Distributed Lossy Source Coding Using BCH-DFT Codes

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Overview

1. Introduction and Background
   - Distributed Source Coding (DSC)
   - Motivation
   - Practical Code Construction

2. Lossy DSC Based on Real-Field Codes
   - The Proposed Approach
   - Rate-Adaptive DSC
   - Numerical Results

3. Other Contributions
   - Distributed Joint Source-Channel Coding
   - Generalized Error Localization

4. Conclusion
   - Summary
   - Future Research Directions
Distributed Source Coding

Problem Statement

- Distributed source coding

\[ \begin{align*}
X & \rightarrow \text{Encoder 1} \rightarrow M_X \\
Y & \rightarrow \text{Encoder 2} \rightarrow M_Y
\end{align*} \]

\[ \begin{align*}
\text{Decoder} & \rightarrow (\hat{X}, D_X) \\
& \rightarrow (\hat{Y}, D_Y)
\end{align*} \]

A communication system with

- Two separate, correlated signals \((X \text{ and } Y)\)
- The sources cannot communicate with each other; thus, encoding is done independently or in a distributed manner
- The receiver, however, can perform joint decoding
Motivation and Applications

Why DSC?
▶ Reduce the data required for storage/transmission
▶ Increase battery life (eliminate power consumption for communication)
▶ Low complexity encoders (shift the complexity to the decoder)

Applications
▶ Sensor networks
▶ Low complexity video coding
Practical Code Construction
Lossless DSC (Slepian-Wolf Coding)

- DSC is essentially a channel coding problem (view \( Y \) as corrupted version of discrete-valued \( X \))

\[ X \rightarrow \text{Encoder} \xrightarrow{\text{syndrome}} \text{Decoder} \rightarrow \hat{X} \]

- The channel code design and its rate depends on the correlation channel
- The correlation is usually modeled as a BSC
- Capacity-approaching channel codes (LDPC and Turbo codes) are asymptotically optimal
Practical Code Construction
Lossy DSC

What if the sources are continuous-valued?

Conventional Approach

- There are quantization loss and binning loss
- Correlation between real-valued signals is translated to binary domain which can bring about further loss
Q: How can we better model the virtual correlation channel?

The Proposed Framework

Motivations

- More realistic correlation channel model
- Lower delay and complexity

References:

BCH-DFT Codes as Channel Codes

Encoding

\[ x \in \mathbb{R}^k \rightarrow DFT \quad X \in \mathbb{C}^k \quad \text{zero padding} \quad C \in \mathbb{C}^n \rightarrow IDFT \quad c \in \mathbb{R}^n \]

- \( G \) consists of \( k \) columns from the IDFT matrix \( (W_n^H) \)
- The remaining \( n - k \) columns of the IDFT matrix form \( H \)
BCH-DFT Codes as Channel Codes

**Encoding**

\[
G
\]

\[
\begin{array}{c}
\text{DFT} \\
W_k
\end{array}
\xrightarrow{\text{zero padding}}
\begin{array}{c}
\Sigma_{n \times k} \\
\end{array}
\xrightarrow{\text{IDFT}}
\begin{array}{c}
\text{IDFT} \\
W^H_n
\end{array}
\]

\[x \in \mathbb{R}^k \rightarrow X \in \mathbb{C}^k \rightarrow C \in \mathbb{C}^n \rightarrow c \in \mathbb{R}^n\]

**Decoding:** let \( r = c + e \) where \( e \) has \( \nu \leq t \) nonzero elements at \( 1 \leq i_1, \ldots, i_\nu \leq n \) with magnitudes \( e_{i_1}, \ldots, e_{i_\nu} \).

- Compute the syndrome of error \( (s = Hr = He) \)
- Form the below syndrome matrix for \( m = \lceil \frac{d^2}{2} \rceil \)

\[
S_m = \begin{bmatrix}
s_1 & s_2 & \ldots & s_{d-m+1} \\
s_2 & s_3 & \ldots & s_{d-m+2} \\
\vdots & \vdots & \ddots & \vdots \\
s_m & s_{m+1} & \ldots & s_d
\end{bmatrix}
\]

