Forward Equation

- Question: where does the term $pf\left(\frac{x}{\rho}, t\right) \frac{1}{\rho}$ in the KFE for $x > x^*$ come from?

- Answer: probability that relative wealth is below $x$ at time $t$ is

$$\Pr(\tilde{x}_t \leq x) = (1 - p\Delta t) \cdot [...] + p\Delta t \cdot \Pr(\rho \tilde{x}_{t-\Delta t} \leq x)$$

$$F(x, t) = (1 - p\Delta t) \cdot [...] + p\Delta t \cdot F(x/\rho, t - \Delta t)$$

$$\Rightarrow f(x, t) = (1 - p\Delta t) \cdot [...] + p\Delta t \cdot f\left(\frac{x}{\rho}, t - \Delta t\right) \frac{1}{\rho}$$
Boundary Conditions

- Density $f(x^*, t)$ determined by

  $$\int_0^\infty f(x, t)dx = 1, \quad \int_0^\infty xf(x, t)dx = 1, \quad \text{all } t.$$

- Point of reinjection $x^*$ determined by budget balance

  $$\tau = p \int_0^\infty \frac{x^*}{\rho} (x^* - \rho x) f(x, t)dx, \quad \text{all } t.$$
Proposition

The stationary distribution \( f(x) \) has the form

\[
 f(x) = \begin{cases} 
 C_1 x^{-\beta_1} & \text{when } x < x^* \\
 C_2 x^{-\beta_2} & \text{when } x > x^* 
\end{cases}
\]

where \( \beta_1 < 1 \) is the smaller root of the characteristic equation

\[
\frac{\kappa^2}{2} \beta^2 - \left( \frac{3}{2} \kappa^2 - (g - \tilde{g}) \right) \beta + \kappa^2 - p - (g - \tilde{g}) = 0
\]

and \( \beta_2 > 2 \) is the larger root of the characteristic equation

\[
\frac{\kappa^2}{2} \beta^2 - \left( \frac{3}{2} \kappa^2 - (g - \tilde{g}) \right) \beta + \kappa^2 - p - (g - \tilde{g}) + q \rho^{\beta-1} = 0.
\]
Stationary Distribution, $f(x)$

See Benhabib and Zhu (2008) for calibration details.
Compare to US Wealth Distribution

Jess Benhabib Shenghao Zhu (New York University)
Age, Luck, and Inheritance
January 11, 2008 3 / 29
Simplifications/Limitations

(1) Perpetual Youth

(2) Weird life insurance business

(3) No Labor Income

(4) Stationary distribution only due to government policy

(5) Returns not Persistent

(6) Everything (bequest motive, savings rate etc) homogeneous

- Benhabib, Bisin and Zhu (2011) improves on (1)-(5).
Benhabib, Bisin and Zhu, ECMA (2011)

- Also introduce capital income taxes, estate taxation.

- Samuelson (1965) describing “Pareto’s law”:

  *In all places and all times, the distribution of income remains the same. Neither institutional change nor egalitarian taxation can alter this fundamental constant of social sciences.*

- Benhabib, Bisin and Zhu (2011): this is not true. Capital income and estate taxation can significantly reduce wealth inequality.
Agent’s Problem

- Agents live for $T$ periods.

- At birth, draw idiosyncratic labor income $y(s)$, and rate of return on capital $r(s)$, fixed over lifecycle.

- Can be correlated with those of parent.

- Agent’s problem is now

$$
\max_{C(s,v)} \int_{s}^{s+T} e^{-\rho(v-s)} u(C(s,v))dv + e^{-\rho T} \phi(W(s,s+T)) \quad \text{s.t} \quad \\
\dot{W}(s,t) = r(s)W(s,t) + y(s) - C(s,t)
$$

with $W(s,s)$ given.
initial earnings over dynasties.
Change of Notation

- More convenient notation:
  - keep track of generations $n = 0, 1, 2, ...$
  - use age $\tau = t - s$

$$\max_{c_n(\tau)} \int_0^T u(c_n(\tau)) d\tau + e^{-\rho T} \phi(w_{n+1}(0)) \quad \text{s.t}$$

$$\dot{w}_n(\tau) = r_n w_n(\tau) + y_n - c_n(\tau)$$

- $(r_n)_n$ and $(y_n)_n$ are stochastic processes

- Estate taxation:

$$w_{n+1}(0) = (1 - b)w_n(T)$$
Evolution of Wealth Across Generations

• With CRRA $u$ and $\phi$, get closed form solution for savings. See Appendix A.

