HANK: Heterogeneous Agent New Keynesian models

- Combine two workhorses of modern macroeconomics:
  - New Keynesian models  Gali, Gertler, Woodford
  - Bewley models  Aiyagari, Bewley, Huggett

- Will present Kaplan-Moll-Violante incarnation, but many others
  - see related literature at end of slides

- Framework for quantitative analysis of aggregate shocks and macroeconomic policy

- Three building blocks
  1. Uninsurable idiosyncratic income risk
  2. Nominal price rigidities
  3. Assets with different degrees of liquidity

- Today: Transmission mechanism for conventional monetary policy
How monetary policy works in RANK

- Total consumption response to a drop in real rates

\[ C \text{ response} = \underbrace{\text{direct response to } r}_{>95\%} + \underbrace{\text{indirect effects due to } Y}_{<5\%} \]

- Direct response is everything, pure intertemporal substitution
How monetary policy works in RANK

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• Direct response is everything, pure intertemporal substitution

• However, data suggest:

  1. Low sensitivity of \( C \) to \( r \)
  2. Sizable sensitivity of \( C \) to \( Y \)
  3. Micro sensitivity vastly heterogeneous, depends crucially on household balance sheets
How monetary policy works in HANK

• Once matched to micro data, HANK delivers realistic:
  • wealth distribution: small direct effect
  • MPC distribution: large indirect effect (depending on $\Delta Y$)
How monetary policy works in HANK

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\[
C \text{ response} = \underbrace{\text{direct response to } r}_{\text{RANK: } >95\%} + \underbrace{\text{indirect effects due to } Y}_{\text{RANK: } <5\%} \quad \underbrace{\text{HANK: } <1/3}_{\text{HANK: } >2/3}
\]
How monetary policy works in HANK

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  - wealth distribution: small direct effect
  - MPC distribution: large indirect effect (depending on $\Delta Y$)

\[ C \text{ response} = \text{direct response to } r + \text{indirect effects due to } Y \]

- RANK: >95%
  - HANK: <1/3
  
- RANK: <5%
  - HANK: >2/3

- Overall effect depends crucially on fiscal response, unlike in RANK where Ricardian equivalence holds
Why does this difference matter?

Suppose Central Bank wants to stimulate $C$

RANK view:

• sufficient to influence the path for real rates $\{r_t\}$
• household intertemporal substitution does the rest

HANK view:

• must rely heavily on GE feedbacks to boost hh labor income
• through fiscal policy reaction and/or an investment boom
• responsiveness of $C$ to $i$ is, to a larger extent, out of CB’s control
Monetary Policy in Benchmark NK Models
Monetary Policy in Benchmark NK Models

Goal:
- Introduce decomposition of $C$ response to $r$ change

Setup:
- Prices and wages perfectly rigid $= 1$, GDP=labor $= Y_t$
- Households: CRRA($\gamma$), income $Y_t$, interest rate $r_t$

\[ C_t(\{r_s, Y_s\}_{s \geq 0}) \]

- Monetary policy: sets time path $\{r_t\}_{t \geq 0}$, special case

\[ r_t = \rho + e^{-\eta t}(r_0 - \rho), \quad \eta > 0 \tag{*} \]

- Equilibrium: $C_t(\{r_s, Y_s\}_{s \geq 0}) = Y_t$

- Overall effect of monetary policy

\[ -\frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} \]
Monetary Policy in RANK

- Decompose $C$ response by totally differentiating $C_0(\{r_t, Y_t\}_{t \geq 0})$

$$dC_0 = \int_0^\infty \frac{\partial C_0}{\partial r_t} dr_t dt + \int_0^\infty \frac{\partial C_0}{\partial Y_t} dY_t dt$$

- Direct response to $r$
- Indirect effects due to $Y$

- Next slide: to understand, do decomposition in 2-period model

- In special case ($\ast$)

$$-\frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} \left[ \frac{\eta}{\rho + \eta} + \frac{\rho}{\rho + \eta} \right]$$

