Studienzentrum Gerzensee Doctoral Program in Economics
Econometrics 2008 Week 4 Take-Home Empirical Exercise

Attached you will find an Excel file PCE.XLS that contains quarterly values of the PCE price deflator for the United States from 1959:1-2007:4. Call these observations P_t. Let $p_t = \ln(P_t)$ denote the logarithm of the price index, $\pi_t = 400(p_t - p_{t-1})$ denote quarterly observations on the rate of price inflation (expressed in percentage points at an annual rate), and $\Delta\pi_t = (1-L)\pi_t$ denote the change in the inflation rate.

1. (a) Estimate an MA(1) for $\Delta\pi_t$, say $\Delta\pi_t = (1-\theta L)a_t$, where a_t is white noise. What is the estimated value of θ and σ_a. Construct a 95% confidence interval for θ.

   (b) Estimate an MA(2) model for $\Delta\pi_t$. Can you reject the null of an MA(1) model versus the alternative that $\Delta\pi_t$ follows an MA(2)? Explain.

In what follows, assume that the MA(1) model is an adequate representation for $\Delta\pi_t$.

2. Suppose that $\pi_t = \tau_t + \varepsilon_t$, where $\tau_t = \tau_{t-1} + e_t$, and where $\{\varepsilon_t\}$ and $\{e_t\}$ are mutually uncorrelated white noise processes with variances σ_ε^2 and σ_e^2.

   (a) Show that $\Delta\pi_t = (1-\theta L)a_t$. Derive an expression for θ and σ_a^2 in terms of σ_ε^2 and σ_e^2.

   (b) Using the estimated MA(1) model in qestion 1, construct estimates of σ_ε^2 and σ_e^2.

3. (a) Use the estimated model in (2) to construct $\tau_{t/t}$ and $P_{t/t}$ (using the Kalman filter), and $\tau_{t/T}$ and $P_{t/T}$ (using the Kalman smoother.) Initialize the Kalman iterations using $\tau_{0/0} = 0$ and $P_{0/0} = 10000$. (Note $P_{0/0} = 10000$ implies that essentially nothing is known about the value of τ_0.)

   (b) Plot the estimates of τ computed in (a) over the period 1965-2007, along with 95% confidence bands ($\pm 2\sqrt{P_{t/t}}$ and $\pm 2\sqrt{P_{t/T}}$). Comment on the evolution of trend inflation in the U.S. over this period.

4. Estimate the spectrum of $\Delta\pi_t$ using an MA(1) parametric spectral estimator. Comment on the shape of the estimated spectra.

5. (a) Provide an expression for the filter weights for an ∞-term band pass filter, $a(L) = \sum_{j=-\infty}^{\infty} a_{|j|}L^j$, for quarterly data for passing periodic components with periods p that satisfy $6 \le p \le 32$. Plot these weights for $-80 \le j \le 80$.

Let $\overline{a}(L) = \sum_{j=-80}^{80} a_{|j|}L^j - \sum_{j=-80}^{80} a_{|j|}$. This is the truncated filter from (a), with the coefficient on lag 0 modified so that $\overline{a}_0 = a_0 - \sum_{j=-80}^{80} a_j$. This modification means that the sum of coefficients in $\overline{a}(L)$ is equal to 0 , that is $\overline{a}(1) = 0$. Let $\pi_t^{BC} = \overline{a}(L)\pi_t$ denote the "business cycle" band pass filtered version of π_t.

(b) Using the representation in question 2, show that the best forecast of π_{T+k}, conditional of $\{\pi_t\}_{t=1}^T$, is given by $\tau_{T/T}$ (the filtered estimate of τ at the end of the sample).

(c) Use the result in (b), and the filter $\overline{a}(L)$, to construct estimates of π_t^{BC} over the 1980:I-2007:IV period. Plot these estimates of π_t^{BC}.

(d) Extra Credit: Let $\hat{\pi}_t^{BC}$ denote the estimates in (c). Compute the standard error of the the estimates (the standard deviation of $(\hat{\pi}_t^{BC} - \pi_t^{BC})$) for $1980{:}1 \le t \le 2007{:}\text{IV}$.

6. (a) Estimate the MA model over the period 1959:II-1984:IV and over the period 1985:I-2007:IV.

(b) Is there evidence that θ has changed? Construct a formal test of the null hypothesis of stability.

(c) Use these split-samples to compute estimates of σ_ε^2 and σ_e^2 for each of the sample periods. Is there evidence that these variances have changed? Explain.