Monte Carlo Results:

Experiments:

*T* = 260

*r*  = 0.5

*q* = 12

Breaks in levels:

*xt* = *μt* + *ut*  where *μt* is the level of the process, which is allowed to change discretely over the sample period and *ut* is a Gaussian *bcd* process.

The *μt* process follows a martingale with increments that are non-zero with probability *p*.

Specifically Δ*μt* = *stδt*  where *st* is Bernoulli(*p*), 

*Break Model 1:* In this experiment *ut* is *I*(0). Note that this is the local-level model (*b* ≠ 0, *c* = 0, and *d* = 1), with non-gaussian increments in the I(*d*) component. In this sense the experiment investigates the accuracy of the Gaussian asymptotic approximation used in the Bayes and MN predictive sets.

*Break Model 2*: In this experiment *ut* follows an AR(1) process with AR coefficient = 0.98. In the asymptotic experiment, *xt* is the sum of an I(1) process (*μ*) and a LTU process (*u*) with *c* = 5.2 ( = 260×(1−0.98)). Note that this model is not nested in the *bcd* framework, so this experiment tests both the accuracy of the Gaussian asymptotic approximation (because *μ* is non-Gaussian) and the ability of the *bcd*-predictive sets control coverage in this non-*bcd* model.

*Calibration:* We carry out experiments with *p* = 1/40 and *p* = 1/260.

For each model and value of *p* we choose two values of *δ*, a “small” and “large” value.

For the first model, these values were chosen to capture level shifts in a time series like the growth rate labor-productivity in the post-WWII U.S. The sample mean of this series is 2.2% and the estimated long-run standard deviation is 3.7%. For this experiment we set *σu* = 3.7 and chose *δ* so that the IQR range of *μT* − *μ*0 was 0.5% (small value of *δ*) or 1.5% (large value of *δ*).

For the second model, these values were chosen to capture level shifts in a time series like nominal interest rates in the post-WWII U.S. The estimated standard deviation of is 0.46 (with long-run standard deviation 0.55). For this experiment we set *var*(*ut* − 0.98*ut*-1) = 0.46 and chose *δ* so that the IQR range of *μT* − *μ*0 was 2.0 (small value of *δ*) or 4.0 (large value of *δ*).

**Values of *δ***

|  |  |
| --- | --- |
| *δ* | IQR |
| a. *p* = 1/40 | |
| 0.0625 | 0.25 |
| 0.1250 | 0.50 |
| 0.1875 | 0.75 |
| 0.375 | 1.5 |
| 0.5 | 2.0 |
| 0.75 | 3.0 |
| 1 | 4.0 |
| 2 | 8.0 |
| c. *p* = 1/260 | |
| 0.125 | 0.25 |
| 0.25 | 0.50 |
| 0.375 | 0.75 |
| 0.75 | 1.5 |
| 1 | 2.0 |
| 1.50 | 3.0 |
| 2.0 | 4.0 |
| 4.0 | 8.0 |

Breaks in Volatility:

We consider two models. The first, allows the volatility of the series to shift discretely, capturing phenomenon like the “Great Moderation”. The second allows the variance and relative variance of components in a local level model to shift. This changes both the variability and persistence in the process and captures phenomenon such as “anchoring” and “unanchoring” of inflation evident in the post-WWII U.S.

Volatility Experiment 1:

Model: *yt* = *σtut* where *ln*(*σt*) follows a martingale Δ*ln*(*σt*) = *stεt* and these follow the same process as above and *ut* is I(0).

*Calibration*: *p* is chosen as 1/40 or 1/260 as above. *δ* is chosen so that the IQR of ln(*σT/σ*0) is 0.25 (small *δ*) or 0.75 (large *δ*). These values can be read from the table above.

Volatility Experiment 2:

Model: *yt* = *u*l*t*  + *σtu*2*t*, where *ln*(*σt*) follows a martingale Δ*ln*(*σt*) = *stεt* and these follow the same process as above and *u*1*t* is I(0) and *u*2*t* is I(1).

Note: With *σt* constant, (1-L)*yt* = (1−*θ*L)*et*, where *θ* depends on the relative variance of the two components and *σe* depends on the values of the variances. Thus, in this experiment, changes in *σt* change both the persistent and volatility of the process.

Calibration: We set var(*u*l*t*) = var(Δ*u*2*t*) = 1 and choose *σ*0 so that the *θ*0 = 0.5, where *θt* denotes the time *t* value of the MA coefficient from the IMA(1,1) representation of the model. We choose *δ* so the IQR of *θT* is 0.5 (small *δ*) or 0.8 (large *δ*).

|  |  |  |  |
| --- | --- | --- | --- |
| *δ* | IQR for *θT* | 25th perc for *θT* | 75th perc for *θT* |
| a. *p* = 1/40 | | | |
| 0.407 | 0.50 | 0.732 | 0.232 |
| 0.806 | 0.80 | 0.869 | 0.069 |
| b. *p* = 1/260 | | | |
| 0.814 | 0.50 | 0.732 | 0.232 |
| 1.612 | 0.80 | 0.869 | 0.069 |