- Direct response to $r$
- Indirect effects due to $Y$

- Reasonable parameterizations ⇒ very small indirect effects, e.g.
  - $\rho = 0.5\%$ quarterly
  - $\eta = 0.5$, i.e. quarterly autocorr $e^{-\eta} = 0.61$

$$\Rightarrow \frac{\eta}{\rho + \eta} = 99\%, \quad \frac{\rho}{\rho + \eta} = 1\%$$
• Just to understand, consider even simpler two-period model
  • households solve
    \[
    \max_{C_0,C_1} U(C_0) + \beta U(C_1) \quad \text{s.t.} \quad C_0 + \frac{C_0}{1+r} = Y_0 + \frac{Y_1}{1+r}
    \]
  • market clearing \( C_0 = Y_0, C_1 = Y_1 \); long-run anchoring \( Y_1 = \bar{Y} \)
  • monetary policy: drop \( r \) from \( \beta(1+r) = 1 \) to \( \beta(1+r) < 1 \)
What if some households are hand-to-mouth?

- "Spender-saver" or Two-Agent New Keynesian (TANK) model

- Fraction $\Lambda$ are HtM "spenders": $C_{t}^{sp} = Y_t$

- Decomposition in special case (*)

\[
- \frac{d \log C_0}{d r_0} = \frac{1}{\gamma \eta} \left[ (1 - \Lambda) \frac{\eta}{\rho + \eta} + \frac{\rho}{\rho + \eta} \right].
\]

- $\Rightarrow$ indirect effects $\approx \Lambda = 20\text{-}30\%$
What if there are assets in positive supply?

• Govt issues debt $B$ to households sector

• Fall in $r_t$ implies a fall in interest payments of $(r_t - \rho)B$

• Fraction $\lambda^T$ of income gains transferred to spenders

• Initial consumption response in special case (*)

$$- \frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} + \frac{\lambda^T B}{1 - \lambda \bar{Y}}.$$  

- fiscal redistribution channel

• Interaction between non-Ricardian households and debt in positive net supply matters for overall effect of monetary policy
HANK
Building blocks

Households
- Face uninsured idiosyncratic labor income risk
- Consume and supply labor
- Hold two assets: liquid and illiquid

Firms
- Monopolistically competitive intermediate-good producers
- Quadratic price adjustment costs à la Rotemberg (1982)

Government
- Issues liquid debt, spends, taxes

Monetary Authority
- Sets nominal rate on liquid assets based on a Taylor rule
Households

\[
\max_{\{c_t, \ell_t, \}\}_{t \geq 0} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} u(c_t, \ell_t) \, dt \quad \text{s.t.}
\]
\[
\dot{b}_t = r^b(b_t)b_t + wz_t\ell_t - c_t
\]

\[
z_t = \text{some Markov process}
\]

\[
b_t \geq -b
\]

- \(c_t\): non-durable consumption
- \(b_t\): liquid assets
- \(z_t\): individual productivity
- \(\ell_t\): hours worked

...no housing – see working paper
Households

\[
\max_{\{c_t, \ell_t, d_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} u(c_t, \ell_t) \, dt \\
\text{s.t.}
\begin{align*}
\dot{b}_t &= r^b(b_t)b_t + wz_t\ell_t - d_t - \chi(d_t, a_t) - c_t \\
\dot{a}_t &= r^a a_t + d_t \\
z_t &= \text{some Markov process} \\
b_t &\geq -b, \quad a_t \geq 0
\end{align*}
\]