• Can obtain wealth at death given wealth at birth

\[ w_n(T) = \sigma_w(r_n, T)w_n(0) + \sigma_y(r_n, T)y_n \]

where $\sigma_w$ and $\sigma_y$ depend on parameters only.

• Let $w_n = w_n(0)$

• Recall $w_{n+1} = (1 - b)w_n(T)$.

• Get a **stochastic** difference equation

\[ w_{n+1} = \alpha(r_n)w_n + \beta(r_n, y_n) \]
Stationary Wealth Distribution

- Can analyze this difference equation
- With $\beta(r_n, y_n) = 0$ this would be a random growth process.
- With $\beta(r_n, y_n) > 0$ this is called a Kesten process
- **Theorem 1**: stationary distribution of initial wealth, $w_n$, has a Pareto tail

  $$\Pr(w_n > w) \sim kw^{-\mu}$$

- Intuition: labor income $\beta(r_n, y_n)$ works as lower bound.
- **Theorem 2**: Stationary distribution of wealth (of all households of all ages from 0 to $T$) also has a Pareto tail with same exponent $\mu$. 
Introduce capital income tax $\zeta$. Let post-tax return be $(1 - \zeta)r_n$.

**Proposition 3**: The tail index $\mu$ increases (i.e. inequality decreases) with the estate tax $b$ and the capital income tax $\zeta$.

Can also examine non-linear taxes $(1 - \zeta(r_n))r_n$.

**Corollary 1**: The tail index $\mu$ increases (i.e. inequality decreases) with the imposition of a nonlinear tax on capital $\zeta(r_n)$.

Capital income and estate taxes reduce wealth inequality by reducing capital income risk, not the average return.
Calibration

- Match US Lorenz curve
- Parameterize $r_n = (.08, .12, .15, .32)$ and

$$
\text{Pr}(r_{n+1}|r_n) = \begin{bmatrix}
.8 + \varepsilon_{low} & .12 - \frac{\varepsilon_{low}}{3} & .07 - \frac{\varepsilon_{low}}{3} & 0.01 - \frac{\varepsilon_{low}}{3} \\
.8 & .12 & .07 & 0.01 \\
.8 & .12 & .07 & 0.01 \\
.8 - \frac{\varepsilon_{high}}{3} & .12 - \frac{\varepsilon_{high}}{3} & .07 - \frac{\varepsilon_{high}}{3} & 0.01 + \varepsilon_{high}
\end{bmatrix}
$$

- $\varepsilon_{low}$ controls persistence of lowest rate of return.
- $\varepsilon_{high}$ controls persistence of highest rate of return.
- “frictions to social mobility”
TABLE II
PERCENTILES OF THE TOP TAIL; $\varepsilon_{low} = .01$

<table>
<thead>
<tr>
<th>Economy</th>
<th>Percentiles</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90th–95th</td>
<td>95th–99th</td>
<td>99th–100th</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>.113</td>
<td>.231</td>
<td>.347</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{high} = 0$</td>
<td>.118</td>
<td>.204</td>
<td>.261</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{high} = .01$</td>
<td>.116</td>
<td>.202</td>
<td>.275</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{high} = .02$</td>
<td>.105</td>
<td>.182</td>
<td>.341</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{high} = .05$</td>
<td>.087</td>
<td>.151</td>
<td>.457</td>
<td></td>
</tr>
</tbody>
</table>
TABLE III
TAIL INDEX, GINI, AND QUINTILES; $\varepsilon_{\text{low}} = .01$

<table>
<thead>
<tr>
<th>Economy</th>
<th>Tail Index $\mu$</th>
<th>Gini</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.49</td>
<td>.803</td>
<td>−.003</td>
<td>.013</td>
<td>.05</td>
<td>.122</td>
<td>.817</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = 0$</td>
<td>1.796</td>
<td>.646</td>
<td>.033</td>
<td>.058</td>
<td>.08</td>
<td>.123</td>
<td>.707</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = .01$</td>
<td>1.256</td>
<td>.655</td>
<td>.032</td>
<td>.056</td>
<td>.078</td>
<td>.12</td>
<td>.714</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = .02$</td>
<td>1.038</td>
<td>.685</td>
<td>.029</td>
<td>.051</td>
<td>.071</td>
<td>.11</td>
<td>.739</td>
</tr>
<tr>
<td>$\varepsilon_{\text{high}} = .05$</td>
<td>.716</td>
<td>.742</td>
<td>.024</td>
<td>.042</td>
<td>.058</td>
<td>.09</td>
<td>.786</td>
</tr>
</tbody>
</table>
### TABLE IX
**TAX EXPERIMENTS—TAIL INDEX \( \mu \)**

