Introduction to Econometrics (4th Edition)

by

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Solutions to Odd-Numbered End-of-Chapter Exercises:
Chapter 5

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5.1 (a) The 95% confidence interval for $\beta_1$ is \{-5.82 \pm 1.96 \times 2.21\}, that is
$$-10.152 \leq \beta_1 \leq -1.4884.$$ 

(b) Calculate the $t$-statistic:

$$t^{act} = \frac{\hat{\beta}_1 - 0}{\text{SE}(\hat{\beta}_1)} = \frac{-5.82}{2.21} = -2.6335.$$ 

The $p$-value for the test $H_0: \beta_1 = 0$ vs. $H_1: \beta_1 \neq 0$ is

$$p\text{-value} = 2\Phi(-|t^{act}|) = 2\Phi(-2.6335) = 2 \times 0.0042 = 0.0084.$$ 

The $p$-value is less than 0.01, so we can reject the null hypothesis at the 5% significance level, and also at the 1% significance level.

(c) The $t$-statistic is

$$t^{act} = \frac{\hat{\beta}_1 - (-5.6)}{\text{SE}(\hat{\beta}_1)} = \frac{0.22}{2.21} = 0.10.$$ 

The $p$-value for the test $H_0: \beta_1 = -5.6$ vs. $H_1: \beta_1 \neq -5.6$ is

$$p\text{-value} = 2\Phi(-|t^{act}|) = 2\Phi(-0.10) = 0.92.$$ 

The $p$-value is larger than 0.10, so we cannot reject the null hypothesis at the 10%, 5% or 1% significance level. Because $\beta_1 = -5.6$ is not rejected at the 5% level, this value is contained in the 95% confidence interval.

(d) The 99% confidence interval for $\beta_0$ is \{520.4 \pm 2.58 \times 20.4\}, that is,
$$467.7 \leq \beta_0 \leq 573.0.$$
5.3. The 99% confidence interval is $1.5 \times (3.94 \pm 2.58 \times 0.31)$ or

$$4.71 \text{ lbs} \leq \text{WeightGain} \leq 7.11 \text{ lbs}.$$
5. 5  (a) The estimated gain from being in a small class is 13.9 points. This is equal to approximately 1/5 of the standard deviation in test scores, a moderate increase.

(b) The $t$-statistic is $t^{act} = \frac{13.9}{2.5} = 5.56$, which has a $p$-value of 0.00. Thus the null hypothesis is rejected at the 5% (and 1%) level.

(c) $13.9 \pm 2.58 \times 2.5 = 13.9 \pm 6.45$.

(d) Yes. Students were randomly assigned to small or regular classes, so that $SmallClass$ is independent of characteristics of the student, including those affecting test scores, that is $u$. Thus $E(u_i | ClassSize_i) = 0$. 
5.7. (a) The $t$-statistic is $\frac{1.2}{1.5} = 2.13$ with a $p$-value of 0.03; since the $p$-value is less than 0.05, the null hypothesis is rejected at the 5% level.

(b) $3.2 \pm 1.96 \times 1.5 = 3.2 \pm 2.94$

(c) Yes. If $Y$ and $X$ are independent, then $\beta_1 = 0$; but the $p$-value in (a) was 0.03. This means that only in 3% of all samples, the absolute value of $t$-statistic would be 2.13 (the value actually observed in this sample) or larger.

(d) $\beta_1$ would be rejected at the 5% level in 5% of the samples; 95% of the confidence intervals would contain the value $\beta_1 = 0$. 
5.9. (a) $\bar{\beta} = \frac{1}{n}(Y_1 + Y_2 + \cdots + Y_n)$ so that it is linear function of $Y_1, Y_2, \ldots, Y_n$.

(b) $E(Y_i | X_1, \ldots, X_n) = \beta_i X_i$, thus

$$E(\bar{\beta} | X_1, \ldots, X_n) = E\left(\frac{1}{X} \frac{1}{n}(Y_1 + Y_2 + \cdots + Y_n) | X_1, \ldots, X_n\right)$$

$$= \frac{1}{X} \frac{1}{n} \beta_1 (X_1 + \cdots + X_n) = \beta_1$$
5.11. Using the results from 5.10, \( \hat{\beta}_0 = \bar{Y}_m \) and \( \hat{\beta}_1 = \bar{Y}_w - \bar{Y}_m \). From Chapter 3, 

\[
SE(\bar{Y}_m) = \frac{s_w}{\sqrt{n_m}} \quad \text{and} \quad SE(\bar{Y}_w - \bar{Y}_m) = \sqrt{\frac{s_w^2}{n_m} + \frac{s_m^2}{n_m}}.
\]

Plugging in the numbers \( \hat{\beta}_0 = 523.1 \) and \( SE(\hat{\beta}_0) = 6.22; \hat{\beta}_1 = -38.0 \)

and \( SE(\hat{\beta}_1) = 7.65. \)
5.13. (a) Yes, this follows from the assumptions in KC 4.3.

(b) Yes, this follows from the assumptions in KC 4.3 and conditional homoskedasticity

(c) They would be unchanged for the reasons specified in the answers to those questions.

(d) (a) is unchanged; (b) is no longer true as the errors are not conditionally homoskedastic.
5.15. Because the samples are independent, $\hat{\beta}_{m,1}$ and $\hat{\beta}_{w,1}$ are independent. Thus
\[
\text{var}(\hat{\beta}_{m,1} - \hat{\beta}_{w,1}) = \text{var}(\hat{\beta}_{m,1}) + \text{var}(\hat{\beta}_{w,1}) \quad \text{Var}(\hat{\beta}_{m,1})
\]
is consistently estimated as $[SE(\hat{\beta}_{m,1})]^2$ and $\text{Var}(\hat{\beta}_{w,1})$ is consistently estimated as
$[SE(\hat{\beta}_{w,1})]^2$, so that $\text{var}(\hat{\beta}_{m,1} - \hat{\beta}_{w,1})$ is consistently estimated by
$[SE(\hat{\beta}_{m,1})]^2 + [SE(\hat{\beta}_{w,1})]^2$, and the result follows by noting the SE is the square root of the estimated variance.