- \(c_t\): non-durable consumption
- \(b_t\): liquid assets
- \(z_t\): individual productivity
- \(\ell_t\): hours worked
- \(a_t\): illiquid assets
- \(d_t\): illiquid deposits (\(\geq 0\))
- \(\chi\): transaction cost function
Households

\[
\max_{\{c_t, l_t, d_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} u(c_t, l_t) \, dt \quad \text{s.t.}
\]
\[
\dot{b}_t = r^b(b_t)b_t + wz_t l_t - d_t - \chi(d_t, a_t) - c_t - \tilde{T}(wz_t l_t + \Gamma) + \Gamma
\]
\[
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\]
\[
z_t = \text{some Markov process}
\]
\[
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- \(\tilde{T}\): income tax/transfer
- \(\Gamma\): income from firm ownership
- no housing – see working paper
Households

\[ \max_{\{c_t, \ell_t, d_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-(\rho + \lambda) t} u(c_t, \ell_t) \, dt \quad \text{s.t.} \]
\[
\dot{b}_t = r^b(b_t) b_t + w z_t \ell_t - d_t - \chi(d_t, a_t) - c_t - \tilde{T}(w z_t \ell_t + \Gamma) + \Gamma \\
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Households

• Adjustment cost function

$$\chi(d, a) = \chi_0 |d| + \chi_1 \left| \frac{d}{\max\{a, a\}} \right|^{\chi_2} \max\{a, a\}$$

• Linear component implies inaction region

• Convex component implies finite deposit rates
• Adjustment cost function

\[ \chi(d, a) = \chi_0 |d| + \chi_1 \left( \frac{d}{\max\{a, a\}} \right)^{\chi_2} \max\{a, a\} \]

• Linear component implies inaction region

• Convex component implies finite deposit rates

• Recursive solution of hh problem consists of:
  1. consumption policy function \( c(a, b, z; w, r^a, r^b) \)
  2. deposit policy function \( d(a, b, z; w, r^a, r^b) \)
  3. labor supply policy function \( \ell(a, b, z; w, r^a, r^b) \)

\[ \Rightarrow \] joint distribution of households \( \mu(da, db, dz; w, r^a, r^b) \)
Firms

Representative competitive final goods producer:

\[ Y = \left( \int_{0}^{1} y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \Rightarrow y_j = \left( \frac{p_j}{P} \right)^{-\varepsilon} Y \]

Monopolistically competitive intermediate goods producers:

- Technology: \( y_j = Z k_j^\alpha n_j^{1-\alpha} \Rightarrow m = \frac{1}{Z} \left( \frac{r}{\alpha} \right)^\alpha \left( \frac{w}{1-\alpha} \right)^{1-\alpha} \)
- Set prices subject to quadratic adjustment costs:
  \[ \Theta \left( \frac{\dot{p}}{p} \right) = \frac{\theta}{2} \left( \frac{\dot{p}}{p} \right)^2 Y \]

Exact NK Phillips curve – see Lecture 2 for derivation

\[ \left( r^a - \frac{\dot{Y}}{Y} \right) \pi = \frac{\varepsilon}{\theta} (m - \bar{m}) + \dot{\pi}, \quad \bar{m} = \frac{\varepsilon-1}{\varepsilon} \]
Determination of illiquid return, distribution of profits

- Illiquid assets = part capital, part equity
  \[ a = k + qs \]
  - \( k \): capital, pays return \( r - \delta \)
  - \( s \): shares, price \( q \), pay dividends \( \omega \Pi = \omega (1 - m)Y \)
Determination of illiquid return, distribution of profits

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- \( s \): shares, price \( q \), pay dividends \( \omega \Pi = \omega (1 - m)Y \)

- Arbitrage:
  
  \[ \frac{\omega \Pi + q}{q} = r - \delta := r^a \]
Determination of illiquid return, distribution of profits

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\[ \frac{\omega \Pi + \dot{q}}{q} = r - \delta := r^a \]

• Remaining \( (1 - \omega)\Pi \)? Scaled lump-sum transfer to hh’s:

\[ \Gamma = (1 - \omega)^{\frac{Z}{Z}} \Pi \]
Determination of illiquid return, distribution of profits

- Illiquid assets = part capital, part equity
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  \[ \Gamma = (1 - \omega) \frac{Z}{\bar{Z}} \Pi \]