- Decoding algorithms have the following major steps:
  1. Detection (determine the number of errors; \( \nu \leq \lfloor \frac{d^2}{2} \rfloor \))
  2. Localization (find the location of errors; \( i_1, \ldots, i_\nu \))
  3. Estimation (calculate the magnitude of errors; \( e_{i_1}, \ldots, e_{i_\nu} \))
Proposed Lossy DSC Based on BCH-DFT Codes

Syndrome Approach

- The decoder computes \( s_x \)
- The encoder finds \( s_e = s_y - s_x \)
  \( (\tilde{s}_e = s_y - \hat{s}_x = s_e - q) \)
- Compression ratio is \( n : n - k \)
Proposed Lossy DSC Based on BCH-DFT Codes

Syndrome Approach

- The decoder computes $s_x$
- The encoder finds $s_e = s_y - s_x$  
  \[ \tilde{s}_e = s_y - \hat{s}_x = s_e - q \]
- Compression ratio is $n : n - k$

Correlation model

\[ Y = X + E, \quad E \sim \begin{cases} \mathcal{N}(0, \sigma^2_0) & \text{w.p. } p_0, \\ \mathcal{N}(0, \sigma^2_1) & \text{w.p. } p_1, \\ 0 & \text{w.p. } 1 - p_0 - p_1, \end{cases} \]

in which $\sigma^2_1 = \sigma^2_i + \sigma^2_0$, $\sigma^2_i \gg \sigma^2_0$, and $p_0 + p_1 \leq 1$.

- $p_0 = 1$ or $p_1 = 1 \implies$ Gaussian correlation model
- $p_0 + p_1 = 1 \implies$ Gaussian-Bernoulli-Gaussian (GBG) model
- $p_0 + p_1 < 1$, $p_0p_1 = 0 \implies$ Gaussian-Erasure (GE) model
Numerical Results

- Channel-error-to-quantization-noise ratio (CEQNR)

\[
\text{CEQNR} \triangleq \frac{\sigma_i^2}{\sigma_q^2},
\]

where \( \sigma_q^2 = \frac{\Delta^2}{12} \).

- Gauss-Markov source \( X \) with \( \sigma_X = 1, \rho = 0.9 \)

- GE correlation model with \( p_1 = 0.04 \)
Rate-Adaptive DSC

The proposed schemes, with short DFT codes, are

- Suitable for low-delay coding
- Vulnerable to the variations of channel
Rate-Adaptive DSC

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▶ Vulnerable to the variations of channel

Solution: Rate-adaptive DSC with feedback
Rate-Adaptive DSC

The proposed schemes, with short DFT codes, are

- Suitable for low-delay coding
- Vulnerable to the variations of channel

![Graph showing the relationship between channel error to quantization noise and MSE for different codes and rates.](image-url)
Rate-Distortion Performance

Parameters:

- Gauss-Markov source $X$ with $\sigma_X = 1$, $\rho = 0.9$
- GBG correlation model with $p_1 = 0.04$, $\sigma_0 = 0.05\sigma_e$
- CEQNR = 25dB (or $\sigma_0 = 0.1282$ and $\sigma_e = 2.5647$ for $b = 4$)
- $R^{GBG}_{X|Y}(D) = \sum_{j=0}^1 p_j R_{X|Y,s_j}(D)$
- $\bar{R}^{GBG}_{X|Y}(D) = R_{X|Y,s_1}(D)$
Other Contributions
Distributed Joint Source-Channel Coding

- Parity-based DSC
- Distributed joint source-channel coding
- Systematic DFT frames and their properties

Other Contributions
Generalized Subspace-Based Error Localization

- Classical decoding with subspace-based approach
- Improved decoding based on extra syndromes
  - Extended Subspace: Increase the number of vectors in the noise subspace
  - Generalized Subspace: Utilize different syndrome matrices for one code

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Summary of Contributions

- A new framework for lossy DSC
  - Syndrome approach
  - Parity approach
- Distributed joint source-channel coding
- Systematic DFT frames
- Rate-adaptive DSC
- Improved decoding for BCH-DFT codes
- Generalized encoding for BCH-DFT codes
Future Research Directions

There are several avenues for future work, mainly revolving around improving the decoding algorithm for DFT codes or extending the developed algorithms to other codes, or fields.

