

Optimization of a Simple Model Explains Some of the Coordination of Throwing

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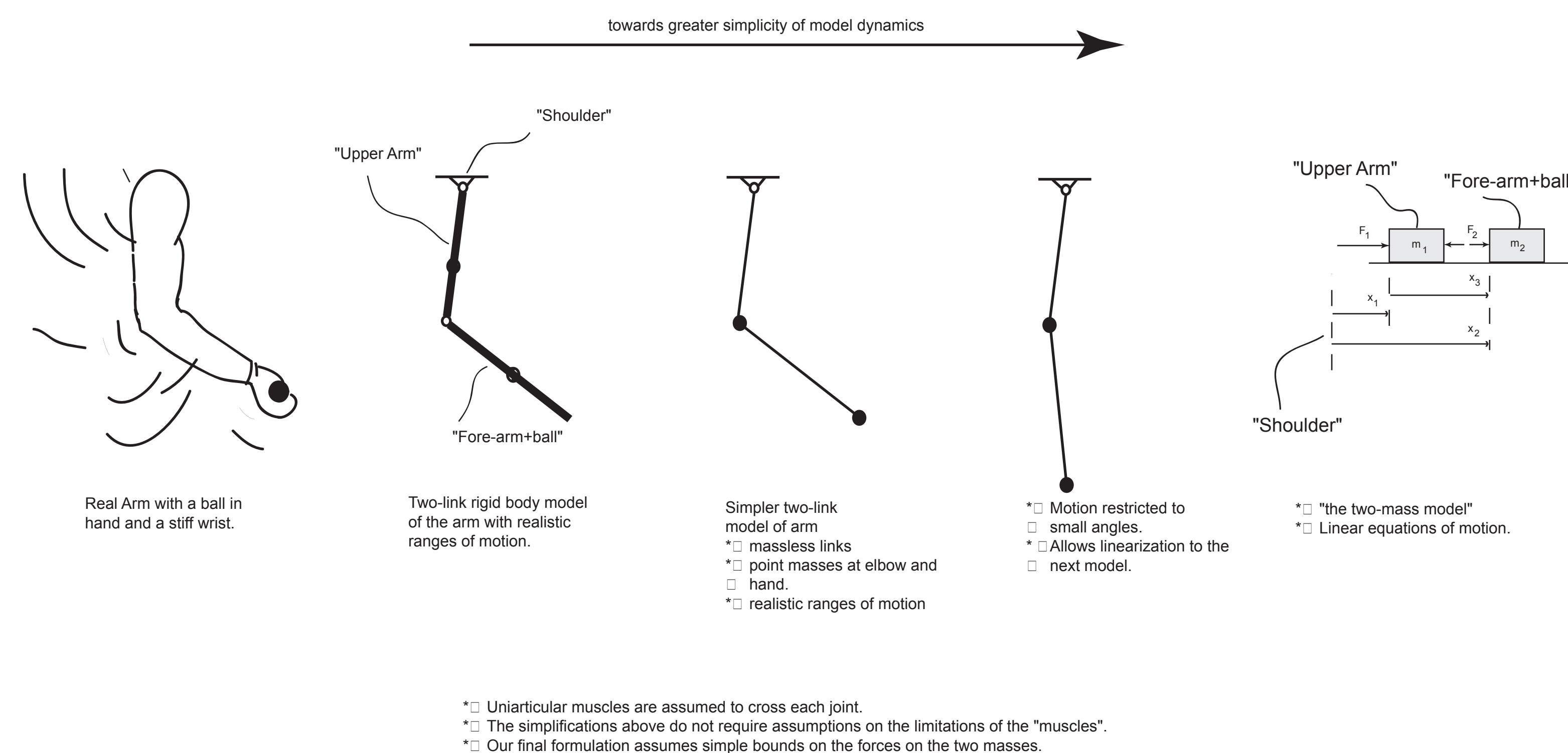
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Introduction:

Researchers [1],[2],[3],[4] have studied sequencing in throwing using models of varying complexity and realism. We complement this research by studying an extremely simple model that contains only a few principal characteristics of throwing. This allows:

- Complete solution without simplifying assumptions about the structure of the optimal solution.
- Formulation of simple rules for sequencing in this model, that might perhaps help understand throwing better.

Two-mass throwing model as a simplification of more realistic models:



Another Motivation for the Model:

Observation: Many types of throwing have similar sequencing (proximal-distal).