- Set \( \omega = \alpha \) (capital share) \( \Rightarrow \) neutralize countercyclical markups
  total illiquid flow = \( rK + \omega \Pi = \alpha mY + \omega (1 - m)Y = \alpha Y \)
  total liquid flow = \( wL + (1 - \omega) \Pi = (1 - \alpha)Y \)
Monetary authority and government

- **Taylor rule**

\[ i = r^b + \phi \pi + \epsilon, \quad \phi > 1 \]

with \( r^b := i - \pi \) (Fisher equation), \( \epsilon \) = innovation (“MIT shock”)

- **Progressive tax** on labor income:

\[ \tilde{T}(wz\ell + \Gamma) = -T + \tau \times (wz\ell + \Gamma) \]

- **Government budget constraint** (in steady state)

\[ G - r^b B^g = \int \tilde{T} \, d\mu \]

- **Transition? Ricardian equivalence fails ⇒ this matters!**
Summary of market clearing conditions

- Liquid asset market
  \[ B^h + B^g = 0 \]

- Illiquid asset market
  \[ A = K + q \]

- Labor market
  \[ N = \int z\ell(a, b, z) d\mu \]

- Goods market:
  \[ Y = C + I + G + \chi + \Theta + \text{borrowing costs} \]
Solution Method
How to “upwind” with two endogenous states

• For simplicity, ignore income risk $z \equiv 1$. HJB equation

$$\rho v(a, b) = \max_c u(c) + v_b(a, b)(w + r^b b - d - \chi(d, a) - c)$$

$$+ v_a(a, b)(d + r^a a)$$

• Again for simplicity, assume $\chi(d, a) = \left(\frac{d}{a}\right)^2 a$: FOC for $d$

$$(1 + \chi_d(d, a))v_b(a, b) = v_a(a, b) \quad \Rightarrow \quad d = \left(\frac{v_a(a, b)}{v_b(a, b)} - 1\right) a$$
How to “upwind” with two endogenous states

• For simplicity, ignore income risk $z \equiv 1$. HJB equation
  \[
  \rho v(a, b) = \max_c u(c) + v_b(a, b)(w + r^b b - d - \chi(d, a) - c) + v_a(a, b)(d + r^a a)
  \]

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  \]

• Applying standard upwind scheme
  \[
  \rho v_{i,j} = u(c_{i,j}) + \frac{v_{i+1,j} - v_{i,j}}{\Delta b}(s_{i,j}^b)^+ + \frac{v_{i,j} - v_{i-1,j}}{\Delta b}(s_{i,j}^b)^+
  \]
  \[
  + \frac{v_{i,j+1} - v_{i,j}}{\Delta a}(s_{i,j}^a)^+ + \frac{v_{i,j} - v_{i,j-1}}{\Delta a}(s_{i,j}^a)^-
  \]
  where e.g. $s_{i,j}^b = w + r^b b_i - d_{i,j} - \chi(d_{i,j}, a_j) - c_{i,j}$

• Hard: $d_{i,j}$ depends on forward/backward choice for $v_b, v_a$
How to “upwind” with two endogenous states

• Convenient trick: “splitting the drift”

\[ \rho v(a, b) = \max_c u(c) + v_b(a, b)(w + r^b b - c) \]
\[ + v_b(a, b)(-d - \chi(d, a)) \]
\[ + v_a(a, b)d \]
\[ + v_a(a, b)r^a a \]

and upwind each term separately

• Can check this satisfies Barles-Souganidis monotonicity condition

• For an application, see

http://www.princeton.edu/~moll/HACTproject/two_asset_nonconvex.pdf
http://www.princeton.edu/~moll/HACTproject/two_asset_nonconvex.m
Subroutines
http://www.princeton.edu/~moll/HACTproject/two_asset_nonconvex_cost.m
http://www.princeton.edu/~moll/HACTproject/two_asset_nonconvex_FOC.m
Parameterization
Three key aspects of parameterization

1. Measurement and partition of asset categories into: 50 shades of K
   - Liquid (cash, bank accounts + government/corporate bonds)
   - Illiquid (equity, housing)
Three key aspects of parameterization