- Improving Error localization (Rate-Distortion) Performance
- Generalized Decoding for DCT and DST Codes
- Lossy DSC Using Oversampled Filter Banks
- Parametric Frequency Estimation
- Spectral Compressive Sensing
Thank You 😊
Backup Slides
Rate Region
Lossless DSC (Slepian-Wolf coding)

Figure: Achievable rate regions for the Slepian-Wolf coding (solid lines) and separate encoding with separate decoding (dashed lines).
BCH-DFT Codes

Encoding

- Encoding scheme for an \((n, k)\) real BCH-DFT

\[ x \in \mathbb{R}^k \quad \rightarrow \quad DFT \quad \rightarrow \quad X \in \mathbb{C}^k \quad \rightarrow \quad \text{zero padding} \quad \rightarrow \quad C \in \mathbb{C}^n \quad \rightarrow \quad IDFT \quad \rightarrow \quad c \in \mathbb{R}^n \]

\[ W_k \quad \rightarrow \quad \Sigma_{n \times k} \quad \rightarrow \quad W_n^H \]

- \(G\) consists of \(k\) columns from the IDFT matrix (\(W_n^H\))
- \(\Sigma\) inserts \(d = n - k\) successive zeros in the transform domain
- \(H\) takes \(n - k\) columns of \(W_n^H\) corresponding to zeros of \(\Sigma\)
- The error correction capacity is \(t = \left\lfloor \frac{d}{2} \right\rfloor = \left\lfloor \frac{n-k}{2} \right\rfloor\)

Example: The \((6,3)\) DFT code

\[
G = \begin{pmatrix}
1 & 0 & 0 \\
\frac{2}{3} & \frac{2}{3} & \frac{-1}{3} \\
0 & 1 & 0 \\
\frac{-1}{3} & \frac{2}{3} & 2 \\
\frac{3}{3} & 0 & 0 \\
\frac{2}{3} & \frac{-1}{3} & 2 \\
\end{pmatrix}
\]
Decoding algorithms for a BCH-DFT code

1. Detection (to determine the number of errors; \( \nu \leq t = \lfloor \frac{n-k}{2} \rfloor \))
2. Localization (to find the location of errors; \( i_1, \ldots, i_\nu \))
3. Estimation (to calculate the magnitude of errors; \( e_{i_1}, \ldots, e_{i_\nu} \))

\[
s = Hr = H(c + e) = He,
\]

\[
s_m = \frac{1}{\sqrt{n}} \sum_{p=1}^{\nu} e_{i_p} X_p^{\alpha - 1 + m}, \quad m = 1, \ldots, d = n - k,
\]

and \( X_p = e^{\frac{j2\pi}{n}i_p}, \ p = 1, \ldots, \nu. \)

\[
S_t = \begin{bmatrix}
s_1 & s_2 & \cdots & s_t \\
s_2 & s_3 & \cdots & s_{t+1} \\
\vdots & \vdots & \ddots & \vdots \\
s_t & s_{t+1} & \cdots & s_{2t-1}
\end{bmatrix}
\]
Error Correction in DFT Codes
Performance with Perfect Error Localization

Figure: The MSE performance of LS estimation for a (17, 9) DFT code with *perfect error localization* for different error patterns.
Error Correction in DFT Codes

Subspace-Based Approach

1. Form the *error-locator matrix* of order $m$ as

$$V_m = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
X_1 & X_2 & \cdots & X_\nu \\
\vdots & \vdots & \ddots & \vdots \\
X_1^{m-1} & X_2^{m-1} & \cdots & X_\nu^{m-1}
\end{bmatrix}.$$ 

2. Define the syndrome matrix (for $m = \lceil \frac{d}{2} \rceil$) by

$$S_m = V_mD V_{d-m+1}^T = \begin{bmatrix}
s_1 & s_2 & \cdots & s_{d-m+1} \\
s_2 & s_3 & \cdots & s_{d-m+2} \\
\vdots & \vdots & \ddots & \vdots \\
s_m & s_{m+1} & \cdots & s_d
\end{bmatrix}.$$ 

where $D$ is a diagonal matrix of size $\nu$ with nonzero diagonal elements $d_p = \frac{1}{\sqrt{n}} e_{ip} X_p^\alpha$, $p = 1, \ldots, \nu$. 
Error Correction in DFT Codes
Subspace-Based Approach

3. Eigen-decompose the covariance matrix $R_m = S_m S_m^H$

$$R_m = [U_e \ U_n] \begin{bmatrix} \Delta_e & 0 \\ 0 & \Delta_n \end{bmatrix} [U_e \ U_n]^H,$$

