

WHY WALK AND RUN: ENERGETIC COSTS AND
ENERGETIC OPTIMALITY IN SIMPLE
MECHANICS-BASED MODELS OF A BIPEDAL
ANIMAL

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