Hypothesis: The sequencing is more a result of the gross common features than the fine details of the dynamics. So a much simpler model with the same gross features will have similar optimal sequencing.

Steps in Modeling:

- List some of the common features of throwing, thereby crudely "defining" throwing.
- Construct a model with a simple realization of each of the features.

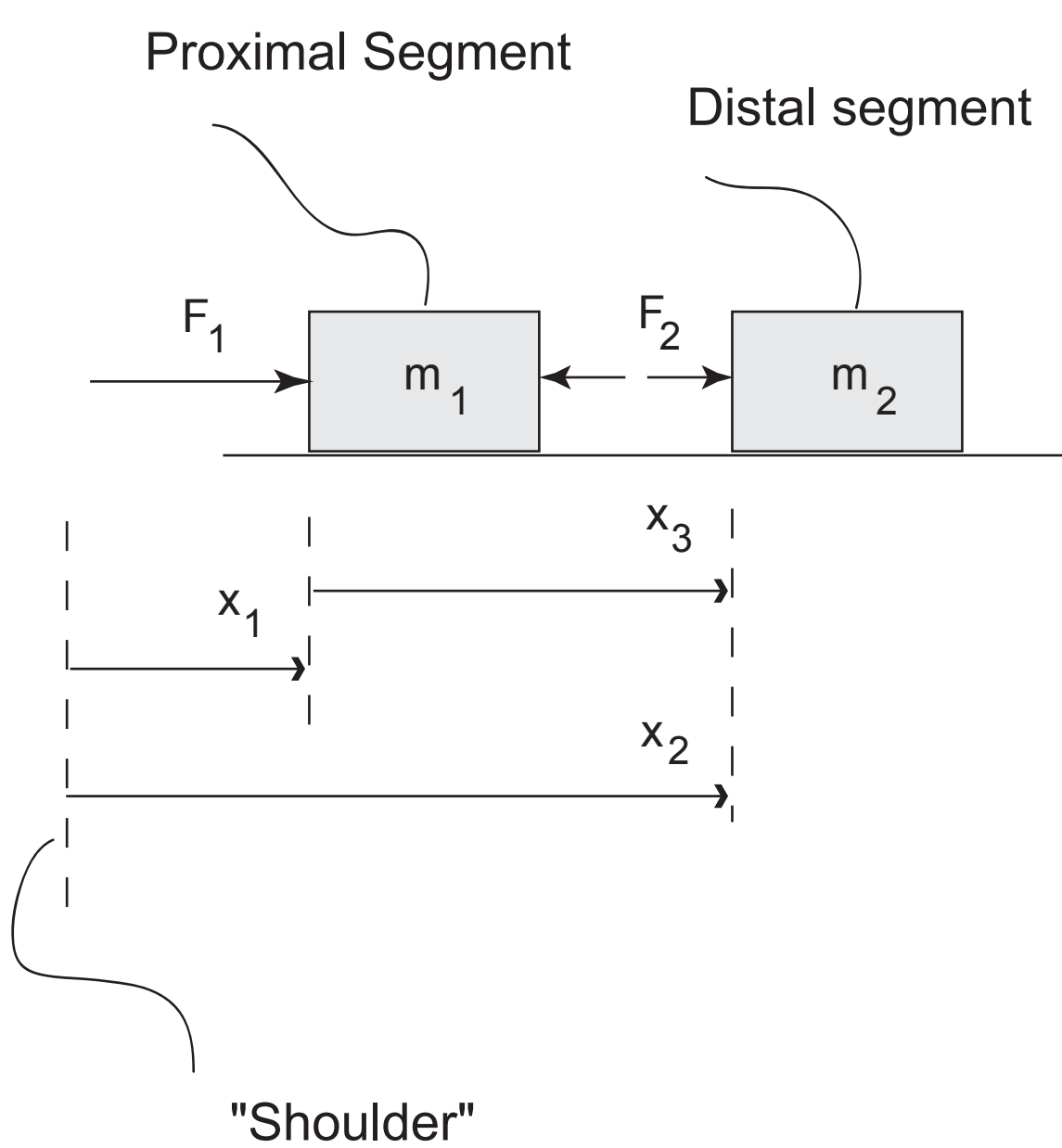
Thus, our throwing model will be a simple member of a class of throwing problems, defined as follows.

Defining a "throw" and Constructing a Simple Model:

What are some of the features common to throwing in its various manifestations? We crudely "define" a throw by these features.