- $\Delta_e$ and $\Delta_n$ contain the $\nu$ largest and $m - \nu$ smallest eigenvalues
- $U_e$ and $U_n$ contain the eigenvectors corresponding to $\Delta_e$ and $\Delta_n$
- The columns in $U_e$ span the channel-error subspace spanned by $V_m$ thus, $U_e^H U_n = 0 \Rightarrow V_m^H U_n = 0$

4. Let $v = [1, x, x^2, \ldots, x^{m-1}]^T$ where $x$ is a complex variable, then

$$F(x) \triangleq \sum_{j=1}^{m-\nu} v^H u_{n,j} = \sum_{j=1}^{m-\nu} \sum_{i=0}^{m-1} f_{ji} x^i.$$

$F(x)$ is sum of $m - \nu$ polynomials $\{f_{ji}\}_{j=1}^{m-\nu}$ of order $m - 1$. 
Error Correction in DFT Codes
Subspace-Based Approach

There are \( m - \nu = \lceil d \rceil - \nu \) polynomials rather than just one, and they have higher degrees of freedom \( \Rightarrow \) Subspace method performs better than the coding-theoretic approach.

**Figure**: Subspace method: graphical representation
Figure: Subspace method: graphical representation
Error Correction in DFT Codes
Subspace-Based Approach

There are \( m - \nu = \lceil d/2 \rceil - \nu \) polynomials rather than just one, and they have higher degrees of freedom \( \Rightarrow \) Subspace method performs better than the coding-theoretic approach.

**Figure:** Subspace method: graphical representation
Subspace vs. coding-theoretic method

There are $m - \nu = \left\lceil \frac{d}{2} \right\rceil - \nu$ polynomials rather than just one, and they have higher degrees of freedom.

$\Rightarrow$ Subspace method performs better than the coding-theoretic approach.
Extended Subspace Decoding

Motivation

Main idea:

Increasing the dimension of the estimated noise subspace ⇒ the number of polynomials with linearly independent coefficients and/or their degree grow.

Construction:

The extended syndrome matrix \( S'_m \) is defined for \( d' > d \), and similar to \( S_m \) it is decomposable as

\[
S'_m = V_m D V_{d'-m+1}^T.
\]

To form \( S'_m \) we need \( d' \) syndrome samples while we only have \( d \) samples.

\[
s'_m = \frac{1}{\sqrt{n}} \sum_{p=1}^{\nu} e_{ip} X_p^{\alpha-1+m}, \quad m = 1, \ldots, d',
\]
Extended Subspace Decoding

Extended Syndrome

\[ s'_m = \begin{cases} 
  s_m, & 1 \leq m \leq d, \\
  \bar{s}_{m-d}, & d < m \leq d', 
\end{cases} \]

where \( \bar{s} = \bar{H}e \), is the extended syndrome of error.

Recall: \( \bar{H} \) consists of those \( k \) columns of the IDFT matrix of order \( n \) not used in \( H \) (used in \( G \)).

Q:
How can we compute \( \bar{s} \)?

Let us try

\[ \bar{H}r = \bar{H}c + \bar{H}e \neq \bar{H}e \]

So to have \( \bar{H}r = \bar{s} \), either \( \bar{H}c \) must vanish or we should remove it.
Extended Subspace Decoding

Extended Syndrome

- \( \tilde{H}c = 0 \) could happen in the special case of rate \( \frac{1}{2} \) codes when all error indices are even
- In general, we need to find a way to remove \( \tilde{H}c \)

We exploit the gain from the extended subspace decoding by transmitting extra samples \( \Rightarrow \) Rate-adaptive DFT codes

1. Rate-adaptive DSC (syndrome & parity approaches)
2. Rate-adaptive channel coding
3. Rate-adaptive distributed joint source-channel coding
Generalized Error Localization

Subspace Approach

1. Eigen-decompose the covariance matrix
   \[ R_m = S_m S_m^H, \quad m = \left\lfloor \frac{d}{2} \right\rfloor = \left\lfloor \frac{n-k}{2} \right\rfloor \]

   \[ R_m = [U_e \ U_n] \begin{bmatrix} \Delta_e & 0 \\ 0 & \Delta_n \end{bmatrix} [U_e \ U_n]^H, \]