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2. Income process with leptokurtic income changes
   - Nature of earnings risk affects household portfolio

3. Adjustment cost function and discount rate
   - Match mean liquid/illiquid wealth and fraction HtM
Three key aspects of parameterization

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Three key aspects of parameterization

1. Measurement and partition of asset categories into:
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2. Income process with leptokurtic income changes
   - Nature of earnings risk affects household portfolio

3. Adjustment cost function and discount rate
   - Match mean liquid/illiquid wealth and fraction HtM
     - Production side: standard calibration of NK models
     - Standard separable preferences: \( u(c, \ell) = \log c - \frac{1}{2} \ell^2 \)
Continuous time earnings dynamics

- Literature provides little guidance on statistical models of high frequency earnings dynamics

- **Key challenge:** inferring within-year dynamics from annual data

- **Higher order moments** of annual changes are informative

- Target key moments of one 1-year and 5-year labor earnings growth from SSA data

- Model generates a thick right tail for earnings levels
Leptokurtic earnings changes (Guvenen et al)
Two-component jump-drift process

- Flow earnings \((y = wz\ell)\) modeled as sum of two components:
  \[
  \log y_t = y_{1t} + y_{2t}
  \]

- Each component is a jump-drift with:
  - mean-reverting drift: \(-\beta y_{it} dt\)
  - jumps with arrival rate: \(\lambda_i\), drawn from \(\mathcal{N}(0, \sigma_i)\)

- Estimate using SMM aggregated to annual frequency

- Choose six parameters to match eight moments:
## Model distribution of earnings changes

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance: annual log earns</td>
<td>0.70</td>
<td>0.70</td>
<td>Frac 1yr change &lt; 10%</td>
<td>0.54</td>
<td>0.56</td>
</tr>
<tr>
<td>Variance: 1yr change</td>
<td>0.23</td>
<td>0.23</td>
<td>Frac 1yr change &lt; 20%</td>
<td>0.71</td>
<td>0.67</td>
</tr>
<tr>
<td>Variance: 5yr change</td>
<td>0.46</td>
<td>0.46</td>
<td>Frac 1yr change &lt; 50%</td>
<td>0.86</td>
<td>0.85</td>
</tr>
<tr>
<td>Kurtosis: 1yr change</td>
<td>17.8</td>
<td>16.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis: 5yr change</td>
<td>11.6</td>
<td>12.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Transitory component:
\[ \hat{\lambda}_1 = 0.08, \quad \hat{\beta}_1 = 0.76, \quad \hat{\sigma}_1 = 1.74 \]

### Persistent component:
\[ \hat{\lambda}_2 = 0.007, \quad \hat{\beta}_2 = 0.009, \quad \hat{\sigma}_2 = 1.53 \]
Model matches key feature of U.S. wealth distribution

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean illiquid assets (rel to GDP)</td>
<td>2.920</td>
<td>2.920</td>
</tr>
<tr>
<td>Mean liquid assets (rel to GDP)</td>
<td>0.260</td>
<td>0.268</td>
</tr>
<tr>
<td>Poor hand-to-mouth</td>
<td>10%</td>
<td>9%</td>
</tr>
<tr>
<td>Wealthy hand-to-mouth</td>
<td>20%</td>
<td>18%</td>
</tr>
</tbody>
</table>
Wealth distributions: Liquid wealth

- Top 10% share: SCF 2004: 86%, Model: 73%
- Top 1% share: SCF 2004: 47%, Model: 16%
- Gini coefficient: SCF 2004: 0.98, Model: 0.85
Wealth distributions: Illiquid wealth

- Top 10% share: SCF 2004: 70%, Model: 87%
- Top 1% share: SCF 2004: 33%, Model: 40%
- Gini coefficient: SCF 2004: 0.81, Model: 0.82
Model generates high and heterogeneous MPCs