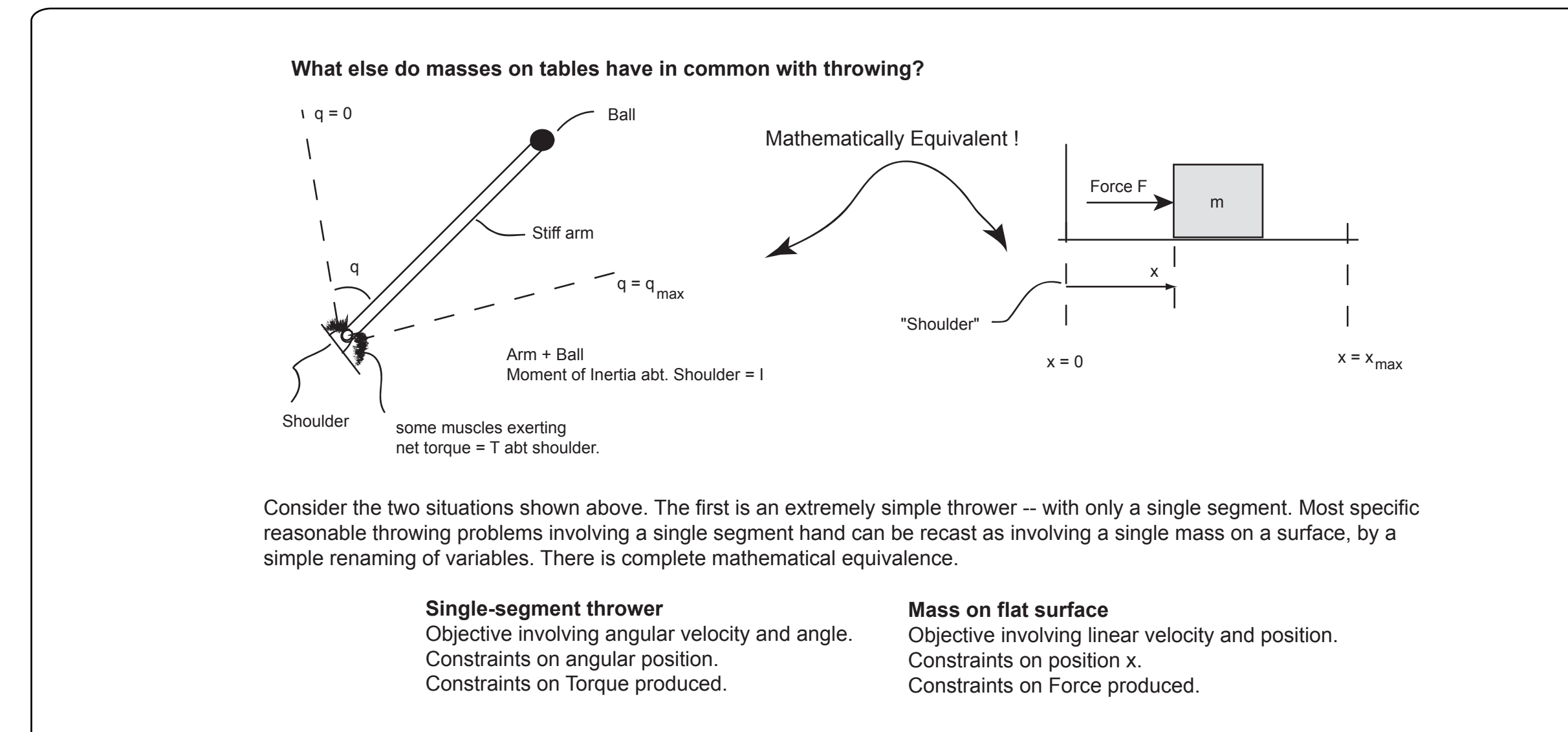
- There is usually more than one segment involved. We consider a model with two segments, one proximal, one distal -- in particular 2 masses on a surface.
- The objective is somewhat related to maximizing the velocity of the most distal segment. We will just consider maximizing the velocity of the second mass as our objective.
- There are muscles -- some force-actuators between consecutive segments and these have some characteristics. One simple constraint is a bound on the force produced.
- The range of motion is limited. There is usually constraint on the relative separation (angle, distance) between a segment and that preceding (more proximal to) it. The constraint on the most proximal segment is relative to a fixed "shoulder". No simplification.