   The columns in \( U_e \) span channel-error subspace spanned by \( V_m \). Thus,
   \[ U_e^H U_n = 0 \Rightarrow V_m^H U_n = 0 \]

2. Let \( v = [1, x, x^2, \ldots, x^{m-1}]^T \) where \( x \) is a complex variable, then

   \[ F(x) \triangleq \sum_{j=1}^{m-\nu} v^H u_{n,j} = \sum_{j=1}^{m-\nu} \sum_{i=0}^{m-1} f_{ji} x^i. \]

   \[ \Rightarrow F(x) \text{ is sum of } m - \nu \text{ polynomials.} \]
Generalized Error Localization
Subspace Approach

1. Eigen-decompose the covariance matrix

$$R_m = S_m S_m^H, \ m = \left\lceil \frac{d}{2} \right\rceil = \left\lceil \frac{n-k}{2} \right\rceil$$

$$R_m = [U_e \ U_n] \begin{bmatrix} \Delta_e & 0 \\ 0 & \Delta_n \end{bmatrix} [U_e \ U_n]^H,$$

- The columns in $U_e$ span channel-error subspace spanned by $V_m$. Thus,

$$U_e^H U_n = 0 \Rightarrow V_m^H U_n = 0$$

2. Let $\mathbf{v} = [1, x, x^2, \ldots, x^{m-1}]^T$ where $x$ is a complex variable, then

$$F(x) \triangleq \sum_{j=1}^{m-\nu} \mathbf{v}^H \mathbf{u}_{n,j} = \sum_{j=1}^{m-\nu} \sum_{i=0}^{m-1} f_{ji} x^i.$$

$$\Rightarrow F(x) \text{ is sum of } m - \nu \text{ polynomials.}$$
Generalized Error Localization

Subspace Approach

1. Eigen-decompose the covariance matrix

\[ R_m = S_m S_m^H, \quad m = \left\lceil \frac{d}{2} \right\rceil = \left\lceil \frac{n-k}{2} \right\rceil \]

\[ R_m = [U_e \ U_n] \begin{bmatrix} \Delta_e & 0 \\ 0 & \Delta_n \end{bmatrix} [U_e \ U_n]^H, \]

- The columns in \( U_e \) span channel-error subspace spanned by \( V_m \). Thus,

\[ U_e^H U_n = 0 \Rightarrow V_m^H U_n = 0 \]

2. Let \( \mathbf{v} = [1, x, x^2, \ldots, x^{m-1}]^T \) where \( x \) is a complex variable, then

\[ F(x) \triangleq \sum_{j=1}^{m-\nu} \mathbf{v}^H \mathbf{u}_{n,j} = \sum_{j=1}^{m-\nu} \sum_{i=0}^{m-1} f_{ji} x^i. \]

\[ \Rightarrow F(x) \text{ is sum of } m - \nu \text{ polynomials.} \]
Generalized Subspace-Based

$$S_m^{[i]} \triangleq \begin{bmatrix} S[0]_n \\ S[i]_n \\ \vdots \\ S[i(m-1)]_n \end{bmatrix} = V_m^{[i]} D V_{d-m+1}^{[i] T}$$

Algorithm

1. Eigendecompose $S_m^{[i]} S_m^{[i] H}$ to find $U_e^{[i]}, U_q^{[i]}$

2. Since the columns in $U_e^{[i]}$ and $V_m^{[i]}$ span the same subspace, $U_e^{[i] H} U_n^{[i]} = 0 \Rightarrow V_m^{[i] H} U_n^{[i]} = 0 \ \forall \ i \in P_n$

3. Define

$$\Gamma(x) \triangleq \sum_{i \in P_n} F^{[i]}(x) = \sum_{i \in P_n} \sum_{j=1}^{m-\nu} \sum_{k=0}^{m-1} f_{jk}^{[i]} x^{ki},$$

and use it for error localization.
Example 1

Consider the \((10, 5)\) code, for which \(P_n = \{1, 3, 7, 9\}\). Then we have

\[
S_3^{[1]} = \begin{bmatrix}
  s_1 & s_2 & s_3 \\
  s_2 & s_3 & s_4 \\
  s_3 & s_4 & s_5
\end{bmatrix},
\]

\[
S_3^{[3]} = \begin{bmatrix}
  s_1 & s_4 & s_7 \\
  s_4 & s_7 & s_{10} \\
  s_7 & s_{10} & s_3
\end{bmatrix},
\]

\[
S_3^{[7]} = \begin{bmatrix}
  s_1 & s_8 & s_5 \\
  s_8 & s_5 & s_2 \\
  s_5 & s_2 & s_9
\end{bmatrix},
\]

\[
S_3^{[9]} = \begin{bmatrix}
  s_1 & s_{10} & s_9 \\
  s_{10} & s_9 & s_8 \\
  s_9 & s_8 & s_7
\end{bmatrix}.
\]
Example 2