Quarterly MPC out of $500 = 16%
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$ Death rate</td>
<td>1/180</td>
<td>Av. lifespan 45 years</td>
</tr>
<tr>
<td>$\gamma$ Risk aversion</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\varphi$ Frisch elasticity (GHH)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\rho$ Discount rate (pa)</td>
<td>4.8%</td>
<td>Internally calibrated</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon$ Demand elasticity</td>
<td>10</td>
<td>Profit share 10 %</td>
</tr>
<tr>
<td>$\alpha$ Capital share</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>$\delta$ Depreciation rate (p.a.)</td>
<td>7%</td>
<td></td>
</tr>
<tr>
<td>$\theta$ Price adjustment cost</td>
<td>100</td>
<td>Slope of Phillips curve, $\epsilon/\theta = 0.1$</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$ Proportional labor tax</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>$T$ Lump sum transfer (rel GDP)</td>
<td>$6,900$</td>
<td>6% of GDP</td>
</tr>
<tr>
<td>$\bar{g}$ Govt debt to annual GDP</td>
<td>0.233</td>
<td>government budget constraint</td>
</tr>
<tr>
<td><strong>Monetary Policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$ Taylor rule coefficient</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>$r^b$ Steady state real liquid return (pa)</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td><strong>Illiquid Assets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^a$ Illiquid asset return (pa)</td>
<td>5.7%</td>
<td>Equilibrium outcome</td>
</tr>
<tr>
<td><strong>Borrowing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^{borr}$ Borrowing rate (pa)</td>
<td>7.9%</td>
<td>Internally calibrated</td>
</tr>
<tr>
<td>$b$ Borrowing limit</td>
<td>$16,500$</td>
<td>$\approx 1 \times$ quarterly labor inc</td>
</tr>
<tr>
<td><strong>Adjustment Cost Function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_0$ Linear term</td>
<td>0.04383</td>
<td>Internally calibrated</td>
</tr>
<tr>
<td>$\chi_1$ Coef on convex term</td>
<td>0.95617</td>
<td>Internally calibrated</td>
</tr>
<tr>
<td>$\chi_2$ Power on convex term</td>
<td>1.40176</td>
<td>Internally calibrated</td>
</tr>
<tr>
<td>$\bar{a}$ Min a in denominator</td>
<td>$360$</td>
<td>Internally calibrated</td>
</tr>
</tbody>
</table>
Results
Transmission of monetary policy shock to $C$

Innovation $\epsilon < 0$ to the Taylor rule: \[ i = \bar{r}^b + \phi \pi + \epsilon \]

- All experiments: $\epsilon_0 = -0.0025$, i.e. $-1\%$ annualized
Transmission of monetary policy shock to $C$

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Transmission of monetary policy shock to $C$

\[ dC_0 = \int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt + \int_0^\infty \left[ \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt \]
Transmission of monetary policy shock to $C$

\[ dC_0 = \int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt + \int_0^\infty \left[ \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt \]

- Direct
- Indirect
Transmission of monetary policy shock to $C$

$$dC_0 = \int_0^\infty \frac{\partial C_0}{\partial r^b_t} dr^b_t dt + \int_0^\infty \left[ \frac{\partial C_0}{\partial r^a_t} dr^a_t + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt$$

\checkmark

Intertemporal substitution and income effects from $r^b \downarrow$

![Graph showing deviation over quarters with total response and direct effect from $r^b$.]
Transmission of monetary policy shock to $C$

\[ dC_0 = \int_0^{\infty} \frac{\partial C_0}{\partial r^b_t} dr^b_t dt + \int_0^{\infty} \left[ \frac{\partial C_0}{\partial r^a_t} dr^a_t + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt \]

\[ \checkmark \]

Portfolio reallocation effect from $r^a - r^b$ ↑

![Graph showing total response, direct, and indirect effects over quarters.](image-url)
Transmission of monetary policy shock to $C$

$$dC_0 = \int_0^\infty \frac{\partial C_0}{\partial r^b_t} dr^b_t dt + \int_0^\infty \left[ \frac{\partial C_0}{\partial r^a_t} dr^a_t + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt$$