The Simple throwing model:



OPTIMIZATION PROBLEM STATEMENT:
Objective: Maximize velocity of the second mass m_2 , subject to:

- Constraints on the forces: The forces F_1 and F_2 are bounded. These bounds are assumed to not depend on the velocity or the "muscle-length".
 $0 \leq F_1 \leq F_1^{max}$ and $0 \leq F_2 \leq F_2^{max}$
- Constraints on the range of motion: The position x_1 of the first mass relative to the shoulder and the position x_3 of the second mass relative to the first mass are bounded.
 $0 \leq x_1 \leq x_1^{max}$ and $0 \leq x_3 \leq x_3^{max}$



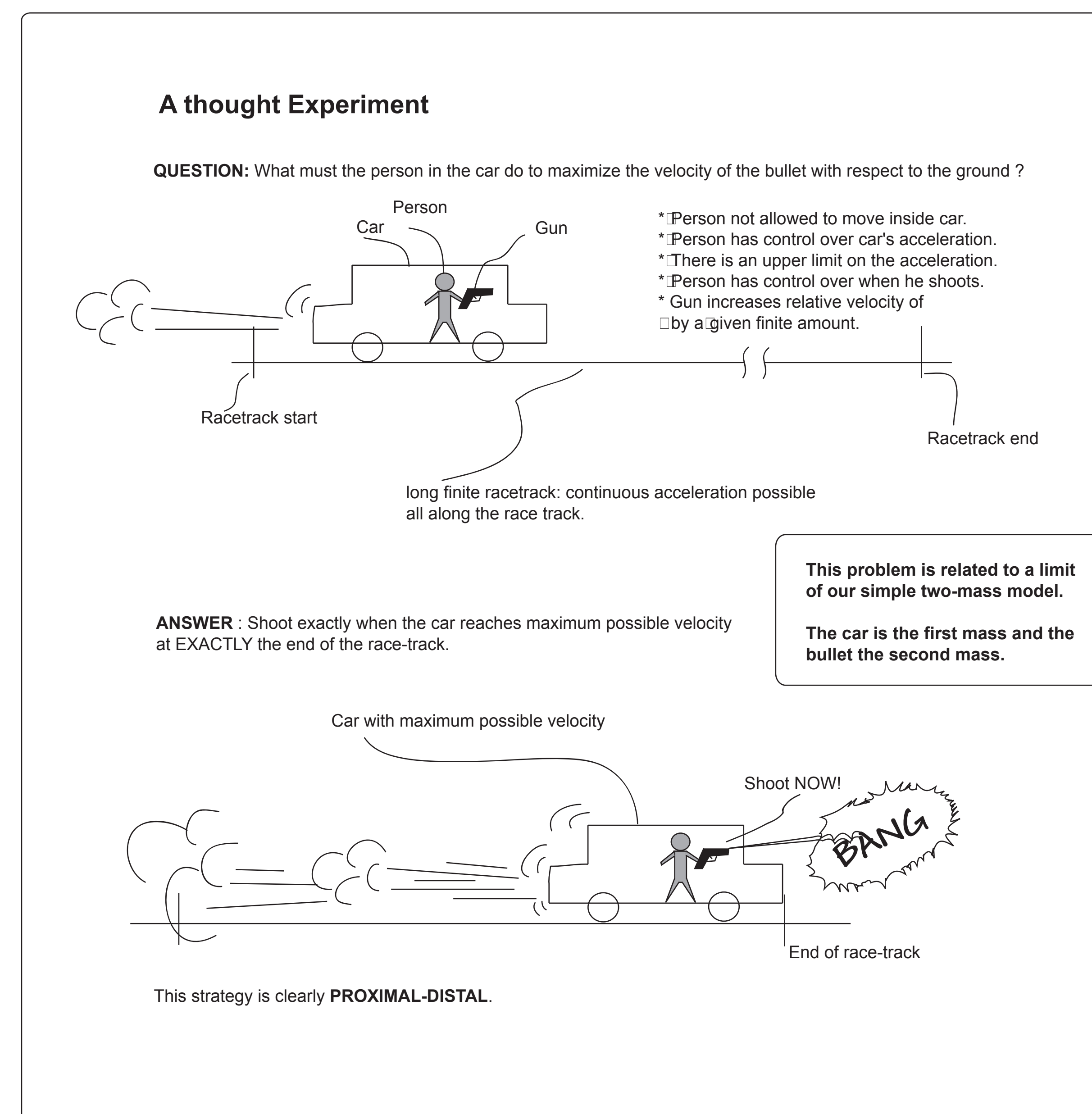
Methods:

We use numerical methods to determine the structure of the solution and use this to obtain closed-form expressions for the optimal solutions.

