Monte Carlo Results:

Experiments:

$T = 260$
$r = 0.5$
$q = 12$

<u>Breaks in levels:</u>

$x_t = \mu_t + u_t$ where μ_t is the level of the process, which is allowed to change discretely over the sample period and u_t is a Gaussian *bcd* process.

The μ_t process follows a martingale with increments that are non-zero with probability p.

Specifically $\Delta\mu_t = s_t\delta_t$ where s_t is Bernoulli(p), $\delta_t \sim iid \begin{cases} +\delta \text{ with probablity } 0.5 \\ -\delta \text{ with probability } 0.5 \end{cases}$

Break Model 1: In this experiment u_t is $I(0)$. Note that this is the local-level model ($b \neq 0$, $c = 0$, and $d = 1$), with non-gaussian increments in the I(d) component. In this sense the experiment investigates the accuracy of the Gaussian asymptotic approximation used in the Bayes and MN predictive sets.

Break Model 2: In this experiment u_t follows an AR(1) process with AR coefficient = 0.98. In the asymptotic experiment, x_t is the sum of an I(1) process (μ) and a LTU process (u) with $c = 5.2$ ($= 260\times(1-0.98)$). Note that this model is not nested in the *bcd* framework, so this experiment tests both the accuracy of the Gaussian asymptotic approximation (because μ is non-Gaussian) and the ability of the *bcd*-predictive sets control coverage in this non-*bcd* model.

Calibration: We carry out experiments with $p = 1/40$ and $p = 1/260$.

For each model and value of p we choose two values of δ, a "small" and "large" value.

For the first model, these values were chosen to capture level shifts in a time series like the growth rate labor-productivity in the post-WWII U.S. The sample mean of this series is 2.2% and the estimated long-run standard deviation is 3.7%. For this experiment we set $\sigma_u = 3.7$ and chose δ so that the IQR range of $\mu_T - \mu_0$ was 0.5% (small value of δ) or 1.5% (large value of δ).

For the second model, these values were chosen to capture level shifts in a time series like nominal interest rates in the post-WWII U.S. The estimated standard deviation of $(1-0.98\,\mathrm{L})R_t^{10\,YearTreasuryBond}$ is 0.46 (with long-run standard deviation 0.55). For this experiment we set $var(u_t - 0.98u_{t-1}) = 0.46$ and chose δ so that the IQR range of $\mu_T - \mu_0$ was 2.0 (small value of δ) or 4.0 (large value of δ).

Values of δ

δ	IQR
a. $p = 1/40$	
0.0625	0.25
0.1250	0.50
0.1875	0.75
0.375	1.5
0.5	2.0
0.75	3.0
1	4.0
2	8.0
c. $p = 1/260$	
0.125	0.25
0.25	0.50
0.375	0.75
0.75	1.5
1	2.0
1.50	3.0
2.0	4.0
4.0	8.0

Breaks in Volatility:

We consider two models. The first, allows the volatility of the series to shift discretely, capturing phenomenon like the "Great Moderation". The second allows the variance and relative variance of components in a local level model to shift. This changes both the variability and persistence in the process and captures phenomenon such as "anchoring" and "unanchoring" of inflation evident in the post-WWII U.S.

Volatility Experiment 1:

Model: $y_t = \sigma_t u_t$ where $ln(\sigma_t)$ follows a martingale $\Delta ln(\sigma_t) = s_t\varepsilon_t$ and these follow the same process as above and u_t is I(0).

Calibration: p is chosen as 1/40 or 1/260 as above. δ is chosen so that the IQR of $\ln(\sigma_T/\sigma_0)$ is 0.25 (small δ) or 0.75 (large δ). These values can be read from the table above.

Volatility Experiment 2:

Model: $y_t = u_{1t} + \sigma_t u_{2t}$, where $ln(\sigma_t)$ follows a martingale $\Delta ln(\sigma_t) = s_t\varepsilon_t$ and these follow the same process as above and u_{1t} is I(0) and u_{2t} is I(1).

Note: With σ_t constant, $(1\text{-L})y_t = (1-\theta\text{L})e_t$, where θ depends on the relative variance of the two components and σ_e depends on the values of the variances. Thus, in this experiment, changes in σ_t change both the persistent and volatility of the process.

Calibration: We set $\text{var}(u_{1t}) = \text{var}(\Delta u_{2t}) = 1$ and choose σ_0 so that the $\theta_0 = 0.5$, where θ_t denotes the time t value of the MA coefficient from the IMA(1,1) representation of the model. We choose δ so the IQR of θ_T is 0.5 (small δ) or 0.8 (large δ).

δ	IQR for θ_T	25th perc for θ_T	75th perc for θ_T
	a. $p = 1/40$		
0.407	0.50	0.732	0.232
0.806	0.80	0.869	0.069
	b. $p = 1/260$		
0.814	0.50	0.732	0.232
1.612	0.80	0.869	0.069