✓

Labor demand channel from $w \uparrow$

![Graph showing deviation (%) over quarters for total response, direct and indirect channels]
Transmission of monetary policy shock to $C$

$$dC_0 = \int_0^\infty \frac{\partial C_0}{\partial r^b_t} dr^b_t dt + \int_0^\infty \left[ \frac{\partial C_0}{\partial r^a_t} dr^a_t + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt$$

Fiscal adjustment: $T \uparrow$ in response to $\downarrow$ in interest payments on $B$
Transmission of monetary policy shock to $C$

$$dC_0 = \int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt + \int_0^\infty \left[ \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt$$

- Direct: $r_t^b$
- Indirect: $r_t^a + q$
- Indirect: $w + \Gamma$
- Indirect: $T$

**Graph:**
- Deviation (%)
- Total Response
- Direct: $r_t^b$
- Indirect: $r_t^a + q$
- Indirect: $w + \Gamma$
- Indirect: $T$
Monetary transmission across liquid wealth distribution

- Total change = \( c \)-weighted sum of (direct + indirect) at each \( b \)
Why small direct effects?

- Intertemporal substitution: (+) for non-HtM
- Income effect: (-) for rich households
- Portfolio reallocation: (-) for those with low but $> 0$ liquid wealth
Role of fiscal response in determining total effect

<table>
<thead>
<tr>
<th></th>
<th>$T$ adjusts (1)</th>
<th>$G$ adjusts (2)</th>
<th>$B^g$ adjusts (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elasticity of $C_0$ to $r^b$</strong></td>
<td>-2.21</td>
<td>-2.07</td>
<td>-1.48</td>
</tr>
<tr>
<td>Share of Direct effects:</td>
<td>19%</td>
<td>22%</td>
<td>46%</td>
</tr>
</tbody>
</table>

- Fiscal response to lower interest payments on debt:
  - $T$ adjusts: stimulates AD through MPC of HtM households
  - $G$ adjusts: translates 1-1 into AD
  - $B^g$ adjusts: no initial stimulus to AD from fiscal side
Monetary transmission in RANK and HANK

\[
\Delta C = \text{direct response to } r + \text{indirect GE response}
\]

- **RANK:** 95%  
  - **RANK view:**
    - High sensitivity of \( C \) to \( r \): intertemporal substitution
    - Low sensitivity of \( C \) to \( Y \): the RA is a PIH consumer

- **HANK:** 2/3  
  - **HANK view:**
    - Low sensitivity to \( r \): income effect of wealthy offsets int. subst.
    - High sensitivity to \( Y \): sizable share of hand-to-mouth agents

⇒ **Q:** Is Fed less in control of \( C \) than we thought?

- **Work in progress:** perturbation methods ⇒ estimation, inference
1. **New Keynesian models with limited heterogeneity**

2. **Bewley models with sticky prices**

- Very useful: Werning’s “as if” result. In benchmark HANK model
  - direct and indirect effects exactly offset each other
  - overall effect same as in RA model
  - true even though incomplete markets ⇒ smaller direct effects
  - same logic as in spender-saver (TANK) model

\[
- \frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} \left[ (1 - \Lambda) \frac{\eta}{\rho + \eta} + (1 - \Lambda) \frac{\rho}{\rho + \eta} + \Lambda \right].
\]

- direct response to \( r \)
- indirect effects due to \( Y \)
Open Questions

• Loads left to do! Just see Janet Yellen’s speech:
  http://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm
  • “the various linkages between heterogeneity and aggregate demand are not yet well understood, either empirically or theoretically.”
  • “More broadly, even though the tools of monetary policy are generally not well suited to achieve distributional objectives, it is important for policymakers to understand and monitor the effects of macroeconomic developments on different groups within society.”

• Two more or less random examples of great questions:
  1. Does inequality affect level of aggregate consumption/saving? some progress in Auclert and Rognlie (2016) “Inequality and Aggregate Demand”

• Particularly useful: empirical evidence but through lens of model