- Numerical Solution:**
- Discretize the problem by approximating the force time-history by a piecewise linear function.
 - Transform the continuous problem to a linear programming problem in terms of the parameters of discretization.
 - Use standard optimization software for optimization.

Relation to Previous Work:

- [1],[2],[3],[4] have all recognised the optimality of a proximal-distal sequence for at least some parameter range.
- All these works assume an underlying structure to the optimal torque-history -- usually a muscle is inactive, maximally active, or in some cases "active in the opposite direction".
- Our solution method is more open-ended but eventually provides partial justification for this assumption -- partial because the solutions are not unique.



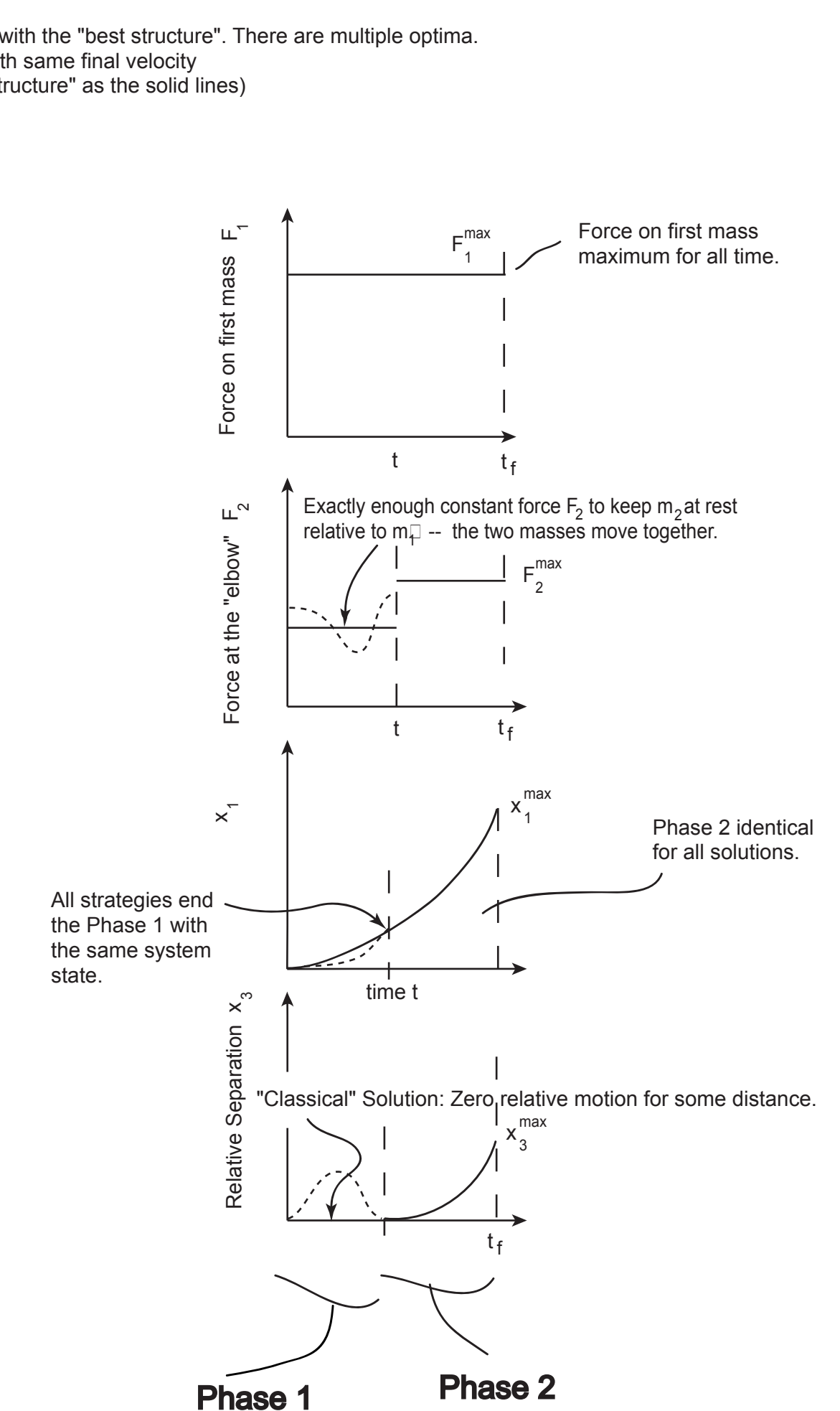
Solutions to our Simple Throwing Problem