► (11, 3) code; \( n = 11 \implies \mathcal{P}_n = \{1, \ldots, 10\} \\
► We can have 10 syndrome matrices for each \( d' \in [8, \ldots, 11] \)
► For \( d' = 11 \), these matrices share the same elements only with different arrangements, e.g.,

\[
S_6'[2] = \begin{bmatrix}
  s_1 & s_3 & s_5 & s_7 & s_9 & s_{11} \\
  s_3 & s_5 & s_7 & s_9 & s_{11} & s_2 \\
  s_5 & s_7 & s_9 & s_{11} & s_2 & s_4 \\
  s_7 & s_9 & s_{11} & s_2 & s_4 & s_6 \\
  s_9 & s_{11} & s_2 & s_4 & s_6 & s_8 \\
  s_{11} & s_2 & s_4 & s_6 & s_8 & s_{10}
\end{bmatrix},
\]

and

\[
S_6'[9] = \begin{bmatrix}
  s_1 & s_{10} & s_8 & s_6 & s_4 & s_2 \\
  s_{10} & s_8 & s_6 & s_4 & s_2 & s_{11} \\
  s_8 & s_6 & s_4 & s_2 & s_{11} & s_9 \\
  s_6 & s_4 & s_2 & s_{11} & s_9 & s_7 \\
  s_4 & s_2 & s_{11} & s_9 & s_7 & s_5 \\
  s_2 & s_{11} & s_9 & s_7 & s_5 & s_3
\end{bmatrix}.
\]
Rate-Adaptive DSC

The proposed schemes, with short DFT codes, are:

- Suitable for low-delay coding
- Vulnerable to the variations of channel

Solution: Rate-adaptive DSC with feedback

Define $\bar{H}$ such that

$$[H_T n - k \times n | \bar{H}_T k \times n] = W_H n$$

Algorithm:

1. Decoder: Request for extra syndrome samples if $\hat{\nu} \geq t$
2. Encoder: Transmit $\bar{s}_x = \bar{H}_x$ sample by sample
3. Decoder: Compute $\bar{s}_y = \bar{H}_y = \bar{s}_x + \bar{s}_e$ and $\bar{s}_e = \bar{s}_y - \bar{s}_x$
4. Decoder: Append $\bar{s}_e$ to $s_e$ and use the extended subspace decoding

Genie-aided

Channel error to quantization noise (dB)

MSE

Quantization error

(10,5) code, $r=0.5$
(10,5) code, $r=0.5037$
(12,5) code, $r=0.5833$
(12,5) code, $r=0.5842$
(15,5) code, $r=0.6667$
(15,5) code, $r=0.6667$

Rate-Adaptive DSC

The proposed schemes, with short DFT codes, are

- Suitable for low-delay coding
- Vulnerable to the variations of channel

**Solution:** Rate-adaptive DSC with feedback

- Define $\bar{H}$ such that $[H^T_{n-k \times n} | \bar{H}^T_{k \times n}] = W^H_n$

- **Algorithm:**
  1. **Decoder:** Request for extra syndrome samples if $\hat{\nu} \geq t$
  2. **Encoder:** Transmit $\bar{s}_x = \bar{H}x$ sample by sample
  3. **Decoder:** Compute $\bar{s}_y = \bar{H}y = \bar{s}_x + \bar{s}_e$ and $\bar{s}_e = \bar{s}_y - \bar{s}_x$
  4. **Decoder:** Append $\bar{s}_e$ to $s_e$ and use the extended subspace decoding
Rate-Adaptive DSC

The proposed schemes, with short DFT codes, are
- Suitable for low-delay coding
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Solution: Rate-adaptive DSC with feedback

![Graph showing MSE vs. Channel error to quantization noise (dB)]
References