<table>
<thead>
<tr>
<th>( b \backslash \zeta )</th>
<th>0</th>
<th>.05</th>
<th>.15</th>
<th>.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.68</td>
<td>.76</td>
<td>.994</td>
<td>1.177</td>
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<tr>
<td>.1</td>
<td>.689</td>
<td>.772</td>
<td>1.014</td>
<td>1.205</td>
</tr>
<tr>
<td>.2</td>
<td>.7</td>
<td>.785</td>
<td>1.038</td>
<td>1.238</td>
</tr>
<tr>
<td>.25</td>
<td>.706</td>
<td>.793</td>
<td>1.051</td>
<td>1.257</td>
</tr>
</tbody>
</table>

### TABLE X
**TAX EXPERIMENTS—GINI**

<table>
<thead>
<tr>
<th>( b \backslash \zeta )</th>
<th>0</th>
<th>.05</th>
<th>.15</th>
<th>.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.779</td>
<td>.769</td>
<td>.695</td>
<td>.674</td>
</tr>
<tr>
<td>.1</td>
<td>.768</td>
<td>.730</td>
<td>.693</td>
<td>.677</td>
</tr>
<tr>
<td>.2</td>
<td>.778</td>
<td>.724</td>
<td>.679</td>
<td>.674</td>
</tr>
<tr>
<td>.3</td>
<td>.754</td>
<td>.726</td>
<td>.680</td>
<td>.677</td>
</tr>
</tbody>
</table>
• Benhabib, Bisin and Zhu (2011,2012): stochastic processes for rate of return on capital exogenously given.

• Cagetti and De Nardi (2006) endogenize this a bit more: entrepreneurship

• Also embed in general equilibrium
Cagetti and De Nardi (2006)

- Construct a quantitative model consistent with observed data.
- Evaluate model along dimensions not matched by construction.
- Study effects of borrowing constraints on aggregates and wealth inequality.
- Results:
  - Model accounts very well for wealth distributions of entrepreneurs and workers.
  - Model generates entry into entrepreneurship consistent with estimates.
  - Model generates entrepreneurial returns consistent with data.
  - More stringent borrowing constraints \(\Rightarrow \) less inequality but also less investment.
  - Voluntary bequests important for wealth concentration.
Cagetti and De Nardi (2006)

- Model features:
  - Two sectors: entrepreneurial and non-entrepreneurial. All action is in former.
  - Individuals heterogeneous in wealth, working ability and entrepreneurial ability.
  - Occupational choice
  - Entrepreneurs operate span of control (i.e. decreasing returns) technology, face borrowing constraints (limited commitment).
- Next few slides: match of wealth distribution
Model Without Entrepreneurs

Fig. 1.—Distribution of wealth, conditional on wealth being positive, for the whole population. Dash-dot line: data; solid line: model without entrepreneurs.
Fig. 2.—Distribution of wealth, conditional on wealth being positive, for the whole population. Dash-dot line: data; solid line: baseline model with entrepreneurs.
Fig. 3.—Distribution of wealth, conditional on wealth being positive, in the baseline model with entrepreneurs. Solid line: workers; dash-dot line: entrepreneurs.
Wealth Distribution of Entrepreneurs Only

Fig. 4.—Distribution of the entrepreneurs’ wealth, conditional on wealth being positive. Dash-dot line: data; solid line: baseline model.
Fig. 6.—Firm size distribution, baseline model with entrepreneurs
Other Related Literature

- **Huggett, Ventura and Yaron (2011)**, “Sources of Lifetime Inequality”
- **Katz and Murphy (1992)**, “Changes in Relative Wages, 1963-87: Supply and Demand Factors”
- **Krueger, Perri, Pistaferri and Violante (2010)**, “Cross-Sectional Facts for Macroeconomists”
  This is a RED special issue see http://www.economicdynamics.org/RED-cross-sectional-facts.htm
- **Quadrini (1999)**, “The Importance of Entrepreneurship for Wealth Concentration and Mobility”
- **Quadrini (2000)**, “Entrepreneurship, Saving and Social Mobility”