Solution for two ranges of parameters: A and B when $F_1^{max} \leq F_2^{max}$

Proximal-Distal Optimal Solution
parameter range A: Sufficiently small x_3^{max} / x_1^{max} for all masses and force maxima given

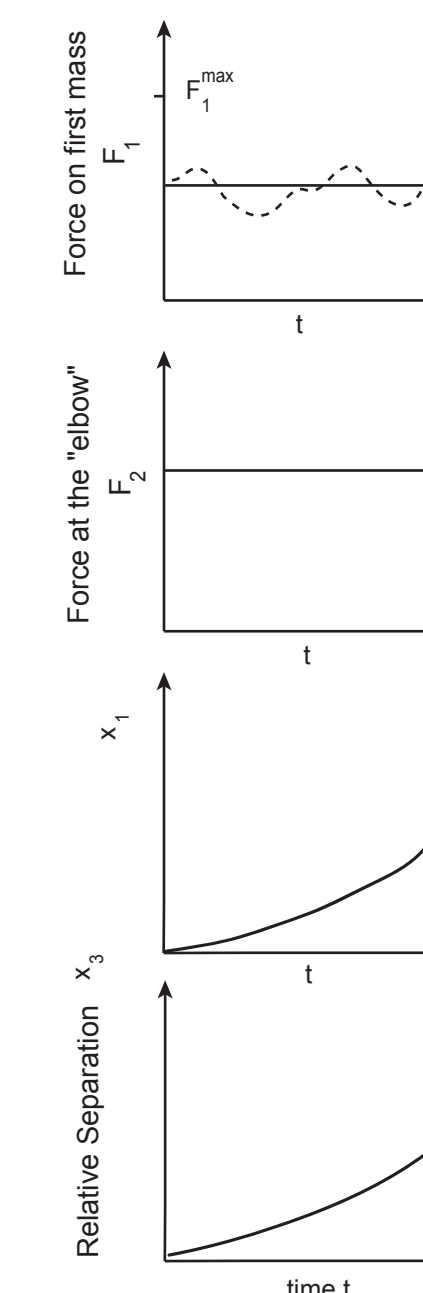
- Description of the "classical" Optimal Solution:**
- Phase 1: the two masses move together for sometime with maximum force on the first mass. The force on the second mass is just enough to maintain zero relative acceleration.
 - Phase 2: after a finite delay, the second mass pulls away from the first mass. In this phase, the force on the second mass is maximum.
 - both the position constraints are hit at the same instant and the maximum velocity is achieved at this instant.

- Other optimal solutions for the same parameters:**
- Other solutions also have two phases of the same duration as above.
 - All solutions including the one described above have identical second phases and hence have features 2 and 3.
 - In the first phase, these solutions involve relative motion of the masses subject to the constraints. The force on the first mass is still maximum. The relative motions are such that the two masses end with the same state as the first phase in (for the second phase and hence the final velocity to identical).



The strategy has the least metabolic cost by ANY reasonable measure. Thus this strategy may be selected when:

- metabolic cost is minimized over strategies that give maximum velocity.



NON-Proximal-Distal Strategy
parameter range B: Sufficiently large x_3^{max} / x_1^{max} for all masses and force maxima, $F_1^{max} \leq F_2^{max}$

- Description of the optimal solutions:**
- Only one phase in any meaningful sense.
 - The force on the second mass is maximum throughout.
 - The force on the first mass is such that both the (maximum) position constraints are hit at the same instant. The maximum velocity is achieved at this instant.

Two More Parameter Ranges: $F_1^{max} \leq F_2^{max}$

- Two more distinct parameter ranges exist.
- Both these are also proximal-distal in the same sense as range A.
- Both involve deceleration of the first mass because of larger F.

More Results:

- Optimal throwing strategies in some throwing models may involve reversing the torques near the end of a throw. Our model does not have such whip-like effects.
- Allowing for negative forces does not change the optimal strategy.
- Making the constraints on the forces have a linear velocity dependence also result in proximal-distal optima for some range of parameters (numerical result).
- Simple heuristic descriptions of the strategies have been devised that help decide which of the two strategies is chosen. These also help extend the solution naturally to problems with more than two masses in some parameter ranges.

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